LOGIC

DEDUCTIVE AND INDUCTIVE

PART II

INDUCTIVE

By

CARVETH READ, M.A.

ANNOTATED INDIAN EDITION

WITH A FOREWORD

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PROGRESSIVE PUBLISHERS

37, COLLEGE STREET : CALCUTTA-12

REPRINT FROM :
FOURTH EDITION ENLARGED AND PARTLY REWRITTEN
ANNOTATED IN INDIA BY ARRANGEMENT

Publication Right in India

By
PROGRESSIVE PUBLISHERS
Second Impression July 1966

Price Rs. 6·00 only

Published by S. K. Basu for PROGRESSIVE PUBLISHERS, 37 College Street, Calcutta-12, and Printed by B. K. Sen, at Rup-Lekha Press, 22, Sitaram Ghosh Street, Cal.-9.

PUBLISHER'S NOTE

This edition of Mr. Read's book uses the text of the fourth edition. It has given marginal notes, supplementary notes, and additional notes; but has not altered the text. It has also added a summary to each chapter along with exercises with hints. In preparing supplementary and additional notes, our only motive has been to elucidate some points of Mr. Read, and for that purpose we have liberally used J. S. Mill's Logic and some modern text books like Dr. Stebbing's. Even a cursory glance at the annotations will show that they will prove useful to the students. Though we have published the book in two volumes we have not changed the numbering and ordering of the chapters. The supplementary notes and appendices in this volume are on the following among other topics : the Syllogism, the processes simulating Induction, kind of hypothesis, Newton's view on hypothesis, Evidence of the Formal ground of Induction, probability, Inverse character of Induction, Classification by series, *non causa-pro causa*, Division by Dichotomy and Definition *per genus et differentiam*.

We are indebted to Prof. Kalikrishna Bannerjee, Lecturer in Philosophy, P. G. Department, Calcutta University, (at present head of the Department of Philosophy, Jadavpur University) who has done the entire work of editing this volume.

We do not claim that the additional notes and elucidatories are faultless. We welcome suggestion from teachers of Logic toward a better edition in future.

PUBLISHER

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INDUCTIVE LOGIC AT A GLANCE

1. INDUCTION IS NECESSARY AS A DISTINCT PROCESS OF REASONING ON THE FOLLOWING GROUNDS :

   (i) *Material truth* in addition to formal consistency is required by us.

   (ii) *Real general* propositions assert more than what have been directly observed. They pass therefore *from the directly observed to the unobserved.*

   (iii) Materially, a syllogism turns upon the resemblance of the Minor to the middle term ; and thus extends the Major premise to new cases.

2. THE DICTUM FOR MATERIAL REASONING HAS BEEN STATED IN DIFFERENT FORMS :

(*a*) whatever can be identified as a specimen of a known substance or kind has the properties of kinds (in the language of kinds). (*b*) whatever relation of events can be identified with the relation of cause and effect is constant. (In the language of causation). (*c*) whatever is constantly related to a phenomenon (cause or kind), determined by certain characters, is related in the same way to any phenomenon, that has the same characters. (In a generalised form)

3. THE USES OF THE SYLLOGISM :

(1) By stating an agreement in the form of a syllogism we can examine the three conditions on which the validity of the argument depends.

(*a*) Are the premises so connected that, if they are true, the conclusion follows ? This depends upon the formal principles of chap. X.

(*b*) Is the minor premise true ?

(*c*) Is the major premise true ?

(2) The syllogistic form makes us fully aware of what an

inference implies. It increases our sense of responsibility for the inference that is seen to imply so much.

(3) In the interest of others an accurate generalisation is a part of the systematic procedure of science. The accuracy of reasoning is capable of being exhibited syllogistically.

4. The uniformity of nature as the formal ground of all reasoning.

The indeterminate notion of uniformity is intelligible as a number of first principles.

(i) The Principles of Contradiction and Excluded Middle.

(ii) Certain Axioms of Mediate Evidence: as, in Mathematics, 'that magnitudes equal to the same magnitude are equal to one another'; and, in Logic, the *Dictum* or its material equivalent.

(iii) That all Times and all Spaces are commensurable.

(iv) Matter and Energy persists in the sense that their quantities remain the same.

(v) Causation is a special mode of the uniformity of nature.

(vi) Certain uniformities of co-existence—(*a*) The Geometrical, (*b*) Relating to Universal coinherences among the properties of concrete things, (*c*) Co-existence due to causation, (*d*) The coinherence of properties in Natural kinds, (*e*) Certain unaccountable cases of the co-existence of properties such as the absence of fragrance from scarlet flowers.

5. The grounds of our belief in the Axioms of Uniformity:

(1) It is the presupposition of every inference.

(2) First taken as dogmas, they are regarded later on as postulates of scientific generalisation and of the architectonic unification of science.

(3) In some measure they are verifiable in practical life and in scientific demonstration.

(4) A natural desire to find in nature a foundation for confidence in our own power.

6. CAUSATION IS THE MOST IMPORTANT ASPECT OF UNIFORMITY IN RELATION TO INDUCTION. WHY ?

- (i) The surest of all marks is a cause.
- (ii) The uniformity of Space, and of Time is involved in causation so far as we conceive causation as essentially matter in motion.
- (iii) The persistence of Matter and Energy is nothing else than causation in the general movement of the world.

7. MILL DEFINES A CAUSE AS THE INVARIABLE UNCONDITIONAL ANTICIDENT OF A GIVEN PHENOMENON.

The Marks of Causation.

(1) A cause is relative to a given phenomenon called the Effect.

(2) The given phenomenon is not a new thing but an event that is a change in something, or in the relative position of things.

(3) The Cause is always an antecedent to the Effect.

(4) The Cause is the invariable antecedent of the Effect. Whenever a given cause occurs it always has the same effect.

(5) The cause is the unconditional antecedent of the Effect in the sense that it is the group of conditions which, without any further condition, is followed by the event in question.

(6) That the cause of an event is an immediate antecedent follows from its being an unconditional one.

(7) Cause and Effect are quantitatively equal to each other. To sum up, the cause of an event has five marks : It is *quantitatively equal* to the Effect, and *qualitatively* the *immediate, unconditional,* and *invariable antecedent* of the Effect. The doctrine of the Plurality of causes means the theory that the same event may be due at different times to different antecedents, and that in fact there may be *vicarious* causes.

An alleged plurality may be removed in one of the following ways :

(i) By considering the effect as a whole we may establish that in different cases we have really different effects.

(ii) By discovering an identical cause shared by the different antecedents.

(iii) By pointing out that the effects alleged to be the same differ in minute details.

(iv) By considering the apparent plurality of causes as the sign of an imperfect scientific investigation.

The advantage of the idea that there may be other causes of effects is that it makes us careful and vigilant in respect of details.

The homogeneous intermixture of Effects is said to operate where the different forms of energy entering into Cause and Effect are severally reducible to units between which equivalents have been established, in short where the antecedents and consequents are fundamentally of the same kind. But where the compound presents very different phenomena from those of its elements, we have the heteropathic intermixture of effects.

Some important concepts :

(1) *Tendency* :—When a cause consists of two or more conditions or forces, we may consider what effect any one of them would have if it operated alone, that is to say, its tendency.

(2) *Resultant* :—If the tendencies are allowed to operate, we have what is known as their Resultant.

(3) *Counteraction* :—When the tendency of a force is combined with another force (not in the same direction) in any resultant, its tendency is said to be counteracted.

(4) *Elimination* :—Elimination is illustrated when the tendency of a certain cause is freed from the counteraction of the counteracting circumstances.

(5) *Resolution* :—This means the analysis of the total effect into its component conditions.

(6) *Reciprocity* :—When several conditions constituting any cause jointly determine the total effect, they may be said to stand in Reciprocity or Mutual influence.

DEFINITION OF INDUCTION : By an Induction we mean the inferential passage to a universal real proposition based on observation in reliance on the uniformity of nature. A proposition is inductively proved when it is proved by facts, not merely deduced from more general premises. By the process of Deduction we mean the method of inductive proof.

Perfect Induction means an induction when all the instances within the scope of the conclusion have been severally examined, and the conclusion has been found true in each case. Imperfect Induction is the method of showing the credibility of a universal real proposition by an examination of some of the instance ; it includes, generally a small fraction of them.

Imperfect Induction may be either methodical or immethodical.

Observations and experiments are the meterial ground of induction.

DEFINITION OF EXPERIMENT : An experiment is an observation made under prepared, and therefore known conditions ; and, when obtainable, it is much to be preferred.

SOME OF THE ADVANTAGES OF EXPERIMENT OVER UNAIDED OBSERVATION :

(i) The repetition of the phenomena to be observed as many times as we like is possible in the case of experiments only.

(ii) Experiments help us to see a phenomenon as removed from the influence of all agents except that whose operation we desire to observe.

(iii) Experiment makes a cool observation possible. But where experiments are impossible observation will be indispensable. Experiments become possible only when some knowledge has already been gained by observation.

The principle of causation is the formal ground of Induction in the sense that the Inductive canons derived from it are means of testing the formal sufficiency of observations to justify the statemeat of a law.

To determine the cause of an effect we cannot however rely on the bare principle of causation. Causation can be determined in two ways :

A. Qualitative Determination

I. (a) That which (without further change) is followed by a given event is its cause.

(b) That which cannot be left out without impairing a phenomenon is a condition of it.

II. (a) That which is not so followed is not the cause.

(b) That which can be left out is not a condition of it.

B. Quantitative Determination

(a) When a cause (or effect) increases or decreases, so does its cffect (or cause).

(b) If two phenomena, having the other marks of cause and effect, seem unequal, the less contains an unexplored factor.

(c) If an antecedent and consequent do not increase or decrease correspondingly, they are not cause and effect, so far as they vary.

Mill's Five Canons of Induction :—Agreement, the Joint Method, Difference, Concomitant Variations and Residues combine variously the propositions stated above. Roughly speaking, the first three are Qualitative Methods, and the last two Quantitative.

(1) The Canon of Agreement :

If two or more instances of a phenomenon under investigation have only one other circumstance (antecedent or consequent) in common, that circumstance is probably the cause (or an indispensable condition) or the effect of the phenomenon, or is connected with it by causation.

Its chief use is to suggest hypotheses as to the cause ; which

must then be used (if possible) experimentally to try if it produces the given effect.

Its chief disadvantages: (a) So far as we recognise the possibility of a plurality of causes, this method of Agreement cannot be quite satisfactory. (b) An accident found preceding an event in the given instances is sometimes liable to pass for a cause of the event.

(2) The Canon of the Joint Method of Agreement in Presence and in Absence.

If (1) two or more instances in which a phenomenon occurs have only one other circumstance (antecedent or consequent) in common, while (2) two or more instances in which it does not occur (though in important points they resemble the former set of instances) have nothing else in common save the absence of that circumstance—the circumstance in which alone the two sets of instances differ throughout (being present in the first set and absent in the second) is probably the effect, or the cause, or an indispensable condition of the phenomenon.

Instances of Presence.	Instances of Absence.
A B C	C H F
p q r	*r x v*
A D E	B D K
p s t	*q y s*
A F G	E G M
p u v	*t f u*

Then A is probably the cause or a condition of *p*, or *p* is dependent upon A: first, by the Canon of Agreement in Presence, as represented by the first set of instances; and secondly, by Agreement in Absence in the second set of instances.

(3) The Canon of Difference.

If an instance in which a phenomenon occurs, and an instance in which it does not occur, have every other circumstance in common save one, that one (whether consequent or antecedent) occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable condition of the phenomenon.

To prove that A is a condition of p, let two instances, such as the Canon requires, be represented thus :

A B C	B C
p q r	q r

Then A is the cause or a condition of p.

(4) THE CANON OF CONCOMITANT VARIATIONS.

Whatever phenomenon varies in any manner whenever another phenomenon (consequent or antecedent) varies in some particular manner [no other change having concurred] is either the cause or effect of that phenomenon [or is connected with it through some fact of causation].

We may illustrate the two cases of the method thus (putting a dash against any letter, A′ or p', to signify an increase or decrease of the phenomenon the letter stands for) : Agreement in Variations (other changes being admissible)—

A B C	A′ D E	A″ F G
p q r	p' s t	p'' u v

Difference in Variations may be symbolically represented thus (no other change having concurred) :

A B	A′ B	A″ B
p q,	p' q,	p'' q

(5 THE CANON OF RESIDUES.

Subduct from any phenomenon such part as previous inductions have shown to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

Antecedents	Consequents
A B C	p q r
B	q
C	r

∴ A is the cause of p.

THE DIRECT DEDUCTIVE METHOD OR THE PHYSICAL METHOD : In cases, when direct observation or experiment is insufficient to resolve an effect into the laws of its conditions, the general method is to calculate what may be expected from a combination of its conditions, as either known or hypothetically assumed, and

to compare this anticipation with the actual Phenomenon. Given any complex mechanical phenomenon, the inquirer considers—(1) what laws already ascertained seem likely to apply to it) in default of known laws, hypotheses are substituted: *cf.* (chap. xviii); he then—(2) computes the effect that will follow from these laws in circumstances similar to the case before him ; and (3) he verifies his conclusion by comparing it with the actual phenomenon.

THE INVERSE OR HISTORICAL METHOD : When the forces determining a phenomenon are too numerous, or too indefinite, to be combined in a direct deduction, we may begin by collecting an empirical law of the phenomenon (as that 'the democracies of City States are arbitrary and fickle'), and then endeavour to show by deductions from "the nature of the case," that is, from a consideration of the circumstances and forces known to be operative (of which, in the above instance, the most important is sympathetic contagion), that such a law was to be expected.

DIFFERENCE BETWEEN THE TWO :

Deduction is thus called into verify a previous induction ; whereas in the 'Physical Method' a deduction was verified by comparing it with an induction or an experiment.

The essential difference between the physical and Historical Methods is that, in the former, whether Direct or Inverse, the deductive process, when complete, amounts to exact demonstration ; whereas, in the latter, the deductions may consist of qualitative reasonings, and the results are indefinite.

POINTS OF SIMILARITY :

They establish—(1) a merely probable connection between the phenomena according to an empirical law (say), between City-democracy and fickle politics) ; (2) connect this with other historical or social generalisations, by showing that they all alike flow from the same causes, namely, from the nature of races of men under certain social and geographical conditions ; and (3) explain why such empirical laws may fail, according to the

differences that prevail among races of men and among the conditions under which they live.

HYPOTHESIS : It is "any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real ; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be, or at least is likely to be, true."

The deduction of known truths from an hypothesis is its Verification ; and when this has been accomplished in a good many cases, and there are no manifest failures, the hypothesis is often called a Theory.

An Hypothesis may be made concerning (1) an Agent, such as the ether; or (2) a Collocation, such as the plan of our solar system—whether geocentric or heliocentric ; or (3) a Law of an agent's operation, as that light is transmitted by a wave-motion of such lengths or of such rates of vibration.

PROOF OF HYPOTHESES :

(1) *If a new agent be proposed, it is desirable that we should be able directly to observe it, or at least to obtain some evidence of its existence of a different kind from the very facts which it has been invented to explain.*

(2) *Whether the hypothetical agent be perceptible or not, it cannot be established as a cause, nor can a supposed law of such an agent be accepted as sufficient to the given inquiry, unless it is adequate to account for the effects which it is called upon to explain, at least so far as it pretends to explain them.*

(3) What is further demanded in the proof of an hypothesis, then, is *not merely that it be shown to agree with the facts, but that every other hypothesis be excluded.*

(4) *An hypothesis must agree with the rest of the laws of Nature ; and, if not itself of the highest generality, must be derivable from primary laws* (chap. xix. § 1).

LAWS CLASSIFIED.

A. Axioms or Principles, that are real, universal and self-evident propositions.

B. Primary Laws of Nature :

C. Secondary Laws : Derivative, having been analysed into, and deduced from, Primary Laws ; or (2) Empirical, those that have not yet been deduced, (though from their comparatively special and complex character, it seems probable they may be, given sufficient time and ingenuity), and that meanwhile rest upon some unsatifactory sort of induction by Agreement or Simple Enumeration.

Secondary Laws, again, are either of succession or of co-existence.

Those of succession are either—(1) of direct causation as 'water quenches fire, or (2) of the effect of a remote cause, as 'Bad harvests tend to raise the price of bread' or (3) of the joint effects of the same cause, as 'Night follows day'.

LAWS OF CO-EXISTENCE ARE OF SEVERAL CLASSES :

(1) One has the generality of a primary law, though it is proved only by Agreement, namely, 'All gravitating bodies are inert.

(2) Next come laws of the Co-existence of those properties which are comprised in the definitions of Natural Kinds.

(3) There are again laws of certain coincidences of qualities not essential to any kind, and sometimes prevailing amongst many different kinds. For example, "white tom-cats with blue eyes are deaf."

(4) Finally, there may be constancy of relative position, as of sides and angles in Geometry.

SCIENTIFIC EXPLNATION : it consists in discovering, deducing, and assimilating the laws of phenomena.

THERE ARE THREE MODES OF SCIENTIFIC EXPANATION : First, the analysis of a phenomenon into the laws of its causes and the concurrence of those causes. Secondly, the discovery of

steps of causation between a cause and its remote effects; the interpolation and concatenation of causes.

Thirdly, the subsumption of several laws is under one more general expression.

LIMITS OF EXPLANATION:

(1) Fundamental states or processes which are distinct and do not resemble one another, and therefore cannot be generalised or subsumed under one explanation.

(2) The first cause of the world, or the uncaused cannot be explained because this involves a neverending process. Further explanation fails where less general laws cannot be subsumed under a more general law.

(3) No *particular* fact can be explained since the causes determining each particular phenomenon are infinite and can never be *computed*.

ANALOGY:

It is used in two senses:—(1) For the resemblance of relations between terms that have little or no resemblance. (2) As a kind of probable proof based upon imperfect similarity between the data of comparison and the subject of our inferencc.

The cogency of an analogical argument depends on the following factors:—(1) The greater the number and importance of the points of agreement, the more probable is the inference. (2) The greater the number and importance of the points of difference, the less probable is the inference. (3) The greater the number of unknown properties in the subject of our argument, the less the value of any inference from those that we do know.

COLLIGATION OF FACTS:

We observe some facts and in a colligation bring them under a concept. The question whether a particular colligation of facts is iductive in character is to be determined by observing whether there is a leap involved in the same. If there is a bare understanding of facts, colligation is a mere summary of facts observed together with an explanatory notion. But where

there is a distinct leap involved, (as in the case of Kepler's theory of planetary motion), colligation accompanies Induction. The functions of the two must, however, be distinguished from each other.

FALLACIES AT A GLANCE

FALLACY : It is any failure to fulfil the conditions of proof (unintentional—> Paralogism. Intentional—> Sophism.)

Formal Fallacies are those fallacies that can be shown to conflict with one or more of the truths of Logic, whether Deductive or Inductive.

Material Fallacies are those that cannot be clearly exhibited as transgressions of any logical principle, but are due to superficial inquiry or confused reasoning.

MAJOR FORMAL FALLACIES OF DEDUCTION.

1. TO MISTAKE THE CONTRARY FOR THE CONTRADICTORY :

(a) A is not greater than B, .·. A is smaller than B.

(b) He is either a fool or a knave. He is a fool, .·. He is not a knave (Disjunctive Syllogism).

2. SIMPLE CONVERSION OF A :

All men are animals .·. All animals are men.

3. CONVERSION OF O :

Some animals are not men .·. Some men are not animals.

4. THE FALLACY OF FOUR TERMS :

Jones is a friend of Stephen.
Henry is a friend of Jones
.·. Henry is a friend of Stephen.

5. TO DISTRIBUTE IN THE CONCLUSION A TERM THAT WAS UNDISTRIBUTED IN THE PREMISES I.E. ILLICIT PROCESS OF (*a*) THE MAJOR OR (*b*) MINOR TERMS.

(a) All birds are mortal.
No men are birds.
.·. No men are mortal.

(b) All Indians are men.
All men are animals.
∴ *All animals are Indians.*

6. THE FALLACY OF UNDISTRIBUTED MIDDLE :

To omit to distribute the Middle term in one or the other premise, as

All verbal propositions are self-evident ;
All axioms are self-evident.
∴ All axioms are verbal propositions.

7. TO SIMPLY CONVERT AN HYPOTHETICAL PROPOSITION, AS

If trade is free, it prospers ;
∴ If trade prospers, it is free.

8. FALLACY OF AFFIRMING THE CONSEQUENT :

If there is a rain, the ground is wet.
The ground is wet.
∴ There is a rain.

9. FALLACY OF DENYING THE ANTECEDENT :

If there is a rain, the ground is wet.
There has been no rain.
∴ The ground is not wet.

10. CONCERNING DILEMMA :

See supplementary notes on chap. xii—part 1.

11. FALLACY OF EQUIVOCATION :

See page xxi part I.

FORMAL FALLACIES OF INDUCTION :

1. To assign the *cause* of anything that is not a concrete event (when you should give the *reason* and not the cause) :—as, *e.g.*, why two circles can touch only in one point.

2. To argue, as if on inductive grounds, concerning the cause of the Universe as a whole. (Fallacy of transcendent inference).

3. To mistake co-existent phenomena for cause and effect—as when the wearing of an amulet is regarded as the cause of escape.

4. *Post hoc, ergo propter hoc.* (After this, therefore, on account of this) : to accept the mere sequence of phenomena, even though often repeated, as proving that the phenomena are cause and effect, or connected by causation.

5. To regard the Co-effets (whether simultaneous or successive) of a common cause as standing in the direct relation of cause and effect : *e.g.* day is the cause of night.

6. To mistake one condition of a phenomenon for the whole cause.

7 To mistake a single consequence of a given cause for the whole effect, is a corresponding error.

8 To demand greater exactness in the estimate of causes or effects than a given subject admits of.

9. To treat an agent or condition remote in time as an unconditional cause : for every moment of time gives an opportunity for new combinations of forces and, therefore, for modifications of the effect.

10. To neglect the negative conditions to which a cause is subject.

11. The neglect of a possible plurality of causes where the effect has been vaguely conceived.

12. The extension of empirical laws beyond adjacent cases.

13. Errors regarding the estimate of analogies and probabilities.

14. Wrong application of the principles of classification, Definition and Division.

15. To rely upon direct Induction where the aid of Deduction may be obtained, or upon observation where experiment may be employed.

Fallacies of observation :

(1) Fallacy of non-observation, where an important point is overlooked.

(2) Fallacy of Malobservation where the observation of such a point is defective.

Petitio principii or begging the question :

(1) Fallacies a priori :—These are committed when mere assertions, pretending to be self-evident, and often sincerely accepted as such by the another and some infatuated disciples, but in which the cool spectator sees either no sense at all, or palpable falsity.

(2) Verbal propositions offered as proof of some matter of fact.

(3) *Circulus in demonstrando* :—the pretence of giving a reason for an assertion, whilst in fact only repeating the assertion itself —generally in other words.

Ignoratio Elenchi

By this we mean the fallacy of mistaking or obscuring the proposition really at issue, whilst proving something else instead.

Argumentum ad hominem : This consists in showing not that a certain proposition is true, but that a person ought to accept it in consistency with his other opinions.

Argumentum ad populum : This is the fallacy of trying to convince people by means of an appeal to their passions without resorting to any reasoning.

Argumentum ad verecundiam : When the favourable opinion of authority is considered sufficient for meeting the need of a clear reasoning.

Argumentum ad scholam :—This is committed when an opponent exhibits the differences of opinion (so far as certain thinkers or theories are concerned) and thereupon pretends to have refuted the theory they agree in supporting.

Fallacy of objections :—To lay stress upon all the considerations against any doctrine or proposal, without any attempt to weigh them against the considerations in its favour.

Argumentum ad Ignorantium : To attempt to refute a statement on the ground that the person who has made the statement, has betrayed his ignorance in respect of details.

Shifting the ground :

(When the original subject matter to be proved is left out and replaced by a different one).

Argumentnm at Baculum : (When coercion is considered as the means of persuasion).

Fallacy of many questions :—

The fallacy consists in putting a number of questions in such a form that a single answer will involve more than what one intends to admit. "He was a fool and a knave—was it or was it not ?" Here if he was a fool but not a knave or if he was a knave but not a fool, Neither 'yes' nor 'no' will suffice. Similar is the case of the inquiry, "Have you left off beating your mother ?"

Fallacies of ellipsis :—

(1) *A dicto secundum quid at dictum simpliciter* :—If in a premise a word is used *secundum quid*, i.e., with the suppressed qualification of including, say, past as well as present labour but in the conclusion used simpliciter i.e. without that qualification so that it means, say, present labour only,—this fallacy is committed.

(2) *A dicto secundum quid ad dictum secundum alterum quid*—is the reverse of (1)

(3) Faltacy of accident. (see Part I p xxi)

CHAPTER XIII

Transition to Induction

§ 1. Formal Consistency and Material Truth : Having now discussed Terms, Propositions, Immediate and Mediate Inferences, and investigated the conditions of formal truth or consistency, we have next to consider the conditions of material truth : whether (or how far) it is possible to arirve at propositions that accurately represent the course of nature or of human life. Hitherto we have dealt with no sort of proof that gives any such assurance. A valid syllogism guarantees the truth of its conclusion, provided the premises be true : but what of the premises ? The relation between the premises of a valid syllogism and its conclusion is the same as the relation between the antecedent and consequent of a hypothetical proposition. If A is B, C is D : grant that A is B, and it follows that C is D ; and, similarly, grant the premises of a syllogism, and the conclusion follows. Again, grant that C is not D, and it follows that A is not B ; and, similarly, if the conclusion of a valid syllogism be false, it follows that one, or other or both of the premises must be false. But, once more, grant that C is D, and it does not follow that A is B ; so neither, if the conclusion of a syllogism be true, does it follow that the premises are. For example :—

The problem of Induction : In syllogism, the premises being true, the conclusion becomes true; but how to know the truth of the premises themselves ?

Sociology is an exact science ;
Mathematics is a branch of Sociology :
∴ Mathematics is an exact science.

Illustration

Here the conclusion is true although the premises are absurd. Or again :—

Mathematics is an exact science ;
Sociology is a branch of Mathematics ;
∴ Socioloy is an exact science.

Here the major premise is true, but the minor is false, and the conclusion is false. In both cases, however, whether the conclusion be true or false, it equally follows from the premises, if there is any cogency in Barbara. The explanation of this is, that Barbara has only formal cogency ; and that whether the conclusion of that, or any other valid mood, shall be true according to fact and experience, depends upon how the form is filled up. How to establish the premises, then, is a most important problem ; and it still remains to be solved.

§ 2. Real General Propositions assert more than has been directly observed : We may begin by recalling the distinction between the denotation and connotation of a general term : the denotation comprising the things or events which the term is a name for ; the connotation comprising the common qualities on account of which these things are called by the same name. Obviously, there are very few general terms whose denotation is exhaustively known ; since the denotation of a general term comprises all the things that have its connotation, or that ever have had, or that ever will have it, whether they exist here, or in Australia, or in the Moon, or in the utmost stars. No one has examined all men, all mammoths, all crystals, all falling bodies, all cases of fever, all revolutions, all stars—nor even all planets, since from time to time new ones are discerned. We have names for animals that existed long before there were men to observe them, and of which we know only a few bones, the remains of multitudinous species ; and for others that may continue to exist when men have disappeared from the earth.

Why so ?

For, denotation of their subjects is comprised of idefinite number of cases.

If, indeed, we definitely limit the time, or place, or quantity of matter to be explored, we may sometimes learn, within

the given limits, all that there is to know : as all the bones of a particular animal, or the list of English monarchs hitherto, or the name of all the members of the House of Commons at the present time. Such cases, however, do not invalidate the above logical truth that few general terms are exhaustively known in their denotation; for the very fact of assigning limits of time and place impairs the generality of a term. The bones of a certain animal may be all examined, but not the bones of all animals, nor even of one species. The English monarchs that have reigned hitherto may be known, but there may be many still to reign.

If the number of cases not indefinite the proposition not a real general one.

The general terms, then, with which Logic is chiefly concerned, the names of Causes and Kinds, such as gravitation, diseases, social events, minerals, plants and animals, stand for some facts that are, or have been, known, and for a great many other similar ones that have not been, and never will be, known. The use of a general term depends not upon our direct knowledge of everything comprised in its denotation, but upon our readiness to apply it to anything that has its connotation, whether we have seen the thing or not, and even though we never can perceive it ; as when a man talks freely of the ichthyosaurus, or of the central heat of planets, or of atoms and ether.

An hence belief in such propositions depends upon something besides direct observation.

Hence Universal Propositions, which consist of general terms, deceive us, if we suppose that their predicates are directly known to be related to all the facts denoted by their subjects. In exceptional cases, in which the denotation of a subject is intentionally limited, such exhaustive direct knowledge may be possible ; as that "all the bones of a certain animal consist of phosphate of lime," or that every member of the present Parliament wears a silk hat. But what predication is possible concerning the hats of all members of Parliament from the beginning ? Ordinarily, then, whilst the relation of predicate to subject has

been observed in some cases, in much the greater number of cases our belief about it depends upon something besides ovservation, or may be said (in a certain sense) to be taken on trust.

Thus the problem of induction: What right have we to assert Universal Propositions?

'All rabbits are harbivorous': why do we believe that? We may have seen a few wild rabbits feeding: or have kept tame ones, and tried experiments with their diet; or have read of their habits in a book of Natural History; or have studied the anatomy and physiology of the digestive system of many sorts of animals: but with whatever care we add testimony and scientific method to our own obervation, it still remains true that the rabbits observed by ourselves and others are few in comparison with those that live, have lived and will live. Similarly of any other universal propositions; that it 'goes beyond the evidence' of direct observation plainly follows from the fact that the general terms, of which such propositions consist, are never exhaustively known in their denotation. What right have we then to state Universal Propositions? That is the problem of Inductive Logic.

Syllogism cannot prove a universal proposition, for this would lead to infinite regress.

§ 3. HENCE, FORMALLY, A SYLLOGISM'S PREMISES SEEM TO BEG THE CONCLUSION: Universal Propositions, of course, cannot always be proved by syllogisms; because to prove a universal proposition by a syllogism, its premises must be universal propositions; and, then, these must be proved by others. This process may sometimes go a little way, thus: *All men are mortal*, because *All animals are*; and *All animals are mortal*, because *All composite bodies are subject to dissolution.* Were there no limit to such sorites, proof would always involve a *regressus ad infinitum*, for which life is too short; but, in fact, prosyllogisms soon fail us.

Again the posi-

Clearly, the form of the Syllogism must itself be misleading if the universal proposition is so: if we think that premises prove the conclusion because they them-

selves have been established by detailed observation, we are mistaken. The consideration of any example will show this. Suppose any one to argue :

tion that a given universal proposition is a conclusion of a syllogism whose universal major itself is established by detailed observation leads immediately to a dilemma.

All ruminants are herbivorous ;
Camels are ruminants :
∴ Camels are herbivorous.

Have we, then, examined all ruminants ? If so, we must have examined all camels, and cannot need a syllogism to prove their herbivorous nature : instead of the major premise proving the conclusion, the proof of the conclusion must then be part of the proof of the major premise. But if we have not exmained all ruminants, having omitted most giraffes, most deer, most oxen, etc., how do we know that the unexamined (say, some camels) are not exceptional ? Camels are vicious enough to be carnivorous ; and indeed it is said that bactrian camels will eat flesh rather than starve, though of course their habit is herbivorous.

Or, again, it is sometimes urged that—

All empires decay :
∴ Britain will decay.

This is manifestly a prediction : at present Britain flourishes, and shows no signs of decay. Yet a knowledge of its decay seems necessary, to justify any one in asserting the given premise. If it is a question whether Britain will decay, to attempt (while several empires still flourish) to settle the matter by asserting that *all* empires decay, seems to be 'a begging of the question.' But although this latter case is a manifest prediction, it does not really differ from the former one ; for the proof that camels are herbivorous has no limits in time. If valid, it shows not only that they are, but also that they will be, herbivorous.

The alleged dilemma.

Hence, to resort to a dilemma, it may be urged : If *all* the facts of the major premise of any syllogism have been examined,

the syllogism is needless; and if *some* of them have not been examined, it is a *petitio principii.* But either all have been examined, or some have not. Therefore, the syllogism is either useless or fallacious.

§ 4. MATERIALLY, A SYLLOGISM TURNS UPON THE RESEMBLANCE OF THE MINOR TO THE MIDDLE TERM; AND THUS EXTENDS THE MAJOR PREMISE TO NEW CASES: A way of escape from this dilemma is provided by distinguishing between the formal and material aspects of the syllogism considerd as a means of proof. It begs the question formally, but not materially; that is to say, if it be a question whether camels are herbivorous, and to decide it we are told that '*all* ruminants are,' laying stress upon the 'all,' as if all had been examined, though in fact camels have not been, then the question as to camels is begged. The form of a universal proposition is then offered as evidence, when in fact the evidence has not been universally ascertained. But if in urging that 'all ruminants are herbivorous' no more is meant than that so many other ruminants of different species are known to be herbivorous, and that the ruminant stomach is so well adapted to a coarse vegetable diet, that the same habit may be expected in other ruminants, such as camels, the argument then rests upon material evidence without unfairly implying the case in question. Now the nature of the material evidence is plainly this, that the resemblance of camels to deer, oxen, *etc.*, in chewing the cud, justifies us in believing that they have a further resemblance in feeding on herbs; in other words, we assume that *resemblance is a ground of inference.*

This difficulty is overcome through a distinction between formal and material aspects of Syllogism.

An Example

Another way of putting this difficulty which we have just been discussing, with regard to syllogistic evidence, is to urge that by the Laws of Syllogism a conclusion must never go beyond the premises, and that therefore no progress in knowledge can ever

The other criticism of Syllogism: an

be established, except by direct observation. Now, taking the syllogism formally, this is true: if the conclusion go beyond the premises, there must be either four terms, or illicit process of the major or minor term. But taking it materially, the conclusion may cover facts which were not in view when the major premise was laid down: facts of which we predicate something not as the result of direct observation, but because they resemble in a certain way those facts which had been shown to carry the predicate when the major premise was formed.

alleged paradox.

Paradox applies only formally, not materially.

'What sort of resemblance is a sufficient ground of interference?' is, therefore, the important question alike in material Deduction and in Induction; and in endeavouring to answer it we shall find that the surest ground of inference is resemblance of causation. For example, it is due to causation that ruminants are herbivorous. Their instincts make them crop the herb, and their stomachs enable them easily to digest it; and in these characters camels are like the other ruminants.

For, the general truth is applied to new particular through resemblance of causation.

§ 5. RESTATEMENT OF THE DICTUM FOR MATERIAL REASONING: In ch. ix, § 3, the *Dictum de omni et nullo* was stated: 'Whatever may be predicated of a term distributed may be predicated of anything that can be identified with that term.' Nothing was there said (as nothing was needed) of the relations that might be implied in the predication. But now that it comes to the ultimate validity of predication, we must be clear as to what these relations are; and it will also be convenient to speak no longer of terms, as in Formal Logic, but of the things denoted. What relations, then, can be determined between concrete facts or phenomena (physical or mental) with the greatest certainty of general truth; and what axioms are there that sanction mediate inferences concerning those relations?

A material translation of Aristotle's dictum.

Mill's corresponding axiom rests (i) on co-existence and again (ii) his ground of induction remains Causation. And here is an inconsistency.

In his *Logic* (B. II. c. 2, § 3) Mill gives as the axiom of syllogistic reasoning, instead of the *Dictum*: "A thing which co-exists with another thing, which other co-exists with a third thing, also co-exists with that third thing." Thus the peculiar properties of Socrates co-exist with the attributes of man, which co-exist with mortality: therefore, Socrates is mortal. But, again, he says that the ground of the syllogism is Induction; that man is mortal is an induction. And, further, the ground of Induction is causation; the law of causation is the ultimate major premise of every sound induction. Now causation is the principle of the succession of phenomena: how, then, can the syllogism rest on an axiom concerning co-existence? On reflection, too, it must appear that 'Man is mortal' predicates causation: the human constitution issues in death.

Explanation of this inconsistency.

The explanation of this inconsistency may perhaps be found in the history of Mill's work. Books I. and II. were written in 1831; but being unable at that time to explain Induction, he did not write Book III. until 1837-8. Then, no doubt, he revised the earliar Books, but not enough to bring his theory of the syllogism into complete agreement with the theory of Induction; so that the axiom of co-existence was allowed to stand.

Mill's diverse application of the axiom of co-existence.

Mill also introduced the doctrine of Natural Kinds as a ground of Induction supplementary, at least provisionally, to causation; and to reasoning about kinds, or Substance and Attribute, his axiom of co-existence is really adapted. Kinds are groups of things that agree amongst themselves and differ from all others in a multitude of qualities: these qualities co-exist, or co-inhere, with a high degree of constancy; so that where some are found others may be inferred. Their co-inherence is not to be considered an ultimate fact; for,

"since everything which occurs is determined by laws of causation and collocations of the original causes, it follows that the co-existences observable amongst effects cannot themselves be the subject of any similar set of laws distinct from laws of causation" (B.III.c.5, §9). According to the theory of evolution (worked out since Mill wrote), Kinds—that is, species of plants, animals and minerals—with their qualities are all due to causation. Still, as we can rarely, or never, trace the causes with any fullness or precision, a great deal of our reasoning, as, *e.g.*, about men and camels, does in fact trust to the relative permanence of natural Kinds as defined by co-inhering attributes.

Causation and Natural kinds.

To see this more clearly, we should consider that causation and natural Kinds are not at present separable; propositions about causation in concrete phenomena (as distinct from abstract 'forces') always involve the assumption of Kinds. For example—'Water rusts iron,' or the oxygen of water combines with iron immersed in it to form rust: this statement of causation assumes that water, oxygen, iron, and iron-rust are known Kinds. On the other hand, the constitution of every concrete thing, and manifestly of every organised body, is always undergoing change, that is, causation, upon which fact its properties depend.

Different Formulations of the principle of mediate reasoning about things (a) taking them as specimens of some kind or other, (b)

How, then, can we frame principles of mediate reasoning, about such things? So far as we consider them as Kinds, it is enough to say: *Whatever can be identified as a specimen of a known substance or Kind has the properties of that Kind.* So far as we consider them as in the relation of causation, we may say: *Whatever relation of events can be identified with the relation of cause and effect is constant.* And these principles may be genralised thus: *Whatever is constantly related to a phenomenon (cause or Kind), determined by certain characters, is related in the same way to any phenomenon, that has the same characters.* Tak-

taking them in the relation of causation, and (c) taking them in a generalised way.

ing this as axiom of the syllogism materially treated, we see that herbivorousness, being constantly related to ruminants , is constantly related ; to camels mortality to man and, therefore, to Socrates ; rusting to the immersion of iron in water generally and, therefore, to this piece of iron. *Nota notoe, nota rei ipsius* is another statement of the same principle ; still another is Mill's axiom, "Whatever has a mark has what it is a mark of." A mark is anything (A) that is never found without something else (B)—a phenomenon constantly related to another phenomenon—so that wherever, A is found, B may be expected : human nature is a mark of mortality.

§ 6. USUES OF THE SYLLOGISM : The Syllogism has sometimes been discarded by those who have only seen that, as formally stated, it is either useless or fallacious : but those who also perceive its material grounds retain and defend it. In fact, great advantages are gained by stating an argument as a formal syllogism. For, in the first place, we can then examine separately the three conditions on which the validity of the argument depends.

Recognition of the material ground can defend Syllogism.

First advantage: three-fold exam. of an argument's validity.

(1) Are the Premises so connected that, *if they are true*, the Conclusion follows ? This depends upon the formal principles of chap. x.

(1) whether the conclusion follows from the premises

(2) Is the Minor Premise true ? This question can only arise when the minor premise is a real proposition ; and then it may be very difficult to answer. Water rusts iron ; but is the metal we are now dealing with a fair specimen of iron ? Few people, comparatively, know how to determine whether diamonds or even gold or silver coins, are genuine. That *Camels are ruminants* is now a verbal proposition to a zoologist, but not to the rest of us ; and to the

(2) whether the minor premise is true

Zoologist the ascertaining of the relation in which camels stand to such ruminants as oxen and deer, was not a matter of analysing words but of dissecting specimens. What a long controversy as to whether the human race constitutes a Family of the Primates! That 'the British Empire is an empire' affords no matter for doubt or inquiry; but how difficult to judge whether the British Empire resembles Assyria, Egypt, Rome, Spain in those characters and circumstances that caused their downfall!

whether the major premise is true

(3) Is the Major Premise true? Are all ruminants herbivorous? If there be any exceptions to the rule, camels are likely enough to be among the exceptions. And here the need of Inductive Logic is most conspicuous: how can we prove our premises when they are universal propositions? Universal propositions, however, are also involved in proving the minor premise: to prove a thing to be iron, we must know the constant reactions of iron.

Second advantage of syllogism is in making clear the logical significance of inference, which lies in its capability of proving general truths.

A second advantage of the syllogism is, that it makes us fully aware of what an inference implies. An inference must have some grounds, or else it is a mere prejudice; but whatever the grounds, if sufficient in a particular case, they must be sufficient for all similar cases, they must admit of being generalised; and to generalise the grounds of the inference, is nothing else than to state the major premise. If the evidence is sufficient to justify the argument that camels are herbivorous *because* they are ruminants, it must also justify the major premise, *All ruminants are herbivorous*; for else the inference cannot really depend merely upon the fact of ruminating. To state our evidence syllogistically, then, must be possible, if the evidence is mediate and of a logical kind; and to state it in this formal way, as depending on the truth of a general principle (the major premise), increases our sense of responsibility for the infer-

ence that is thus seen to imply so much; and if any negative instances lie within our knowledge, we are the more likely to remember them. The use of syllogisms therefore tends to strengthen our reasonings.

Third advantage of syllogistic formulation is accurate communication of the argument

A third advantage is, that to formulate an accurate generalisation may be useful to others: it is indeed part of the systematic procedure of science. The memoranda of our major premises, or reasons for believing anything, may be referred to by others, and either confirmed or refuted. When such a memorandum is used for further inferences, these inferences are said, in the language of Formal Logic, to be drawn *from* it, as if the conclusion were contained in our knowledge of the major premise; but, considering the limited extent of the material evidence, it is better to say that the inference is drawn *according to* the memorandum or major premise, since the grounds of the major premise and of the conclusion are in fact the same (Mill: *Logic*, B. II. c. 3). Inductive proofs may be stated in syllogisms, and inductive inferences are drawn *according* to the Law of Causation.

Uniformity of nature analysed

§ 7. Analysis of the Uniformity of Nature considered as the formal ground of all reasoning: To assume that resemblance is a ground of inference, and that substance and attribute, or cause and effect, are phenomena constantly related, implies belief in the Uniformity of Nature. The Uniformity of Nature cannot be defined, and is therefore liable to be misunderstood. In many ways Nature seems not to be uniform: there is great variety in the sizes, shapes, colours and all other properties of things: bodies falling in the open air—pebbles, slates, feathers—descend in different lines and at different rates; the wind and weather are proverbially uncertain; the course of trade or of politics, is full of surprises.

The principle of uniformity is tacitly assumed even in our practical life.

Yet common maxims, even when absurd, testify to a popular belief that the relations of things are constant: the doctrine of St. Swithin and the rhyme beginning 'Evening red and morning grey,' show that the weather is held to be not wholly impredictable; as to human affairs, it is said that 'a green Yule makes a fat churchyard,' that 'trade follows the flag,' and that 'history repeats itself'; and Superstition knows that witches cannot enter a stable-door if a horse-shoe is nailed over it, and that the devil cannot cross a threshold inscribed with a perfect pentagram. But the surest proof of a belief in the uniformity of nature is given by the conduct of men and animals; by that adherence to habit, custom and tradition, to which in quiet times they chiefly owe their safety, but which would daily disappoint and destroy them, if it were not generally true that things may be found where they have been left and that in similar circumstances there are similar events.

This principle is the ground of all inference; and it is itself beyond any proof.

Now this general belief, seldom distinctly conceived, for the most part quite unconscious (as a principle), merely implied in what men do, is also the foundation of all the Sciences; which are entirely occupied in seeking the Laws (that is, the Uniformities) of Nature. As the uniformity of nature cannot be defined, it cannot be proved; the most convincing evidence in its favour is the steady progress made by Science whilst trusting in it. Nevertheless, what is important is not the comprehensive but indeterminate notion of Uniformity so much as a number of First Principles, which may be distinguished in it as follows:

The principle comprehends within itself a number of First Principles:

(1) Contradiction and Excluded Middle.

(1) The Principles of Contradiction and Excluded Middle (ch. vi. § 3) declare that in a given relation to a given phenomenon any two or more other phenomena are incompatible (*B is not A and a*); whilst the given phenomenon either stands related to another phenomenon or not (*B is either A or a*).

It is not only a matter of Logic but of fact that, if a leaf is green, it is not under the same conditions red or blue, and that if it is not green it is some other colour.

(2) Some Mathematical axioms.

(2) Certain Axioms of Mediate Evidence: as, in Mathematics, 'that magnitudes equal to the same magnitude are equal to one another'; and in Logic, the *Dictum* or its material equivalent.

(3) Commensurability of all times and all spaces

(3) That all Times and all Spaces are commensurable; although in certain relations of space (as π) the unit of measurement must be infinitely small.—If Time really trotted with one man and galloped with another, as it seems to; if space really swelled in places, as De Quincey dreamed that it did; life could not be regulated, experience could not be compared and science would be impossible. The mathematical Axioms would then never be applicable to space or time, or to the objects or processes that fill them.

(4) Persistance of Matter & Encrgy

(4) The Persistence of Matter and Energy: the physical principle that, in all changes of the universe, the quantities of Matter and Energy (actual and potential, so-called) remain the same. —For example, as to matter, although dew is found on the grass at morning without any apparent cause, and although a candle seems to burn away to a scrap of blackened wick, yet every one knows that the dew has been condensed from Vapour in the air, and that the candle has only turned into gas and smoke. As to energy, although a stone thrown up to the housetop and resting there has lost actual energy, it has gained such a position that the slightest touch may bring it to the earth again in the same time as it took to travel upwards; so on the house-top it is said to have potential energy. When a boiler works an engine, every time the piston is thrust forward (mechanical energy), an equivalent in heat (molecular energy), is lost. But

for the elucidation of these principles, readers must refer to treatises of Chemistry and Physics.

(5) Causation

(5) Causation, a special form of the foregoing principles of the persistence of matter and energy, we shall discuss in the next chapter. It is not to be conceived of as anything occult or noumenal, but merely as a special mode of the uniformity of Nature or experience.

(6) Certain co-existential Uniformities

(6) Certain Uniformities of Co-existence; but for want of a general principle of Co-existence, corresponding to Causation (the principle of Succession), we can only classify these uniformities as follows :

(a) Geometrical

(a) The Geometrical; as that, in a four-sided figure, if the opposite angles are equal, the opposite sides are equal and parallel.—Countless similar uniformities of co-existence are disclosed by Geometry. The co-existent facts do not cause one another, nor are they jointly caused by something else; they are mutually involved : such is the nature of space.

(b) Universal co-inherences among the properties of concrete things.

(b) Universal co-inherences among the properties of concrete things.—The chief example is the co-inherence of gravity with inertia in all material bodies. There is, I believe, no other entirely satisfactory case; but some good approximations to such uniformity are known to physical science.

(c) Causal co-existence

(c) Co-existence due to Causation; scuh as the positions of objects in space at any time.—The houses of a town are where they are, because they were put there; and they remain in their place as long as no other causes arise strong enough to remove or destroy them. Similarly, the relative positions of rocks in geological strata, and of trees in a forest, are due to causes.

(d) Co-inherence of properties in Natural kinds.

(d) The co-inherence of properties in Natural Kinds; which we call the constitution, defining characters, or specific nature of such things.—Oxygen, Platinum, sulphur and the other elements; water, common salt, alcohol and other compounds; the various species of plants and animals: all these are known to us different groups of co-inherent properties. It may be conjectured that these groupings of properties are also due to causation, and sometimes the causes can be traced: but very often the causes are still unknown; and, until resolved into their causes, they must be taken as necessary data in the investigation of nature. Laws of the co-inherence of the properties of Kinds do not, like laws of causation, admit of methodical proof upon their own principles, but only by constancy in experience and statistical probability (c. xix, § 4).

(e) Other minor co-existences

(*e*) There are also a few cases in which properties co-exist in an unaccountable way, without being co-extensive with any one species, genus, or order: as most metals are whitish, and scarlet flowers are wanting in fragrance. (On this § 7, see Venn's *Empirical Logic*, c. 4.)

Fonr grounds of belief; in Uniformity

§ 8. Grounds of our belief in Uniformity: Inasmuch as Axioms of Uniformity are ultimate truths, they cannot be deduced; and inasmuch as they are universal, no proof by experience can ever be adequate. The grounds of our belief in them seem to be these:

(1) Every inference takes for granted an order of Nature corresponding with it; and every attempt to explain the origin of anything assumes that it is the transformation of something else: so that uniformity of order and conservation of matter and energy are necessary presuppositions of reasoning.

(2) On the rise of philosophic reflection, these tacit presuppositions are first taken as dogmas, and later as postulates of scientific generalisation, and of the architectonic unification of science. Here they are indispensable.

(3) The presuppositions or postulates are, in some measure, verifiable in practical life and in scientific demonstration, and the better verifiable as our methods become more exact.

(4) There is a cause of this belief that cannot be said to contain any evidence for it, namely, the desire to find in Nature a foundation for confidence in our own power to foresee and to control events.

N.B. The student is advised to go through Appendix on 'Inductive Syllogism' before beginning the next Chapter.

SUMMARY

Neither a syllogism nor any deductive inference examines the truth of its premises. It accepts the premises and attempts to find out what follows from them. So the problem arises : how are the premises, to be established ?

To state the problem in another way. The general terms of which universal propositions consist, are never exhaustively known in their denotation. So the problem arises, what right have we to make such propositions ? This is the problem of induction.

Universal proposition cannot always be established by Syllogisms. The Universal premise of one syllogism can be established by another syllogism, the universal premise of which can be established by a fresh syllogism, but there must be a limit to such sorites. Moreover, the form of the syllogism is such that it either involves *petitio principii* or becomes needless if it attempts to establish universal propositions.

That is, if we think that the premises of a syllogism prove the conclusion as they have been established by detailed observation, we encounter a dilemma. If all the facts of the major premise of any syllogism have been examined, the syllogism is needless and if some of them have not been examined, it is a *petitio principii.* But either all have been examined or some have not. So the syllogism is either useless or fallacious.

We can escape from the dilemma by distinguishing between the formal and material aspects of the syllogism. Thus a syllogism begs the question formally but not materially, as resemblance is the real ground of inference here.

In this way, we can also dispose of the other criticism of syllogism which is to the effect, that as in a valid syllogism, the conclusion does not go beyond the premises, so, a syllogism does not give us new knowledge.

Hence, the important question is, what sort of resemblance is a sufficient ground of inference? And in endeavouring to answer it we find that the surest ground of inference is resemblance of causation.

In Aristotle's dictum nothing is said of the relations that might be implied in the predication.

So, Mill gives instead of the dictum the following as the axiom of syllogistic reasoning: "A thing which co-exists with another thing, which other co-exists with a third thing, also co-exists with that third thing." But there is an inconsistency involved here. For, the law of causation is the ground of sound induction and it is the principle of the succession of phenomena of syllogism. But the axiom as formulated by Mill is of co-existence and is different from an induction. And so how can induction be the ground of deduction?

So we are also to hold that the doctrine of natural kinds is also a ground of induction. We can rarely trace the causes with any fullness, and so a great deal of reasoning about the real events of the world does in fact trust to the relative permanence of natural kinds as defined by co-inhering attributes.

The principle of mediate reasoning about things considered as kinds, is (1) whatever can be identified as a specimen of a known substance of kind has the properties of that kind; and considered in the relation of causation, is (2) whatever relation of events can be identified with the relation of cause and effect is constant; and the generalised form of (1) and (2) is (3) whatever is constantly related to a phenomenon (cause of kind) determined by certain characters is related in the same way to any phenomenon that has the same characters.

When the material grounds of the syllogism are perceived, it is found to have the following advantages:—(1) In the first place we can examine separately the three conditions on which the validity of the argument depends:—(a) Are the premises so connected that, *if they are true*, the conclusion follows? (b) Is the minor premise true? (c) Is the major premise true? (2) A second advantage of the syllogism is that it makes us fully aware of what an inference implies. An inference must have

grounds, but whatever the grounds, if sufficient in a particular case, they must be sufficient for all cases, they must admit of being generalised. Now, to generalise the grounds of the inference is nothing else than to state the major premise. So to state our evidence syllogistically must be possible, if the evidence is mediate and of a logical kind; and to state it in this formal way, as depending on the truth of a general principle, increases our sense of responsibility for the inference that is thus seen to imply so much. (3) A third advantage is that to formulate an accurate generalisation may be useful to others: it is indeed part of the systematic procedure of science.

The statements made above concerning resemblance, co-inherence and causation implies belief in uniformity of Nature. Its real meaning is liable to be misunderstood. For, in many ways nature seems not to be uniform. Yet common maxims, even when absurd, testify to a popular belief that the relations of things are constant. And the surest proof of a belief in uniformity of nature is given by the conduct of men and animals, by that adherence to habit, custom and tradition, to which in quiet times they chiefly owe their safety, but which would daily disappoint and destroy them, if it were not generally true that things may be found where they have been left and that in similar circumstances there are similar events.

Now strictly speaking uniformity of Nature can neither be defined nor proved; the most convincing evidence in its favour is the steady progress made by science. In the indeterminate notion of uniformity, a number of first principles may be distinguished as follows :—

(1) The principle of Contradiction and Excluded Middle.
(2) Some Mathematical axioms.
(3) The principle of commensurability of all Times and all Spaces.
(4) The Principle of Persistence of Matter & Energy.
(5) The principle of Causation.
(6) Certain principles of co-existential uniformity:
  (*a*) Geometrical.
  (*b*) Universal co-inherences among the properties of concrete things.
  (*c*) Causal co-existence.
  (*d*) Co-inherence of properties in Natural Kinds.
  (*e*) Other minor co-existences.

The grounds of our belief in the Axioms of Uniformity are as following :—

(1) Every inference takes for granted an order of Nature corresponding with it ; and every attempt to explain the origin of anything assumes that it is the transformation of something else : so that uniformity of order and conservation of matter and energy are necessary presuppositions of reasoning.

(2) On the rise of philosophic reflection, these tacit presuppositions are first taken as dogmas, and later as postulates of scientific generalisation, and of the architectonic unification of science. Here they are indispensable.

(3) The presuppositions or postulates are, in some measure, verifiable in practical life and in scientific demonstration and the better variable as our methods become more exact.

(4) There is a cause of this belief that cannot be said to contain any evidence for it, namely, the desire to find in Nature a foundation for confidence in our own power to foresee and to control events.

EXERCISES WITH HINTS

1. *Explain clearly the transition from deduction to induction.*

[See sec. 1-4.]

2. *Show that deductive reasoning can give material truth only when it is aided by induction. Discuss in this connection if the truth of a syllogism can be proved by pro-syllogisms.*

[See sec. 1-4.]

3. *'Induction is the inverse process of Deduction'* *Discuss.*

[This is the view of Jevons. He maintained that the essence of induction consists in the framing and verification of a hypothesis. That is, in Induction we start with particular facts and from them aided by insight or imagination arrive at a hypothesis or general proposition. To establish this general proposition we descend by deduction to particular facts. Now, the inference lies in this second movement, *i.e.*, in the descending from the general to the particular. So the Inductive inference is really speaking deductive. The only difference between deduction and induction lies in starting point. Whereas Deduction starts

straight from a general proposition. Induction starts from particular facts and ascends to universal proposition by means of insight or imagination (*i.e.*, non-inferrentially) and then like deduction descends to particular. So Induction is inverse deduction.

4. *'The difference between deduction and induction is not one of principle but of starting point'—discuss.*

[The essence of inference lies in exhibiting the manner in which particular facts are connected together in a system. The world, that is to say, is a whole of interrelated facts. Every fact is necessarily connected with other facts. And inference is the consciousness of this necessary connection of things. The aim of it therefore is to show the connection between facts and facts or laws. And this aim is achieved both by deduction and induction. Deduction starts with the law or the whole, and tries to show how the facts or the parts come under it. Induction starts with facts or the parts and tries to show how the law or the whole is of them. "In induction reality presents itself in concrete and partially isolated instances, and the task of inference is to discern the universal which is more or less hidden in the instances. In deduction, on the other hand, reality presents itself in its universal aspect and the task of inference is to trace the presence of that universal in the differing and complex instances of its manifestation. The distinction therefore is solely one of the order in which the two aspects of reality are presented to us, and this difference can have no effect upon either reality itself or our final conception of it when we know it in both its aspects." (Welton)

(See ans. to Ex. 3 also).]

5. *Explain the distinction between inductive and deductive reasoning.*

[It is often supposed that induction and deduction are opposed to each other. Now, this is a mistaken view. For, they presuppose each other and hence cannot be opposed to each other. Deduction pre-supposes induction as unaided by induction it cannot have the whole or the universal from which it is to start. And induction pre-supposes deduction as unaided by deduction it cannot be verified. So deduction and induction are not opposed to each other. One makes the other possible. The mistaken view owes its origin to either of the following assumptions : (1) hypothesis is not essential to induction and (2) observation is not necessary for knowledge. In other words,

some inductive logicians fail to recognise the importance of hypothesis in induction. They do not see that the essence of induction lies in the framing and verification of a hypothesis. Had they seen this, they would have also seen that as deduction is necessary for verification, so it can be neither unnecessary nor opposed to induction. So, the view that induction is opposed to deduction is misleading as it is merely the conseqence of a misleading view concerning the nature and importance of hypothesis. Similarly some Formal logicians fail to note that if there were no sense-experience there would have been no knowledge and consequently entertain the view that the universal with which deduction starts is an innate or self-evident truth. Now, the majority of the universals are not so. Observation is necessary for discovering and proving them and hence induction is presupposed by deduction. These Formal logicians who fail to follow this thing that induction is unnecessary or opposed to deduction. See also ans. to Ex. 3 & 4.]

6. *'The uniformity of nature can neither be defined nor proved'—Discuss.*

[See secs. 7 & 8,]

7. *'In the indeterminate notion of uniformity a number of first principles may be distinguished'—Comment upon it.*

[See sec. 7.]

8. *What are grounds of our belief in the principle of uniformity of nature ?*

[See sec. 8.]

9. *What grounds are there for holding that the sun will rise tomorrow ?*

[See sec. 8.

We hold that the sun will rise tomorrow as we believe in the principle of uniformity of nature.]

10. *What grounds are there for saying that the future will resemble the past ?*

[See sec. 8.]

11. *Discuss whether induction precedes deduction or deduction precedes induction.*

[The question rests upon the assumption that induction and deduction are opposed to each other so that one does not complete and make the other possible. But such a view concerning the relation of induction and deduction is mistaken ; and hence

the question is also misleading. However, those Formal and Material Logicians, who on account of their bias have failed to see the real relation between deduction and induction have argued either the priority of deduction or the priority of induction. The prejudiced Material Logicians have argued that as the universal premise of deduction is supplied by induction, so induction must be regarded as prior to deduction. Similarly the prejudiced Formal Logicians have said that as deduction is necessary for establishing a general proposition, so deduction must precede induction. But these views are mistaken. For, there can be no induction apart from deduction and similarly apart from induction there can be no deductive inference relating to the concrete reality. So the question of one preceding the other does not arise].

12. *Explain the problem of induction and bring out the interdependence of induction and deduction. (M.U. '41).*

13. *Expand the central assumption of induction. Can it be proved? If so, how? (M.U., '42).*

[The central assumption is the principle of uniformity of nature.]

14. *What is the problem of induction? Discuss the view that all inference is essentially deductive. (M.U., '43).*

[For the second part of the question see Ex. 3.]

15. *Can the provinces of induction and deduction be kept separate.*

[See Ex. 3, 4 and 5.]

16. *Can Induction be expressed as a form of Syllogism?*

[See Appendix.]

CHAPTER XIV

Causation

Uniformity of causation specially important:

§ 1. THE MOST IMPORTANT ASPECT OF UNIFORMITY IN RELATION TO INDUCTION IS CAUSATION: For the theory of Induction, the specially important aspect of the Uniformity of Nature is Causation. For (1) the principles of Contradiction and Excluded Middle are implied in all logical operations, and need no further explication.

(2) That one thing is a mark of another or constantly related to it, must be established by Induction; and the surest of all marks is a Cause. So that the application of the axiom of the Syllogism in particular cases requires, when most valid, a previous appeal to Causation.

(3) The uniformity of Space and of Time is involved in Causation, so far as we conceive Causation as essentially matter in motion—for motion is only known as a traversing of space in time; and so far as forces vary in any way according to the distance between bodies; so that if space and time were not uniform, causation would be irregular. Not that time and space are agents, but they are conditions of every agent's operation.

(4) The persistence of Matter and Energy, being nothing else than Causation in the general movement of the world, is applied under the name of that principle in explaining any particular limited phenomenon, such as a soap-buble, or a thunderstorm, or the tide.

(5) As to co-existences, the Geometrical do not belong to Logic: those involved in the existence of plants, animals, and inorganic bodies, must, as far as possible, be traced to causes; and so, of course, must the relative positions of objects in space at any time: and what Co-existences remain do not admit of

methodical inductive treatment: they will be briefly discussed in chap. xix.

Causation a name for phenomena in a particular relation to other phenomena

Causation, then, is that mode or aspect of the Uniformity of Nature which especially concerns us in Induction; and we must make it as definite as possible. It is nothing occult, but merely a convenient name for phenomena, called their effect. Similarly, if the word 'force' is sometimes used for convenience in analysing causation, it means nothing more than something in time and space, itself moving, or tending to move, or hindering or accelerating other things. If any one does not find these words convenient for the purpose, he can use others.

Mill's definition of Cause

§ 2. Definition of "Cause" explained: five marks of Causation—A Cause, according to Mill, is "the invariable unconditional antecedent" of a given phenomenon. To enlarge upon this:

(1) Cause relative to a given phenomenon the effect

(1) A Cause is *relative to a given phenomenon, called the Effect.* Logic has no method for investigating the cause of the universe as a whole, but only of a part or epoch of it: we select from the infinite continuum of Nature any portion that is neither too large nor too small for a trained mind to comprehend. The magnitude of the phenomenon may be a matter of convenience. If the cause of disease in general be too wide a problem, can fevers be dealt with; or, if that be too much, is typhus within the reach of inquiry? In short, how much can we deal with accurately?

(2) The given phenomenon always an event

(2) The given phenomenon is always *an event*; that is to say, not a new thing (nothing is wholly new), but a change is something, or in the relative position of things. We may ask the cause of the phases of the moon, of the freezing of water, of the kindling of a match, of a deposit of chalk, of the differentiation of species. To inquire the cause of France being

a republic, or Russia an autocracy, implies that these countries were once otherwise governed, or had no government : to inquire the cause of the earth being shaped like an orange, implies that the matter of the earth had once another shape.

(3) Antecedent to the effect

(3) The Cause is *antecedent* to the Effect, which accordingly is often called its *consequent.* This is often misunderstood and sometimes disputed. It has been said that the meaning of 'cause' implies an 'effect,' so that until an efiect occurs there can be no cause. But this is a blunder; for whilst the word 'cause' implies 'effect' it also implies the relative futurity of the effect; and effect implies the relative priority of the cause. The connotation of the words, therefore, agrees well enough with Mill's doctrine. In fact, the danger is that any pair of contrasted words may suggest too strongly that the phenomena denoted are separate in Nature; whereas every natural process is continuous. If water, dripping from the roof wears away a stone, it fell on the roof as rain; the rain came from a condensing cloud; the cloud was driven by the wind from the sea, whence it exhaled; and so on. There is no known beginning to this, and no break in it. We may take any one of these changes, call it an effect and ask for its cause; or call it a cause, and ask for its effect. There is not in Nature one set of things called causes and another called effects; but every change is both cause (or a condition) of the future and effect of the past; and whether we consider an event as the one or the other, depends upon the direction of our curiosity or interest.

For cause is essentially motion that takes time

Still, taking the event as effect, its cause is the antecedent process; or, taking it as a cause, its effect is the consequent process. This follows from the conception of causation as essentially motion; for that *motion takes time* is (from the way our perceptive powers grow) an ultimate intuition. But, for the same reason, there is no interval of time between cause and effect; since all the time is filled up with motion.

Nor must it be supposed that the whole cause is antecedent to the effect as a whole: for we often take the phenomenon on such a scale that minutes, days, years, ages, may elapse before we consider the cause as exhausted (*e.g.*, an earthquake, a battle, an expansion of credit, natural selection operating on a given variety, and all that time the effect has been accumulating.) But we may further consider such a cause as made up of moments or minute factors, and the effect as made up of corresponding moments; and then the cause, taken in its moments, is antecedent throughout to the effect, taken in its corresponding moments.

The whole cause is not antecedent to the effect as a whole

(4) The Cause is the *invariable* antecedent of the effect; that is to say, whenever a given cause occurs it always has the same effect: in this, in fact, consists the uniformity of causation. Accordingly, not every antecedent of an event is its Cause: to assume that it is so, is the familiar fallacy of arguing '*post hoc ergo propter hoc.*' Every event has an infinite nmber of antecedents that have no ascertainable connection with it: if a picture falls from the wall in this room, there may have occurred, just previously, an earthquake in New Zealand, an explosion in a Japanese arsenal, a religious riot in India, a political assassination in Russia and a vote of censure in the House of Commons, besides millions of other less noticeable events, between none of which and the falling of the picture can any direct causatian be detected; though, no doubt, they are all necessary occurrences in the general world-process, and remotely connected. The cause, however, was that a door slammed violently in the room above and shook the wall, and that the picture was heavy and the cord old and rotten. Even if two events invariably occur one after the other, as day follows night, or as the report follows the flash of a gun, they may not be cause and effect, though it is highly probable that they are closely connected by causation; and in each of these two examples the events are co-effects of a

(4) The invariable antecedent

ommon cause, and may be regarded as elements of its total effect. Still, whilst it is not true that every antecedent, or that every invariable antecedent, of an event is its cause, the cause is conceived of as some change in certain conditions, or some state and process of things, such that should it exactly recur the same event would invariably follow. If we consider the antecedent state and process, of things very widely or very minutely, it never does exactly recur; nor does the consequent. But the purpose of induction is to get as near the truth as possible within the limits set by our faculties of observation and calculation. Complex causal instances that are most unlikely to recur as a whole, may be analysed into the laws of their constituent conditions.

(5) The Uncondional Antecedent

(5) The Cause is the Unconditional Antecedent. A cause is never simple, but may be analysed into several conditions; and 'Condition' means any necessary factor of a Cause: any thing or agent that exerts, absorbs, transforms, or deflects energy; or any relation of time or space in which agents stand to one another. A positive condition is one that cannot be omitted without frustrating the effect; a negative condition is one that cannot be introduced without frustrating the effect. In the falling of the picture, *e.g.*, the positive conditions were the picture (as being heavy), the slamming of the door, and the weakness of the cord: a negative condition was that the picture should have no support but the cord. When Mill, then, defines the Cause of any event as its "unconditional" antecedent, he means that it is that group of conditions (state and process of things) which without any further condition, is followed by the event in question: it is the least antecedent that suffices, positive conditions being present and negative absent.

The antecedent that can be left

Whatever item of the antecedent can be left out, then, without affecting the event, is no part of the cause. Earthquakes have happened in New Zealand and votes of censure in the House of Commons with-

a picture's falling in this room: they were unconditional antecedents; something else needed to bring down a picture. Unconditionality also distinguishes a true cause from an invariable antecedent that is only a co-effect: for when day follows night something else happens; the Earth rotates upon her axis: a flash of gunpowder is not an unconditional antecedent of a report; the powder must be ignited in closed chamber.

out, i.e., is unconditional is not the cause. The invariable co-effect, is not cause.

By common experience, and more precisely by experiment, it is found possible to select from among the antecedents of an event a certain number upon which, so far as can be perceived, it is dependent, and to neglect the rest; to purge the cause of all irrelevant antecedents is the great art of inductive method. Remote or minute conditions may indeed modify the event in ways so refined as to escape our notice. Subject to the limitations of our human faculties, however, we are able to many cases to secure an unconditional antecedent upon which a certain event invariably follows. Everybody takes this for granted: if the gas will not burn, or a gun will not go off, we wonder 'what can be wrong with it,' that is, what positive condition is wanting, or what negative one is present. No one now supposes that gunnery depends upon those "remotest of all causes," the stars, or upon the sun being in Sagittarius rather than in Acquarius, or that he shoots straightest with a silver bullet, or after saying the alphabet backwards.

The great art of inductive method is to find out the unconditional antecedent.

(6) That the Cause of any event is an Immediate Antecedent follows from its being an unconditional one. For if there are three events, A B C causally connected, it is plain that A is not the unconditional antecedent of C, but requires the further condition of first giving rise to B. But that is not all; for the B that gives rise to C is never merely the effect of A; it involves

(6) Immediate antecedent

something further. Takc such a simple case as the motion of the earth round the sun (neglecting all other conditions, the other planets, *etc.*), and let the earth's motion at three successive moments be A B C : A is not the whole cause of B in velocity and direction; we must add relation to the sun, say x. But then, again, the cause of C will not be merely Bx, for the relation to the sun will have altered; so that we must represent it as Bx^1. The series, therefore, is Ax Bx^1 C. What is called a "remote cause" is, therefore, doubly conditional; first, because it supposes an intervening cause; and secondly, because it only in part determines the conditions that constitute this intervening cause.

The immediacy of a cause being implied in its unconditionalness, is an important clue to it; but as far as the detection of causes depends upon sense-perception, our powers (however aided by instruments) are unequal to the subtlety of Nature. Between the event and what seems to us the immediate antecedent many things (molecular or etherial changes) may happen in Chemistry or physics. The progress of science would be impossible, were not observation supplemented by hypothesis and calculation. And where phenomena are treated upon a large scale, as in the biological and social sciences, immediacy, as a mark of causation, must be liberally interpreted. So far, then, as to the qualitative character of Causation.

Quantitatively the cause equal to the effect

(7) But to complete our account of it, we must briefly consider its quantitative character. As to the Matter contained, and as to the Energy embodied, Cause and Effect are conceived to be *equal*. As to matter, indeed, they may be more properly called identical; since the effect is nothing but the cause redistributed. When oxygen combines with hydrozen to form water, or with mercury to form red precipitate, the weight of the compound is exactly equal to the weight of the elements combined in it; when a shell explodes and knocks down a wall, the materials of the shell and wall are scattered about. As to energy, we see that in the heavenly bodies, which meet with no

sensible impediment, it remains the same from age to age : with things 'below the moon' we have to allow for the more or less rapid conversion of the visible motion of mass into other forms of energy, such as sound and heat. But the right understanding of this point involves physical considerations of some difficulty, as to which the reader must refer to appropriate books such as Balfour Stewart's on *The Conservation of Energy*.

Cause, sum of Moving Power and a Collocation of circumstances.

The comprehension of the quantitative aspect of causation is greatly aided by Bain's analysis of any cause into a 'Moving or an Inciting Power, and a 'Collocation' of circumstances. When a demagogue by making a speech stirs up a mob to a riot, the speech is the moving or inciting power ; the mob already in a state of smouldering passion, and a street convenient to be wrecked, are the collocation. When a small quantity of strychnine kills a man, the strychnine is the inciting power ; the nature of his nervo-muscular system, apt to be thrown into spasms by that drug, and all the organs of his body dependent on that system, are the collocation. Now any one who thinks only of the speech, or the drug, in these cases, may express astonishment at the disproportion of cause and effect :

"*What great events from trivial causes spring?*"

But, remembering that the whole cause of the riot included the excited mob, every one sees that its muscular power is enough to wreck a street; and remembering that breathing depends upon the normal action of the intercostal muscles, it is plain that if this action is stopped by strychnine, a man must die. Again, a slight rise of temperature may be a sufficient inciting power to occasion extensive chemical changes in a collection of elements otherwise stable ; a spark is enough to explode a powder magazine. Hence, when sufficient energy to account for any effect cannot be found in the inciting power, or manifestly active condition, we must look for it in the collocation which is often supposed to be passive.

The distinction between agent and patient a merely relative distinction

And that reminds us of another common mis-apprehension, namely, that in Nature some things are passive and others active: the distinction between 'agent' and 'patient.' This is a merely relative distinction: in Nature all things are active. To the eye some things seem at rest and others in motion; but we know that nothing is really at rest, that everything palpitates with molecular change, and whirles with the plannet through space. Everything that is acted upon reacts according to its own nature; the quietest-looking object (say, a moss-covered stone), if we try to push or lift it, pushes or pulls us back, assuring us that 'action and reaction are equal and opposite.' 'Inertia' does not mean want of vigour, but may be metaphorically described as the inexpugnable resolve of everything to have its own way.

Equality of cause and effects defines and interprets the unconditionality of causation.

The equality of cause and effect defines and interprets the unconditionality of causation. The cause, we have seen, is that group of conditions which, without any further condition, is followed by a given event. But how is such a group to be conceived? Unquantified, it admits only of a general description: quantified, it must mean a group of conditions equal to the effect in mass and energy, the essence of the physical world. Apparently, a necessary conception of the human mind; for if a cause seem greater than its effect, we ask what has become of the surplus matter and energy; or if an effect seem greater than its cause; we ask whence the surplus matter and energy has arisen. So convinced of this truth is every experimenter, that if his results present any deviation from it, he always assumes that it is he who has made some mistake or oversight, never that there is indeterminism or discontinuity in Nature.

The transformation of matter and energy, then, is the essence of causation: because it is continuous, causation is immediate; and because in the same circumstances the transformation al-

ways follows the same course, a cause has invariably the same effect. If a fire be lit morning after morning in the same grate, with coal, wood, and paper of the same quality and similarly arranged, there will be each day the same flaming of paper, crackling of wood and glowing of coal, followed in about the same time by the same reduction of the whole mass partly to ashes and partly to gases and smoke that have gone up the chimney. The flaming, crackling and glowing are, physically, modes of energy ; and the change of materials into gas and ashes is a chemical and physical redistribution : and, if some one be present, he will be aware of all this; and then, besides the physical changes, there will be sensations of light, sound and heat; and these again will be always the same in the same circumstances.

This also explain the immediacy and the invariability of the cause.

The Cause of any event, then, when exactly ascertainable, has five marks: it is (quantitatively) *equal* to the effect, and (qualitatively) *the immediate, unconditional, invariable antecedent of the effect.*

The concluding definition

§ 3. How strictly the conception of Cause can be applied depends upon the subject under investigation: This scientific conception of causation has been developed and rendered definite by the investigations of those physical sciences that can avail themselves of exact experiments and mathematical calculation ; and it is there, in Chemistry and Physics, that it is most at home. The conception can indeed be carried into the Biological and Social Sciences, even in its quantitative form, by making the proper allowances. For the limbs of animals are levers, and act upon mechanical principles ; and digestion and the aeration of the blood by breathing are partly chemical processes. There is a quantitative relation between the food a man eats and the amount of work he can do. The numbers of any species of plant or animal depend upon

This scientific view of causation is most at home in Physics and Chemistry and can be carried into the Biological and Social sciences.

the food supply. The value of a country's imports is equal to the value of its exports and of the services it renders to foreigners. But, generally, the less experiment and exact calculation are practicable in any branch of inquiry, the less rigorously can the conception of causation be applied there, the more will its application depend upon the qualitative marks, and the more need there will be to use it judiciously. In every inquiry the greatest possible precision must be aimed at; but it is unreasonable to expect in any case more precise proof than the subject admits of in the existing state of culture.

It is difficult to apply it in Psychology

Wherever mental action is involved, there is a special difficulty in applying the physical notion of causation. For if a Cause be conceived of as matter in motion, a thought, or feeling, or volition can be neither cause nor effect. And since mental action is involved in all social affairs, and in the life of all men and animals, it may seem impossible to interpret social or vital changes according to laws of causation. Still, animals and men are moving bodies; and it is recognised that their thoughts and feelings are so connected with their movements and with the movements of other things acting upon them, that we can judge of one case by another; although the connection is by no means well understood, and the best words (such as all can agree to use) have not yet been found to express even what we know about it. Hence, a regular connection being granted, I have not hesitated, to use biological and social events and the laws of them, to illustrate causation and induction; because, though less exact than chemical or mechanical examples, they are to most people more familiar and interesting.

In practical affairs, it is felt that everything depends upon causation; how to play the fiddle, or sail a yacht or get one's living, or defeat the enemy. The price of pig-iron six months hence, the prospects of the harvest, the issue in a Coroner's Court, Home Rule and Socialism, are all questions of causation. But, in such cases, the conception of a cause is rarely applied in

its full scientific acceptation, as the unconditional antecedent, or 'all the conditions' (neither more nor less) upon which the event depends. This is not because men of affairs are bad logicians, or incapable of scientific comprehension; for very often the reverse is conspicuously true; but because practical affairs call for promptitude and a decisive seizing upon what is predominantly important. How learn to play the fiddle? "Go to a good teacher." (Then, beginning young enough, with natural aptitude and great diligence, all may be well). How defeat the enemy? "Be two to one at the critical juncture." (Then, if the men are brave, disciplined, well armed and well fed, there is a good chance of victory). Will the price of iron improve? "Yes: for the market is oversold": (that is, many have sold iron who have none to deliver, and must at some time buy it back; and that will put up the price—if the stock is not too great, if the demand does not fall off, and if those who have bought what they cannot pay for are not in the meanwhile obliged to sell). These prompt and decisive judgments (with the parenthetic considerations unexpressed) as to what is the Cause, or predominantly important condition, of any event, are not as good as a scientific estimate of all the conditions when this can be obtained; but, when time is short, the insight of trained sagacity may be much better than an imperfect theoretical treatment of such problems.

The popular view of cause—any antecedent not necessarily the unconditional one

§ 4. Scientific conception of Effect. Plurality of Causes: To regard the Effect of certain antecedents in a narrow selective way, is another common mistake. In the full scientific conception of an Effect it is the sum of the unconditional consequences of a given state and process of things: the consequences immediately flowing from that situation without further conditions. Always to take account of all the consequences of any cause would no doubt be impracticable; still the practical, as well as the scientific interest,

Scientifically the effect is the sum of the unconditional consequences

often requires that we should enlarge our views of them; and there is no commoner error in private effort or in legislation than to aim at some obvious good, whilst overlooking other consequences of our action, the evil of which may far outweigh that good. An important consequence of eating is to satisfy hunger, and this is the ordinary motive to eat; but it is a poor account of the physiological consequences. An important consequence of firing a gun is the propulsion of the bullet or shell; but there are many other consequences in the whole effect, and one of them is the heating of the barrel, which, accumulating with rapid firing, may at last put the gun out of action. The tides have consequences to shipping and in the wear and tear of the coast that draw every one's attention; but we are told that they also retard the rotation of the earth, and at last may cause it to present always the same face to the sun, and, therefore, to be uninhabitable. Such concurrent consequences of any cause may be called its Co-effects; the Effect being the sum of them.

When the effect is not so considered, the erroneous doctrine of Plurality of Causes, (that the same effect may be due at different times to different causes) is formulated

The neglect to take account of the whole effect (that is, of all the co-effects) in any case of causation is perhaps the reason why many philosophers have maintained the doctrine of a "Plurality of Causes": meaning not that more than one condition is operative in the antecedent of every event (which is true), but that the same event may be due at different times to different antecedents, that in fact there may be *vicarious* causes. If, however, we take any effect as a whole, this does not seem to be true. A fire may certainly be lit in many ways: with a match or a flint and steel, or by rubbing sticks together, or by flash of lightning: have we not here a plurality of causes? Not if we take account of the whole effect; for then we shall find it modified in each case according to the difference of the cause. In one case there will be a burnt match, in another a warm flint, in the last a changed state of electrical tension.

And similar differences are found in cases of death under different conditions, as stabbing, hanging, cholera; or of shipwreck from explosion, scuttling, tempest. Hence a Coroner's Court expects to find, by examining a corpse, the precise cause of death. In short, if we knew the facts minutely enough, it would be found that there is only one Cause (sum of conditions) for each Effect (sum of co-effects), and that the order of events is as uniform backwards as forwards.

Though erroneous, the doctrine of Plurality of Causes is necessary in practical affairs and in unmanageable scientific investigations.

Still, as we are far from knowing events minutely, it is necessary in practical affairs, and even in the more complex and unmanageable scientific investigations, especially those that deal with human life, to acknowledge a possible plurality of causes for any effect. Indeed, forgetfulness of this leads to many rash generalisations; as that 'revolutions always begin in hunger'; or that 'myths are a disease of language.' Then there is great waste of ingenuity in reconciling such propositions with the recalcitrant facts. A scientific method recognises that there may be other causes of effects thus vaguely conceived, and then proceeds to distinguish in each class of effects the peculiarities due to different causes.

If not understood as the sum of conditions, cause cannot be understood as immediate.

§ 5. Some condition, but not the whole Cause, may long precede the Effect; and some co-effect, and not the whole effect, may long survive the Cause: The understanding of the complex nature of Causes and Effects helps us to overcome some other difficulties that perplex the use of these words. We have seen that the true cause is an *immediate* antecedent; but if the cause is confounded with *one* of its constituent conditions, it may seem to have long preceded the event which is regarded as its effect. Thus, if one man's death is ascribed to another's desire of revenge, this desire may have been entertained for years before the assassination occured: similarly, if a ship-

wreck is ascribed to a sunken reef, the rock was waiting for ages before the ship sailed that way. But, of course, neither the desire of revenge nor the sunken rock was 'the sum of the conditions' on which the one or the other event depended: as soon as this is complete the effect appears.

If not understood as the sum of unconditional consequence the effect cannot be understood as the initial tendency of the cause.

We have also seen the true effect of any state and process of things is the immediate consequence; but if the effect be confounded with *one* of its constituent factors, it may seem to long outlive the cessation of the cause. Thus, in nearly every process of human industry and art, one factor of the effect—a road, a house, a tool, a picture—may, and generally does, remain long after the work has ceased: but such a result is not the whole effect of the operations that produce it. The other factors may be, and some always are, evanescent. In most of such works some heat is produced by hammering or friction, and the labourers are fatigued; but these consequences soon pass off. Hence the effect as a whole only momentarily survives the cause. Consider a pendulum which having been once set agoing swings to and fro in an arc, under the joint control of the shaft, gravitation and its own inertia: at every moment its speed and direction change; and each change may be considered as an effect, of which the antecedent change was one condition. In such a case as this, which, though a very simple, is a perfectly fair example of all causation, the duration of either cause or effect is quite insensible: so that, as Dr. Venn says, *an Effect, rigorously conceived, is only "the initial tendency"* of its Cause.

Intermixture of effects: Mecha-

§ 6. Mechanical Causes and the homogeneous Intermixture of Effects; Chemical Causes and the heteropathic Intermixture of Effects: Mill contrasted two forms under which causation appears to us: that is to say, the conditions constituting a cause may be modified, or 'inter mixed' in the effect, in two ways,

which are typified respectively by Mechanical and Chemical action. In mechanical causation, which is found in Astronomy and all branches of Physics, the effects are all reducible to modes of energy, and are therefore commensurable with their causes. They are either directly commensurable, as in the cases treated of in the consideration of the mechanical power; or, if different forms of energy enter into cause and effect, such as mechanical energy, electrical energy, heat, these different forms are severally reducible to units, between which equivalents have been established. Hence Mill calls this *the "homogeneous intermixture of effects,"* because the antecedents and consequents are fundamentally of the same kind.

nical Causation, homogeneous intermixture of effects.

In chemical causation, on the other hand, cause and effect (at least, as they present themselves to us) differ in almost every way: in the act of combination the properties of elements (except weight) disappear, and are superseded by others in the compound. If, for example, mercury (a heavy, silvery liquid) be heated in contact with oxygen (a colourless gas), oxide of mercury is formed (red precipitate, which is a powder). This compound presents very different phenomena from those of its elements; and hence Mill called this class of cases "the heteropathic intermixture of effects." Still, in chemical action, the effect is not (in Nature) heterogeneous with the cause: for the weight of a compound is equal to the sum of the weights of the elements that are merged in it, and an equivalence has been ascertained between the energy of chemical combination and the heat, light, *etc.*, produced in the act of combination.

Chemical causasation and organic processes, heteropathic intermixture of effects

The heteropathic intermixture of effects is also found in organic processes (which, indeed, are partly chemical): as when a man eats bread and milk, and by digestion and assimilation converts them into nerve, muscle and bone. Such phenomena may make us wonder that people should ever have believed that 'effects

resemble their causes,' or that 'like produces like.' A dim recognition of the equivalence of cause and effect in respect of matter and motion may have aided the belief; and the resemblance of offspring to parents may have helped : but it is probably a residuum of magical rites ; in which to whistle may be regarded as a means of raising the wind, because the wind whistles ; and rain-wizards may make a victim shed tears that the clouds also may weep.

§ 7. Tendency, Resultant, Counteraction, Elimination, Resolution, Analysis, Reciprocity : Another consideration arises out of the complex character of causes and effects. When a cause consists of two or more conditions or forces, we may consider what effect any one of them would have if it operated alone, that is to say, its *Tendency*. This is best illustrated by the Parallelogram of Forces: if two forces acting upon a point, but not in the same direction be represented by straight lines drawn in the direction of the forces, and in length proportional to their magnitudes, these lines, meeting in an angle, represent severally the tendencies of the forces; whilst if the parallelogram be completed on these lines, the diagonal drawn from the point in which they meet represents their *Resultant* or *effect*.

Tendency

Resultant or effect

Again, considering the tendency of any force if it operated alone, we may say that, when combined with another force (not in the same direction) in any resultant, its tendency is *counteracted* : either partially, when the direction of the resultant is different ; or wholly when, the other force being equal and opposite, the resultant is equilibrium. If the two forces be in the same direction, they are merely added together. Counteraction is only one mode of combination ; in no case is any force destroyed.

Counteraction

Sometimes the separate tendencies of combined forces can only be theoretically distinguished : as when the motion of a projectile is analysed into a tendency to travel in the straight

line of its discharge, and a tendency to fall straight to the ground. But sometimes a tendency can be isolated: as when,—after dropping a feather in some place sheltered from the wind, and watching it drift to and fro, as the air, offering unequal resistances to its uneven surface, counteracts its weight with varying success, until it slowly settles upon the ground,—we take it up and drop it again in a vacuum, when it falls like lead. Here we have the tendency of a certain cause (namely, the relation between the feather and the earth) free from counteraction: and this is called the *Elimination* of the counteracting circumstances. In this case indeed there is physical elimination; whereas, in the case of projectile, when we say that its actual motion is resolvable (neglecting the resistance of the air) into two tendencies, one in the line of discharge, the other earth-wards, there is only theoretical elimination of either tendency, considered as counteracting the other; and this is more specifically called the *Resolution* or Analysis of the total effect into its component conditions. Now, Elimination and Resolution may be said to be the essential process of induction in the widest sense of the term, as including the combination of Induction with Deduction.

Elimination

Resolution or Analysis

The several conditions constituting any cause, then, by aiding or counteracting one another's tendencies, jointly determine the total effect. Hence, viewed in relation one to another, they may be said to stand in *Reciprocity* or mutual influence. This relation at any moment is itself one of co-existence, though it is conceived with reference to a possible effect. As Kant says, all substances, as perceived in space at the same time, are in reciprocal activity. And what is true of the world of things at any moment (as connected, say, by gravity), is true of any selected group of circumstances which we regard as the particular cause of any event to come. The use of the concept of reciprocity, then, lies in the analysis of a cause: we must not

Reciprocity or Mutual influence

think of reciprocity as obtaining in the succession of cause and effect, could turn back upon its cause; for as the effect arises its cause disappears, and is irrecoverable by Nature or Magic. There are many cases of rhythmic change and of moving equilibria, in which one movement or process produces another, and this produces something closely resembling the former, and so on in long series : as with the swing of a pendulum or the orbit of a planet : but these are series of cause and effect, not of reciprocity.

SUMMARY

In chap. XIII we have seen that in the indeterminate notion of uniformity a number of first principles may be distinguished. Of these various first principles, the principle of causal uniformity is specially important for the theory of Induction. The principles of Contradiction and Excluded Middle are implied in logical operations. That one thing is a mark of another or constantly related to it must be established by induction or an appeal to causation. The uniformity of space and of time is involved in causation as causation is essentially matter in motion. The persistence of Matter and Energy is nothing else than causation in the general movement of the world. As regards co-existence we have to say that while geometrical co-existences do not belong to Logic, those involved in the concrete world of matter and life, must as far as possible be traced to causes. Thus the principle of causal uniformity is specially important and we must make it as definite as possible.

A cause is always *relative* to a given phenomenon called the effect. That is, the names cause and effect are relative names. We should not suppose that in the world there are some events which are causes while the others are effects, as we should not suppose that in a society there are some men who are fathers while the others are sons. As A may be both a father and a son, i. e., father in relation to B and son in relation to C, similarly the same event may be both a cause and an effect. Again, the cause is relative to a *given* phenomenon. That is, the sciences are not interested in ascertaining the cause of the whole universe.

They are interested in ascertaining the cause of a part of it, a part neither too large nor too small for a trained mind to comprehend.

Now, this given phenomenon, or the part of the universe whose cause, a science is interested in ascertaining is an event in time. If there were no change either in the thing or in the relative position of things, the names cause and effect would have been meaningless. This aspect of causation, viz., that causation is essentialiy motion which takes time, distinguishes the causal relation from the relation of the ground and the consequent.

Thus causation is essentially motion. Now motion takes time and hence the cause is antecedent to the effect, which accordingly is called the eonsequent. While the word cause implies the effect, it also implies the relative futurity effects. Now, this should not be taken to suggest that nature is discontinuous. Nature is rather to be conceived of as a continuum of events in which the causes are the preceding or antecedent events while the effects are the succeeding or consequent ones. There is no interval of time between cause and effect, since all the time is filled up with motion. Again, it must not be supposed that the whole cause is antecedent to the effect as a whole. But still the cause taken in its moments is antecedent throughout to the effect in its corresponding moments.

The cause then it may be said, is an antecedent event. But any antecedent event is not the cause. For, the cause is an invariable antecedent. That antecedent is said to be invariable which being present the consequent follows or which is absent while the consequent is absent. So, whenever a cause occurs it always has the same effect, though it does not follow from this that whenever the same effect happens it is caused by the same cause. Now, to forget that the cause is the invariable antecedent is to commit the fallacy of *post hoc ergo propter hoc* (after this therefore on account of this). In other words, to take any antecedent event to be a cause is to make a mistake.

But the cause is not merely an invariable antecedent. That is, even if two events invariably occur one after the other, as day follows night or as the report follows the flash of a gun, they may not be cause and effect. For, the cause is the unconditional antecedent. It is not conditioned by, i.e., is not dependent upon anything else to produce the effect. That is, a

cause is never simple, but may be analysed into several conditions or the necessary parts of it. Now, such conditions may be either positive or negative. A positive condition must be present i.e., must not be absent to produce the effect, while the negative condition must be absent i.e., must not be present to produce the effect. And when the cause of any event is called its unconditional antecedent, what is meant is that it is that group of conditions, positive as well as negative, which without any further condition is followed by the event in question.

Now, because, the cause is the unconditional antecedent so it is also the immediate antecedent. In other words, the cause would not have been an unconditional antecedent i.e., the least antecedent that suffices if it were not also immediate.

Again, quantitatively the cause is equal to the effect. This follows from the principles of uniformity of nature and conservation of energy. That is, if the cause were not uniformly equal to the effect, it would be either sometimes greater and sometimes less than effect, or always greater than the effect or always less than the effect. Now, it cannot be supposed that the first alternative is true for, that will amount to the denial of the principle of uniformity. Neither can we suppose that the second and third alternatives are true, for that will amount to the denial of the principle of conservation of energy.

The comprehension of the quantitative aspect of causation is aided by Bain's analysis of any cause into a 'Moving or Inciting Power' and a 'Collocation' of circumstances. Again, this defines and interprets the unconditionality and the immediacy and the invariability of the cause. Unquantified, the unconditional antecedent admits only of a general description, but quantified, it means a group of conditions equal to the effect in mass and energy. And because the transformation of matter and energy is the essence of causation and because in the same circumstances the transformation always follows the same course the cause is immediate and invariable. So scientifically the cause of any event when exactly ascertainable has five marks : it is (quantitatively) equal to the effect and (qualitatively) the immediate unconditional, invariable antecedent of the effect. But for common sense it is one of the prominent antecedent that may or may not form a part of the scientific cause.

As scientifically the cause is the unconditional antecedent so scientifically the effect is the sum of the unconditional con-

sequences of a given state and process of things. When the effect is not so regarded the erroneous doctrine of plurality of cause that the same effect is due at different times to different causes is formulated. Now, the doctrine of plurality of causes, though erroneous, is necessary in practical affairs and in unmanageable scientific investigations.

Thus, neither the cause nor the effect is simple ; and the conditions constituting a cause may be modified or intermixed in two ways. There may be either a homogeneous or a heterogeneous intermixture. In mechanical causation, homogeneous intermixture is illustrated. Here the antecedents and the consequents are fundamentally of the same kind. But in chemical causation and organic processes, cause and effect differ almost in every way ; in the act of combination the properties of elements (except weight) disappear and are suppressed by others in the compound. This intermixture is known as heteropathic intermixture.

EXERCISES WITH HINTS

1. *What exactly is meant when it is said that A is the cause of B.*

[See sec. 2·]

2. *How would you define a cause ? Should the relation of logical dependence be regarded as identical with that of causation ?*

[See sec. 2 and add—There is a controversy among the logicians over the relation between the causal relation and the relation of logical dependence. According to some logicians the essence of the causal relation is a necessity. When it is stated that the cause is unconditional, this aspect of the cause is emphasised. And, so it is not essential that the cause should be regarded as essentially a motion that takes time. "Cause," says Bosanquet ; "as an event in time is an imperfect conception..... Cause is always taken to be more or less of a complication of relations and circumstances, and these as acknowledged to bear on an another are not mere events in time." In other words, "the temporal succession which seems the natural differentia of causation disappears in the reference of the effect to positive and continuous system. Mere temporal ralation is negative, is nothing. It is only the unity behind the temporal relation that

can bind cause to its effect." (Bosanquet). Thus some thinkers, do not consider the temporal aspect of cause as essential, and identify the logical relation of ground and consequent with the material relation of cause and effect. But there are other thinkers who consider change or temporality as essential, for we cannot obtain the effect by analysing the cause. They maintain that this at least is the scientific conception of it. Undoubtedly the causal relation is a necessary relation, but any necessary relation is not causal. We cannot say that a triangle is the cause of its three interior angles being equal to two right angles, though we do say that it is a ground of it. Hence necessity or uniformity in time is causality. And the causal relation is synthetic and different from the relation of logical dependence which is analytic. The difference between these two schools of logicians is due to their respective philosophic standpoints. The empiricists consider temporality to be essential while the rationalists do not think so. And it is almost impossible to decide which school is right.]

3. Distinguish between a cause and a condition, with the help of an illustration.

(See para 8 of see. 2)

4. Why is it that one should not regard night as the cause, not even a universal condition of day ?

(See para 7 of sec. 2)

5. How would you define antecedent, cause, effect and consequent ?

[An antecedent is that which is before. The cause is an antecedent. But any antecedent is not the cause, only the unconditional antecedent is so. Similarly the consequent is that which comes after. The effect is a consequent. But any consequent is not the effect. Only the unconditional consequent is so.]

6. Explain and examine the doctrine of plurality of Causes.

[See sec. 4 and note : This doctrine is due to an inadequate conception of the effect. If the effect be regarded as the sum of the unconditional consequences of a given state and process of things, the question of holding that the same effect may be due at different times to different causes, does not arise. If the distinctive character of each effect is taken into account, i.e. if the effect is specialised, the plurality of causes is eliminated. Similarly if the cause is generalised, i.e. if the general

feature of the so-called vicarious causes is taken into account, it also is eliminated. Some people think that the definition of cause as the invariable antecedent stands against this doctrine. But this is a mistake. For what exactly is meant when it is said that the A is the invariable antecedent of B is that when B is absent, A also is absent, and not that when B is present A also is present. Smoke is invariably related with fire, and this does not mean that wherever there is fire, there is smoke, but that where fire is absent smoke also is absent.]

7. *How would you distinguish conjunction of causes from plurality of causes.*

[For conjunction of causes see sec. 6.

There is conjunction of causes when A&B&C is the cause of X and there is plurality of causes when A or B or C is the cause of X.]

8. *"Science does not recognise any plurality of causes." Analyse the scientific conception of cause and comment upon it.*

[Scientifically viewed the cause is the unconditional antecedent while the effect is the unconditional consequent. So science cannot rccognise any plurality of cause.]

9. *Write notes on : Tendency, Resultant, Reciprocity, Elimination, Resolution and Counteraction.*

(See sec. 7.)

10. *Explain the principle of conservation of energy and discuss its bearing upon the conception of cause.*

[According to the scientists, the total amount of energy in the universe is constant. There is merely a transformation of energy. No new energy ever comes into existence nor some old energy goes out of existence. And as the effect is the causal energy transformed, it is quantitatively equal to the cause. See sec. 2 para 13.]

11. *Discuss if sequence or continuity is the essential aspect of causation.*

[Cause is essentially a motion that takes time. If there were no change in nature, the name cause would have been unintelligible. Now, the question is how are we to conceive of the nature of change that nature involves. If nature is conceived as a procession of discontinuous events, sequence becomes the essential aspect of causation. But if nature is conceived of as

a continuous process, sequence ceases to be the essential aspect of causation. Now it is often maintained, that if the cause is regarded as the temporal antecedent to the effect, nature is assumed to be a series of discontinuous, more or less impermanent events, and sequence becomes the essential aspect of causation, (see para 4 sec. 2). But nature is a continuous process. It is very difficult, if not impossible to decide where the cause ends, and the effect begins. The cause is indeed antecedent to the effect. But they are, as Mellone has said, "divided by a simple mathematical line—a line destitute of breadth—which is thrown by our thought across the current of events ; on the one side we have the cause, on the other the effect. There is no pause in reality ; the whole process is continuous ; the immediate cause comes into full action only at the very moment when the effect begins to be produced. The point to be borne in mind is the continuity of cause and effect." (Mellone, Logic, p 239].

12. Explain and illustrate, Moving power, collocation, mutuality of cause and effect, Remote cause, Kinetic and potential energy, Agent and patient, and Inertia.

[Moving power and collocation, see para 14 sec. 2. Mutuality of cause and effect :—It may be the case that there is a causal relation between two events but still it is impossible to decide which of them is the cause and which is the effect, for they determine each other. Thus, habits of industry produce wealth, and the acquisition of wealth promotes industry. Similarly education improves nature and nature facilitates education. Here we have what is called the mutuality of cause and effect.]

Remote cause : see para 11 of sec. 2.

Kinetic and potential energy :—Kinetic energy is energy possessed by matter in motion while potential energy is energy possessed by matter in position.

Agent, patient and inertia :—see para 15 sec. 2).]

13. What exactly is meant when it is said that the cause is a group of antecedents necessary to and sufficient for the effect ?

[It is meant that cause is unconditional.]

14. 'Science has no part in the search for causes'. Explain and comment. (M.U. '47).

[Science does not believe in plurality of cause and so does not attempt to find out the many causes of a phenomenon. It intends to find out the cause which undoubtedly is extremely

complex and consists of many conditions. So science attempts to find out the many conditions and the cause of the effect.]

15. *Induction is only possible on the assumption that things not only* are *together but* belong *together. Explain* (M.U.)

[Belong together, means, are causally connected.]

16. *'Plurality of causes is due to failure in analysis and disappears with scientific investigation !' Explain and discuss.* (M.U.)

17. *'Inductive reasoning assumes that reality is systematic and coherent' critically examine.* (M.U.)

[If there is no necessary connection between two events, *i.e.*, if the events are, not members of a whole that defines and is defined by their nature, there can be no induction. So induction assumes that events are members of a coherent whole, and that causation is not a subjective idea.]

18. *What is the Law of Causation ? Examine the definition of cause as the invariable and unconditional antecedent.* (M.U.)

19. *'The relation of cause and effect is not one of mere time sequence' Discuss. Indicate the nature of the causal relationship.* (M.U.)

20. *'Cause is the invariable and uncoditional antecedent'—Examine* (M.U.)

21. *Mill's conception of* cause *as the invariable and unconditional antecedent, relates to a static (wooden) universe which is wholly non-existent. Explain and comment.* (M.U.)

[The critics of Mill argue that he considered the cause to be a temporal antecedent only because he conceived of the change that nature involves as a procession of discontinuous events. But this criticism is rather mistaken. Mill undoubtedly may be accused of entertaining what may be called the alphabetical view of the universe. But,—and this is important—the conception of cause as the unconditional antecedent does not imply that nature is such a wooden affair. It is quite compatible with the view that she is a cotinuous process. See sec. 2 Para 4, and hints to Ex. 11.]

22. *What is a cause from the standpoints of* (1) *common sense,* (2) *science and* (3) *conservation of energy.*

23. *What is an effect scientifically ? Can such an effect be due to different causes at different times ?*

24. *Can the effect outlive the cause ?*

[See sec. 5 Para 2.]

CHAPTER XV
Inductive Method

§ 1. Outline of Inductive investigation: It is necessary to describe briefly the process of investigating laws of causation, not with the notion of teaching any one the Art of Discovery, which each man pursues for himself according to his natural gifts and his experience in the methods of his own science, but merely to cast some light upon the contents of the next few chapters. Logic is here treated as a process of proof; proof supposes that some general proposition or hypothesis has been suggested as requiring proof; and the search for such propositions may spring from scientific curiosity or from practical interests.

Not suggestion but proof of universal proposition is the problem of Inductive Logic.

We may, as Bain observes (*Logic*: B. iii. ch. 5), desire to detect a process of causation either (1) amidst circumstances that have no influence upon the process but only obscure it; as when, being pleased with a certain scent in a garden, we wish to know from what flower it rises; or, being attracted by the sound of some instrument in an orchestra, we desire to know which it is: or (2) amidst circumstances that alter the effect from what it would have been by the sole operation of some cause; as when the air deflects a falling feather; or in some more complex case, such as a rise or fall of prices that may extend over many years.

Bain's analysis of the conditions of suggestion.

To begin with we must form definite ideas as to what the phenomenon is that we are about to investigate; and in a case of any complexity this is best done by writing a detailed description of it: *e.g.*, to investigate the cause of a recent fall of prices,

Conditions of inductive proof of a cause:

we must describe exactly the course of the phenomenon, dating the period over which it extends, recording the successive fluctuations of prices, with their maxima and minima, and noting the classes of goods or securities that were more or less affected, *etc.*

(i) definition or detailed description of the phenomenon under investigation.

Then the first step of elimination (as Bain further observes) is "to analyse the situation mentally," in the light of analogies suggested by our experience or previous knowledge. Dew, for example, is moisture formed upon the surface of bodies from no apparent source. But two possible sources are easily suggested by common experience: is it deposited from the air, like the moisture upon a mirror when we breathe upon it; or does it exude from the bodies themselves, like gum or turpentine? Or, again, as to a fall of prices, a little experience in business, or knowledge of Economics, readily suggests two possible explanations; either cheaper production in making goods or carrying them; or a scarcity of that in which the purchasing power of the chief commercial nations is directly expressed, namely, gold.

(ii) Analysis of the situation mentally—the first step of elimination.

Having thus analysed the situation and considered the possibility of one, two, three, or more possible causes, we fix upon one of them for further investigation; that is to say, we frame an hypothesis that this is the cause. When an effect is given to find its cause, an inquirer nearly always begins his investigations by thus framing an hypothesis as to the cause.

(iii) Framing of the hypothesis, as to cause.

The next step is to try to *verify* this Hypothesis. This we may sometimes do by *varying circumstances* of the phenomenon, according to the Canons of direct Inductive Proof to be discussed in the next chapter; that is to say, by *observing* or *experimenting* in such a way as to get rid of or eliminate the obscuring or disturbing conditions. Thus, to find out which flower in a

(iv) Verification of the hypothesis—

(a) in simple

cases, by varying the circumstances—i.e., by observation and experiment.

garden gives a certain scent, it is usually enough to rely on observation, going up to the likely flowers one after the other and smelling them: at close quarters, the greater relative intensity of the scent is sufficiently decisive. Or we may resort to a sort of experiment, plucking a likely flower, as to which we frame the hypothesis (this is the cause), and carrying it to some place where the air is free from conflicting odours. Should observation or experiment disprove our first hypothesis we try a second; and so on until we succeed, or exhaust the known possibilities.

(b) in complex cases, by deduction i.e. by indirect induction.

But if the phenomenon is so complex and extensive as a continuous fall of prices, direct observation or experiment is a useless or impossible method; and we must then resort to Deduction; that is, to indirect Induction. If, for example, we take the hypothesis that the fall is due to a scarcity of gold, we must show that there is a scarcity; what effect such a scarcity may be expected to have upon prices from the acknowledged laws of prices, and from the analogy of other cases of an expanded or restricted currency; that this expectation agrees with the statistics of recent commerce: and finally, that the alternative hypothesis that the fall is due to cheaper production is not true; either because there has not been a sufficient cheapening of general production; or because, if there has been, the result to be rationally expected from it are not such as to agree with the statistics of recent commerce. (Ch. xviii.)

When a hypothesis cannot be framed, we wait for the phenomenon to

But now suppose that, a phenomenon having been suggested for explanation, we are unable at the time to think of any cause—to frame any hypothesis about it; we must then wait for the phenomenon to occur again, and, once more observing its course and accompaniments and trying to recall its antecedents, do our best to conceive an hypothesis, and

proceed as before. Thus, in the first great epidemic of influenza, some doctors traced it to a deluge in China, others to a volcanic eruption near Java; some thought it a mild form of Asiatic plague, and others caught a specific microbe. As the disease often recurred, there were fresh opportunities of framing hypothesis; and the microbe was identified.

occur again.

Again, the investigation may take a different form: given a supposed Cause to find its Effect; *e.g.*, a new chemical element, to find what compounds it forms with other elements; or, the spots on the sun—have they any influence upon our weather?

Proof of an effect may take a different form.

Here, if the given cause the under control, as a new element may be, it is possible to try experiments with it according to the Canons of Inductive Proof. The inquirer may form some hypothesis or expectation as to the effects, to guide his observation of them, but will be careful not to hold his expectation so confidently as to falsify his observation of what actually happens.

When the effect is under control, the proof is by experiment and hypothesis.

But if the cause be, like the sun-spots, not under control, the inquirer will watch on all sides what events follow their appearance and development; he must watch for consequences of the new cause he is studying in many different circumstances, that his observations may satisfy the canons of proof. But he will also resort for guidance to deduction; arguing from the nature of the cause, if anything is known of its nature, what consequences may be expected, and comparing the results of this deduction with any consequent which he suspects to be connected with the cause. And if the results of deduction and observation agree, he will still consider whether the facts observed may not be due to some other cause.

When not under control, by careful observation and deduction.

When under control but too dangerous to experiment with, by deduction, mild experiment etc.

A cause, however, may be under control and yet be too dangerous to experiment with; such as the effects of a poison—though, if too dangerous to experiment with upon man, it may be tried upon animals ; or such as a proposed change of the constitution by legislation ; or even some minor Act of Parliament, for altering the Poor Law, or regulating the hours of labour. Here the first step must be deductive. We must ask what conse quences are to be expected from the nature of the change (comparing it with similar changes), and from the laws of the special circumstances in which it is to operate ? And sometimes we may partially verify our deduction by trying experiments upon a small scale or in a mild form. There are conflicting deductions as to the probable effect of giving Home Rule to Ireland ; and experiments have been made in more or less similar cases, as in the Colonies and in some foreign countries. As to the proposal to make eight hours the legal limit of a day's labour in all trades, we have all tried to forecast the consequences of this and by way of verification we might begin with nine hours ; or we might induce some other country to try the experiment first. Still, no verification by experiments on a small scale, or in a mild form, or in somewhat similar yet different circumstances, can be considered logically conclusive. What proofs are conclusive we shall see in the following chapters.

Real Induction defined.

§ 2. Induction defined : To begin with the conditions of direct Induction.—An Induction is an universal real proposition, based on observation, in reliance on the uniformity of Nature : when well ascertained, it is called a Law. Thus, that all life depends on the presence of oxygen is (1) an universal proposition ; (2) a real one, since the 'presence of oxygen' is not connoted by 'life' ; (3) it is based on observation ; (4) it relies on the uniformity of Nature, since all cases of life have not been examined.

Such a proposition is here called 'an induction,' when it is inductively proved; that is, proved by facts, not merely deduced from more general premises (except the premise of Nature's uniformity): and by the 'proces of induction' is meant the method of inductive proof. The phrase 'process of induction' is often used in another sense, namely for the inference or judgment by which such propositions are arrived at. But it is better to call this 'the process of hypothesis,' and to regard it as a preliminary to the process of induction (that is, proof), as furnishing the hypothesis which, if it can stand the proper tests, becomes induction or law.

Hypothesis and Induction—the process of Induction.

§ 3. "PERFECT INDUCTION": Inductive proofs are usually classed as Perfect and Imperfect. They are said to be perfect when all the instances within the scope of the given proposition have been severally examined, and the propositon has been found true in each case. But we have seen (chap. xiii.§2) that the instances included in universal propositions concerning Causes and Kinds cannot be exhaustively examined: we do not know all planets, all heat, all liquids, all life, *etc.*; and we never can, since a man's life is never long enough. It is only where the conditions of time, place, etc., are arbitrarily limited that examination can be exaustive. Perfect induction might show (say) that every member of the present House of Commons has two Christian names. Such an argument is sometimes exhibited as a Syllogism in Darapti with a Minor premise in U., which legitimates a Conclusion in A., thus:

Perfect and Imperfect Induction.

A. B. to Z have two Christian names;

A. B. to Z are all the present M.P.'s:

∴ All the present M. P.'s have two Christian names. But in such an investigation there is no need of logical method to find the major premise; it is mere counting: and to carry out the syllogism is a hollow formality. Accordingly, our definition of Induction excludes the kind unfortunately called Perfect,

by including in the notion of Induction a reliance on the uniformity of Nature; for this would be superfluous if every instance in question had been severally examined. Imperfect Induction, then, is what we have to deal with: the method of showing the credibility of an universal real proposition by an examination of *some* of the instances it includes, generally a small fraction of them.

Imperfect Induction is proper Induction.

§ 4. Imperfect Induction methodical or immethodical: Imperfect Induction is either Methodical or Immethodical. Now, Method is procedure upon a principle; and if the method is to be precise and exclusive, the principle must be clear and definite.

Imperfect Induction: Methodical and Immethodical.

There is a Geometrical Method, because the axioms of Geometry are clear and definite, and by their means, with the aid of definitions, laws are deduced of the equality of lines and angles and other relations of position and magnitude in space. The process of proof is purely Deductive (the axioms and definitions being granted). Diagrams are used not as facts for observation, but merely to fix our attention in following the general argument; so that it matters little how badly they are drawn, as long as their divergence from the conditions of the proposition to be proved is not distracting. Even the appeal to "superposition" to prove the equality of magnitudes (as in Euclid I. 4), is not an appeal to observation, but to our judgment of what is implied in the foregoing conditions. Hence no inference is required from the special case to all similar ones; for they are all proved at once.

Geometry, methodical but deductive.

There is also, as we have seen, a method of Deductive Logic resting on the Principles of Consistency and the *Dictum de omni et nullo*. And we shall find that there is a method of Inductive Logic, resting on the principle of Causation.

Propositions resting on the Principles of

Consistency etc., are so. Inductions resting on Causation are inductive and methodical.

But there are a good many general propositions, more or less trustworthy within a certain range of conditions, which cannot be methodically proved for want of a precise principle by which they may be tested; and they, therefore, depend upon Immethodical Induction, that is, upon the examination of as many instances as can be found, relying for the rest upon the undefinable principle of the Uniformity of Nature, since we are not able to connect them with any of its definite modes enumerated in chap. xiii. § 7. To this subject we shall return in chap. xix., after treating of Methodical Induction, or the means of determining that a relation of events is of the nature of cause and effect, because the relation can be shown to have the marks of causation, or some of them.

Other proper Indnctions are immethodical.

§ 5. Observation and Experiment, the material ground of Induction, compared: Observations and Experiments are the *material* grounds of Induction. An experiment is an observation made under prepared, and therefore known, conditions; and, when obtainable, it is much to be preferred. Simple observation shows that the burning of the fire depends, for one thing, on the supply of air; but it cannot so us that it depends on oxygen. To prove this we must make experiments; as by obtaining pure oxygen and pure nitrogen (which, mixed in the proportion of one to four, form the air) in separate vessels, and then plunging a burning taper into the oxygen—when it will blaze fiercely; and again plunging it into the nitrogen—when it will be extinguished. This shows that the greater part of the air does nothing to keep the fire alight, except by diminishing its intensity and so making it last longer. Experiments are more perfect the more

Observation and experiment—material grounds of Induction.

Their nature defined.

Observation cannot, but experiment can, prove cause.

carefully they are prepared, and the more completely the conditions are known under which the given phenomenon is to be observed. Therefore, they become possible only when some knowledge has already been gained by observation; for else the preparation which they require could not be made.

Experiments depend on observation.

Observation, then, was the first material ground of Induction, and in some sciences it remains the chief ground. The heavenly bodies, the winds and tides, the strata of the earth, and the movements of history, are beyond our power to experiment with. Experiments upon the living body or mind are indeed resorted to when practicable, even in the case of man, as now in all departments of Psychology; but, if of a grave nature, they are usually thought unjustifiable. And in political affairs experiments are hindered by the reflection, that those whose interests are affected must bear the consequences and may resent them. Hence, it is in physical and chemical inquiries and in the physiology of plants and animals (under certain conditions) that direct experiment is most constantly practised.

Observation the first material ground.

Its merits

Where direct experiment is possible, however, it has many advantages over unaided observation. If one experiment does not enable us to observe the phenomenon satisfactorily, we may try again and again; whereas the mere observer, who wishes to study the bright spots on Mars, or a commercial crisis, must wait for a favourable opportunity. Again, in making experiments we can vary the conditions of the phenomenon, so as to observe its different behaviour in each case; whereas he who depends solely on observation must trust the bounty of nature to supply him with a suitable diversity of instances. It is a particular advantage of experiment that a phenomenon may sometimes be 'isolated,' that is, removed from the influence of all agents

Experiment: its merits and advantages.

except that whose operation we desire to observe, or except those whose operation is already known: whereas a simple observer, who has no control over the conditions of the subject he studies, can never be quite sure that its movements or changes are not due to causes that have never been conspicuous enough to draw his attention. Finally, experiment enables us to observe coolly and circumspectly and to be precise as to what happens, the time of its occurrence, the order of successive events, their duration, intensity and extent.

But whether we proceed by observation or experiment, the utmost attainable exactness of measurements and calculation is requisite; and these presuppose some Unit, in multiples or divisions of which the result may be expressed. This unit cannot be an abstract number as in Arithmatic, but must be one something—an hour, or a yard, or a pound—according to the nature of the phenomenon to be measured. But what is an hour, or a yard or a pound? There must in each case be some constant Standard of reference to give assurance that the unit may always have the same value. "The English pound is defined by a certain lump of platinum preserved at Westminister." The unit may be identical with the standard or some division or multiple of it; and, in measuring the same kind of phenomena, different units may be used for different purposes as long as each bears a constant relation to the standard. Thus, taking the rotation of the earth as the standard of Time, the convinient unit for long periods is a year (which is a multiple); for shorter periods, a day (which is identical); for shorter still, an hour (which is a division), or a second, or a thousandth of a second. (See Jevons' *Principles of Science*, ch. 14.).

Exact measurement necessary everywhere.

§ 6. THE PRINCIPLE OF CAUSATION IS THE FORMAL GROUND OF INDUCTION: The principle of Causation is the *formal* ground of Induction; and the Inductive Canons derived from it are means of testing the formal sufficiency of observations

Principle of causation the formal ground of Induction. Inductive canons derived from it.

to justify the statement of a Law. If we can observe the process of cause and effect in nature we may generalise our observation into a law, because that process is invariable. First, then, can we observe the course of cause and effect? Our power to do so is limited by the refinement of our senses aided by instruments, such as lenses, thermometers, balances, *etc.* If the causal process is essentially molecular change, as in the maintenance of combustion of oxygen, we cannot directly observe it; if the process is partly cerebral or mental, as in social movements which depend on feeling and opinion, it can but remotely be inferred; even if the process is a collision of moving masses (billiard-balls), we cannot really observe what happens, the elastic yielding, and recoil and the internal changes that result; though no doubt photography will throw some light upon this, as it has done upon the galloping of horses and the impact of projectiles. Direct observation is limited to the effect which any change in a phenomenon (or its index) produces upon our senses; and what we believe to be the causal process is a matter of inference and calculation. The meagre and abstract outlines of Inductive Logic are apt to foster the notion, that the evidence on which Science rests is simple; but it is amazingly intricate and cumulative.

On causation generalisation depends. But direct observation of cause and effect very limited.

Secondly, so far as we can observe the process of nature, how shall we judge whether a true causal instance, a relation of cause and effect, is before us? By looking for the five marks of Causation. Thus, in the experiment above descrived, showing that oxygen supports combustion, we find—(1) that the taper which only glowed before being plunged into the oxygen, bursts into flame when there —Sequence; (2) that this begins to happen at once without

Again, only the five marks of cansation can be observed and proved.

perceptible interval—Immediacy; (3) that no other agent or disterbing circumstance was present (the preparation of the experiment having excluded any such thing)—Unconditionalness; (4) the experiment may be repeated as often as we like with the same result—Invariableness. Invariableness, indeed, I do not regard as formally necessary to be shown, supposing the other marks to be clear; for it can only be proved within our experience; and the very object of Induction is to find grounds of belief beyond actual experience. However, for material assurance, to guard against his own liability to error, the inquirer will of course repeat his experiments.

Invariableness of course, not formally necessary to be shown. The inquirer should repeat the experiments.

The above four are the qualitative marks of Causation: the fifth and quantitative mark is the Equality of Cause and Effect; and this, in the above example, the Chemist determines by showing that, instead of the oxygen and wax that have disappeared during combustion, an equivalent weight of carbon dioxide, water, *etc.*, has been formed.

Here, then, we have all the marks of causation; but in the ordinary judgments of life, in history, politics, criticism, business, we must not expect such clear and direct proofs; in subsequent chapters it will appear how different kinds of evidence are combined in different departments of investigation.

§ 7. The Inductive Cannons are derived from the principle of Causatian, the more readily to detect it in facts observed: The Inductive Canons, to be explained in the next chapter, describe the character of observations and experiments that justify us in drawing conclusions about causation; and, as we have mentioned, they are derived from the principle of Causation itself. According to that principle, cause and effect are invariably, immediately and unconditionally antecedent and consequent, and are equal as to the matter and energy embodied.

Determination of the unconditionality of cause, the great object of the Inductive Methods.

Invariability can only be observed, in any of the methods of induction, by collecting more and more instances, or repeating experiments. Of course it can never be exhaustively observed.

Immediacy, too, in direct Induction, is a matter for observation the most exact that is possible.

Succession, or the relation itself of antecedent and consequent, must either be directly observed (or some index of it); or else ascertained by showing that energy gained by one phenomenon has been lost by another, for this implies succession.

But to determine the unconditionality of causation, or the indispensability of some condition, is the great object of the methods, and for that purpose the meaning of unconditionality may be further explicated by the following rules for the determination of a Cause.

A. Qualitative Determination

I.—For Positive Instances.

To prove a supposed Cause: (*a*) Any agent whose introduction among certain conditions (without further change) is followed by a given phenomenon; or, (*b*) whose removal is followed by the cessation (or modification) of that phenomenon, is (so far) the cause or an indispensable condition of it.

Meaning of unconditionality further explicated.

To find the Effect: (*c*) Any event that follows a given phenomenon, when there is no further change; or, (*d*) that does not occur when the conditions of a former occurrence are exactly the same, except for the absence of that phenomenon, is the effect of it (or is dependent on it).

II.—For Negative Instances.

To exclude a supposed Cause: (*a*) Any agent that can be introduced among certain conditions without being followed by a given phenomenon (or that is found without that phenomen-

on); or (*b*) that can be removed when that phenomenon is present without impairing it (or that is absent when that phenomenon is present), is not the cause, or does not complete the cause, of that phenomenon in those circumstances.

To exclude a supposed Effect: (*c*) Any event that occurs without the introduction (or presence) of a given phenomenon or (*d*) that does not occur when that phenomenon is introduced (or is present), is not the effect of that phenomenon.

Subject to the conditions thus stated, the rules may be briefly put as follows:

I. (*a*) That which (without further change) is followed by a given event is its cause.

II. (*a*) That which is not so followed is not the cause.

I. (*b*) That which cannot be left out without impairing a phenomenon is a condition of it.

II. (*b*) That which can be left out is not a condition of it.

B. Quantitative Determination

The Equality of Cause and Effect may be further explained by these rules:

III. (*a*) When a cause (or effect) increases or decreases, so does its effect (or cause).

III. (*b*) If two phenomena, having the other marks of cause and effect, seem unequal, the less contains an unexplored factor.

III. (*c*) If an antecedent and consequent do not increase or decrease correspondingly, they are not cause and effect, so far as they vary.

It will next be shown that these propositions are variously combined in Mill's five Canons of Induction: Agreement, the Joint Method, Difference, Variations, Residues. The first three are sometimes called Qualitative Methods, and the two last Quantitative; and although this grouping is not quite accurate,

seeing that Difference is often used quantitatively, yet it draws attention to an important distinction between a mere description of conditions and determination by exact measurement.

To avoid certain misunderstandings, some slight alterations have been made in the wording of the Canons. It may seem questionable whether the Canons add anything to the above propositions: I think they do. They are not discussed in the ensuing chapter merely out of reverence for Mill, or regard for a nascent tradition; but because, as describing the character of observations and experiments that justify us in drawing conclusions about causation, they are guides do the analysis of observations and to the preparation of experiments. To many eminent investigators the Canons (as such) have been unknown; but they prepared their work effectively so far only as they had definite ideas to the same purport. A definite conception of the conditions of proof is the necessary antecedent of whatever preparations may be made for proving anything.

SUMMARY

The inductive sciences discover and prove universal propositions. This discovery is rather the work of insight and springs from scientific curiosity or practical iterests, and hence the conditions of it cannot be the subject-matter of a science. But the conditions of proof can be so, and the problem of Inductive Logic is the problem of the proof of universal propositons. Now, when a causal relation is established, a universal proposition is said to be proved. And hence the question arises—what are the conditions of the inductive proof of a cause?

The first step of the inductive proof of a cause is definiton or detailed description of the phenomenon under investigation. The second step is the analysis of the situation mentally in the light of analogies suggested by our experience or previous knowledge. (This is the first step of elimination). The third step of inductive proof is the framing of the hypothesis as to cause while the fourth step is to try to varify this hypothesis. This

may sometimes be done by varying the circumstances of the phenomenon according to the canons of direct Inductive proof, *i,e.*, by observing or experimenting in such a way as to get rid of or eliminate the obscuring or disturbing conditions. But if the phenomenon is so complex and extensive as a continuous fall of prices, direct observation or experiment is a useless or impossible method; and we must resort to Deduction; that is to indirect Induction. Now, when a phenomenon having been suggested for explanation we are unable at the time to think of any cause—to frame any hypothesis about it, we must wait for the phenomenon to occur again and once more observing its course and accompaniments and trying to recall its antecedents, do our best to conceive an hypothesis and proceed as before.

Now, the proof of an effect may take a different form. If the given cause be under control it is possible to try experiments with it according to the canons of Inductive Proof. But if the cause be like the sun-spots, not under control, the inquirer will watch on all sides what events follow their appearance and development. He will also resort for guidance to deduction. And if the results of deduction and observation agree, he will still consider whether the facts observed may not be due to some other cause.

Again, the cause may be under control and yet be too dangerous to experiment with. Here we must resort to deduction and mild experiment.

Induction is a universal real proposition based on observation in reliance on the uniformity of nature; when well ascertained it is called a Law. Thus, that all life depends on the presence of oxygen is (1) an universal proposition; (2) a real one, since the presence of oxygen is not connoted by life; (3) it is based on observation; (4) it relies on the uniformity of nature since all cases of life have not been examined.

Now, Inductive proofs are usually classed as Perfect and Imperfect. They are said to be perfect when all the instances within the scope of the given proposition have been severally examined. But the instances included in universal propositions concerning causes and kinds cannot be exhaustively examined. It is only where the conditions of time, place etc., are arbitrarily limited that examination can be exhaustive. Accordingly our definition of Induction excludes the kind, unfortunately called perfect, by including in the notion of Induction a reli-

ance on the uniformity of nature. Imperfect Induction then is what we have to deal with.

Imperfect induction is either methodical or immethodical. The geometrical method is methodical but deductive. Propositions resting on the principles of consistency and the *dictum de omni et nullo* are also methodical but deductive. Inductions resting on causation are inductive and methodical while the other proper inductions are immethodical, though inductive.

Observations and experiments are the material grounds of induction. An experiment is an observation made under prepared and therefore known conditions and when obtainable, it is much to be preferred. For, while simple observation cannot, experiment can prove causation. Experiments are more perfect, the more carefully they are prepared and the more completely the conditions are known under which the given phenomenon is to be observed. So observation is the first material ground of Induction, and in some sciences, it remains the chief ground. But where direct experiment is possible it has many advantages over unaided observation. Thus, experiment enables us to observe the phenomenon as many times and from as many points of view as we like. Again, in making experiments we can vary the conditions of the phenomenon, so as to observe its different behaviour in each case. Further, it is a particular advantage of experiment, that a phenomenon may sometimes be 'isolated.' Finally, experiment enables us to observe coolly and circumspectly, and to be precise as to what happens. But whether we proceed by observation or experiment, the utmost exactness of measurements and calculation is requisite.

The principle of causation is the formal ground of Induction and the Inductive canons are derived from it.

Now, though on causation generalisation depends yet direct observation of cause and effect is very limited. If the causal process is essentially molecular change, we cannot directly observe it ; if the process is partly cerebral or mental we can but remotely infer it; even if the process is a collision of moving masses, we cannot really observe what happens. Direct observation is limited to the effect which any change in a phenomenon produces upon our senses. The evidence on which science rests is not simple but amazingly intricate and cumulative.

Again, only the five marks of causation can be observed and proved, though of course, invariableness is not formally neces-

sary to be shown. The great object of the Inductive methods is the determination of the unconditionality of the cause.

EXERCISES WITH HINTS

1. *Is perfect Induction really a form of Induction?* (C. U. '52). [See sec. 3 and hints to Ex. 3].

2. *What are the processes simulating induction? Why are they not Induction proper?*

[According to Mill "Induction is the process by which we conclude that what is true of certain individuals of a class is true of the whole class or that what is true at certain times will be true in similar circumstances at all times." That is, "Induction is a process of inference; it proceeds from the known to the unknown; and any operation involving no inference, any process in which what seems the conclusion is no wider than the premises from which it is drawn does not fall within the meaning of the term." So what are called perfect Induction, and several processes used in mathematics and colligation of facts are not inductions proper but processes that simulate Induction.

For perfect Induction see sec. 3.

Several processes used in mathematics:—There are several processes used in mathematics which pretend to be, but are not inductions proper. "For example when we have proved with respect to the circle, that a straight line cannot meet it in more than two points and when the same thing has been successively proved of the elipse, the parabola and the hyperbola it may be laid down as an universal property of the sections of the cone." "It would be difficult to refuse to the proposition arrived at the name of a generalisation, since there is no room for any generalisation beyond it. But there is no induction because there is no inference: the conclusion is a mere summing up of what was asserted in the various propositions from which it is drawn."

"A case somewhat similar is the proof of a geometrical theorem by means of a diagram. Whether the diagram be on the paper or only in imagination, the demonstration does not prove directly the general theorem; it proves only that the conclusion is true of the particular triangle or circle exhibited in the diagram; but since we perceive that in the same way in which we

have proved it of that circle, it might also be proved of any other circle, we gather up into one general expression all the singular propositions susceptible of being thus proved and embody them in all universal proposition." Here therefore, "the characteristic quality of Induction is wanting, since the truth obtained, though really general, is not believed on the evidence of particular instances. We do not conclude that all triangles have the property because some triangles have, but from the ulterior demonstrative evidence which was the ground of our conviction in the particular instances."

Similarly what is called Mathematical Induction is not induction proper. That is, when a mathematician after calculating a sufficient number of the terms of the algebrical or arithmetical series ascertains the law of the series and unhesitatingly fills up any number of the succeeding terms without repeating the calculations, he does not infer inductively. For "he only does so when it is apparent from a *priori* considerations, that the mode of formation of subsequent terms, each from that which preceded it, must be similar to the formation of the terms which have been already calculated." This also is a case of induction by parity of reasoning. (See also sec. 4).

Colligation of Facts : It is a mere description by general terms of a set of observed facts. When there is a phenomenon consisting of parts that are capable of being separately observed, there is a convenience in obtaining a representation of the whole by colligating or piecing together the separate observations of the parts. And so, such piecing together is often done in daily life and in science. Such piecing together is called Colligation of Facts. "A navigator sailing in the midst of the ocean discovers land; he cannot at first or by one observation determine whether it is a Continent or an island; but he coasts along it and after a few days finds himself to have sailed completely round it: he then pronounces it an island." This is an example of Colligation of facts. And according to Mill, this is not a case of induction, for the chief characteristic mark of induction *viz.*, the passage from the known to the unknown is wanting here. As for example, in the case of the sailor "there was no particular time or place of observation at which he could perceive that this land was entirely surrounded by water; he ascertained the fact by a succession of partial observations, and then selected a general expression which summed up in two or three words, the whole of what he so observed. But is there anything of the na-

ture of an induction in this process? Did he infer anything that had not been observed, from something else which had? Certainly not. He had observed the whole of what the proposition asserts. That the island in question is an island, is not an inference from the partial facts which the navigator saw in the course of his circumnavigation; it is the facts themselves; it is a summary of those facts; the description of a complex fact, to which the simpler ones are as the parts of a whole." So though Whewell thinks that the colligation of facts is a real proposition and so induction proper, Mill considers it to be nothing more than a mere description by general terms. But still Mill admits that "there is between Colligation and Induction a real correlation. Colligation is not always induction; but Induction is always Colligation."

Some contemporary logicians like Joseph believe that Whewell was right in having taken colligation of facts to be a real proposition i.e. not merely a binding together of facts but a mental rearrangement of them, but still Mill is right so far as he contends that it is not induction proper. For though colligation of facts is generalisation, yet it cannot be called *proof*. It is a hypothesis merely, which to be elevated to the rank of induction, ought to be proved. Or as Mill puts it, "there is such a thing as proof and inductions differ altogether from descriptions in their relation to that element. Induction is proof; it is inferring something unobserved from something observed; it requires therefore an appropriate test of proof. When we merely collate known observations, and in Dr. Whewell's phraseology connect them by means of a new conception; if the conception does serve to connect the observations we have all we want. It neither needs nor admits of proof." So Dr. Whewell fails to distinguish between two important things *viz.*, invention and proof. "Invention may be required in any operation, but is the essence of none. A new conception may be introduced for descriptive purposes, and so it may be for inductive purposes. But it is so far from constituting induction, that induction does not necessarily stand in need of it."]

N.B. The argument as in this last paragraph is more important and to the point than what has been stated before.

3. "*Perfect induction rests upon the principle of uniformity of nature and hence is a form of real induction.*"—Discuss.

[Dr. P. K. Roy holds such a view. According to him when

we argue that all the known planets shine by sun's light we have a case of perfect Induction. And here we rely upon the principle of uniformity of nature and pass from the known to the unknown in as much as we mean that the planets in future will shine by the sun's light. Now, this view is mistaken. For, pure perfect Induction has nothing to do with future. It is concerned only with what has so far been observed. We can say that the planets in future will shine by sun's light, only when we have established a necessary connection between being a planet and shinning by sun's light, and so go beyond perfect Induction.]

4. *Is perfect induction without any scientific value? Can it establish a necessary proposition? Is the conclusion of Perfect Induction really universal?*

[*Value of Perfect Induction*: Perfect induction is not without any scientific value. It is as useful as most form of abridged notations are. As Jevons puts it, "it is of great importance and requires to be continually used in science and common life. The power of expressing a great number of particular facts in a very brief space is essential to the progress of science. Perfect Induction is absolutely necessary to enable us to deal with a great number of particular facts in a very brief space." Indeed it is as Johnson says, a summary induction, and is as useful as any summary is.

Necessary proposition: Jevons thinks that Perfect Induction gives a necessary or certain conclusion. But this is not true. It can give us a true conclusion, but it can never give us a necessary conclusion. For enumeration cannot yield necessity. As Grumley puts it: "even when counting is complete it does not give us scientific knowledge. A characteristic that is found to belong to every individual of a class may be no more than an inseparable accident."

Universal Conclusion: The conclusion of Perfect Induction is neither a universal real proposition nor really a universal proposition. It is not a universal real proposition as here there is no passage from the known to the unknown. It is not really a universal proposition as it is nothing but an abridged form of some singular propositions. The summary of the particulars does not make the universal, and so the universal proposition is not the summary of some singular propositions. Hence the universal proposition is a pseudo-universal.]

5. *What is meant by 'material' and 'formal' grounds of Induction? Distinguish between them.* (C.U. '47).

[The principle of casual uniformity is the formal ground of Induction. It determines the form of Induction. That is, in induction we pass from some to all, and this passage from some to all is due to our belief in the principle of casual uniformity. If we had not believed in this principle, it woul not have been possible for us to pass from some to all. But, this principle is merely the formal ground. That is, it is what may be called the negative condition of Inductive inference. If we do not believe in it, we cannot pass from some to all. But the mere belief in it, does not enable us to infer inductively. Because we believe in it, we cannot say that all life depends on oxygen. What we can say is that if life depends on oxygen today, it will always do so under similar circumstances. Now, if life depends on oxygen today is to be ascertained by means of observation and experiment. So the matter of induction is supplied by observation and experiment, and they constitute the material grounds of induction.]

6. *Explain the advantages and disadvantages of Observation and Experiment in investigation.* (C.U. '47).

[See sec. 5.]

7. *Explain observation and experiment. Is the distinction between them absolute?* (C.U. '49).

[See sec. 5 and add: Many logicians think that there is a qualitative distiction between observation and experiment. Thus some of them maintain that while observation is passive experiment is active. But this statement is sufficiently vague and resists all comment. To comment upon it we should first of all find out the precise sence of it. It may thus mean that in observation we depend upon nature, but in experiment we ourselves prepare the event. That is, in mere obvervation we do not prepare the event, the event is prepared, so to say, in the laboratory of nature. But in experiment we ourselves prepare the event and observe it. Now if this be the sense of the statement, the contention that the distinction between observatson and experiment is qualitative or absolute turns out to be inadequate, as experiment is still then a particular kind of observation. Similarly, if the statement means, that in observation, we do not control the circumstances and so are passive, while in experiment we control the circumstances, and so are active, the statement becomes quite legitimate but fails to support the view

that the distinction between observation and experiment is absolute. Indeed, no body intends to deny the truth of the statement that observation is rather finding a fact while experiment is making one. But,—and this is important—this does not justify us in holding that observation and experiment differ qualitatively. If we are to hold such a view, we must declare that in mere observation the mind is passive. But mind in mere observation is not passive. For it involves selection and rejection, classification and definition, articulation and judgment—in short interpretation. So mere observation is not passive, and consequently the distinction between observation and experiment is not one of kind but of degree. Of course, it may be granted, that this difference in degree in some cases is so great, that almost qualitative difference emerges, and so while experiment can give certain conclusions, observation cannot.]

8. *Enunciate the Law of Causation and the principle of uniformity of nature. Why are these two regarded as constituting the formal ground of induction ?* (C.U. '51).

9. *Explain the nature of observation and experiment and determine their place in scientific knowledge.* (C.U. '46).

10. *Why is it said that the principle of causation is the formal ground of induction ?* (C.U. '44).

11. *Discuss the value of Perfect Induction and show how colligation of facts falls short of induction trully so called ?* (C.U. '43).

12. *Observation and experiment are said to be meterial grounds of Induction. Explain what precisely you understand by this and discuss the relative value of observation and experiment.* (C.U. '43).

13. *Distinguish between perfect and imperfect induction. Is it true to say that perfect Inductions have no scientific value whatsoever ?* (C.U. '42).

14. *"The distinction between experiment and observation is never absolute." Discuss. What are the advantages of experiment over observation ?* (C.U. '42).

15. *Distinguish between Formal and Material grounds of Induction and examine the view that the ground itself is an Induction.* (C.U. '41).

16. *Discuss fully the statement that observation and experiment are aids to elimination.* (C.U. '41).

17. *What are inductions improperly so called ?* (C.U. '40)
[See hints to Ex. 3.]

18. *Is it correct to say that we use observation for the purpose of description and experiment for that of explanation ?* (M.U. '47).

[As there is no qualitative distinction between observation and experiments, so this way of stating the relation between the two is unsatisfactory. But, still it must be admitted that while experiment can, observation cannot exhibit the necessary connection between two phenomena. Consequently, observation at best can only describe, and thereby suggest a highly probable explanation.]

19. *What is the goal of Induction ? Is it complete enumeration ? Is the value of piling up instances anything more than rhetorical ?* (M.U. '47).

[Enumeration has no necessary connection with Scientific Induction. One observation can establish a universal real proposition while thousands of observations may fail to do this. So scientific induction is concerned with the nature and not with the number of instances. But still repeated observation is necessary. The value of piling up instances is not merely rhetorical. Its real significance lies in demonstrating that the postulated cause is not non-unconditional or non-invariable. As is maintained in Indian Logic, the purpose of repeated ovservation (*bhuyodarsanā*) is to nullify the possibility of contary instances *vyabhicārāsānkā*).

20. *"Observation and experiment are aids to the elimination of the errelevant"*.—Explain how. (M.U. '48).

[We usually eliminate the irrelevant by varying the circumstances. But to vary the circumstances is to overve, either by mere ovservation or by experiment, the event in different circumstances. So observation and experiment are aids to elimination. But they are not merely this. They are also the material grounds of induction.]

21. *What is observation ? What are the coditions of good observation ? Does observation imply selection and interpritation ?*

[The mind is active even in mere observation. If we do not attend, we cannot observe. While we observe nature, we also interrogate her. That the mind is active in observation, is shown by the fact, that an observation may become erroneous. Again, observation involves selection and interpritation. Two men of different taste or interest may look at the same thing and yet they will not observe the same thing. Further it in-

volves classification. Hence observation is not a mere looking at facts. And there can be good obvervation only when the mind is free from anxiety, fear and such other states, and the physical conditions like light are good. Moreover, the observer must have the moral courage of obesrving and recording even those instances that may disprove his theories.]

22. *Is generalisation the only business of natural science ?*

[No ; the science not only generalise but also verify the generalisations.]

23. *Explain the inductive process and illustrate the different steps involved.*

[See secs. 2—4.]

CHAPTER XVI

THE CANONS OF DIRECT INDUCTION

§ 1. The Canon of Agreement : Let me begin by borrowing an example from Bain (*Logic* : B. III. *c.* 6). The North-East wind is generally detested in this country ; as long as it blows few people feel at their best. Occasional well-known causes of a wind being injurious are violence, excessive heat or cold, excessive dryness or moisture, electrical condition, the being laden with dust or exhalations. Let the hypothesis be that the last is the cause of the North-East wind's unwholesome quality ; since we know it is a ground current setting from the pole toward the equator and bent westward by the rotation of the earth ; so that, reaching us over thousands of miles of land, it may well be fraught with dust, effluvia, and microbes. Now, examining many cases of North-East wind, we find that this is the only circumstance in which all the instances agree : for it is sometimes cold, sometimes hot ; generally dry, but sometimes wet ; sometimes light, sometimes violent, and of all electrical conditions. Each of the other circumstances, then, can be omitted without the N. E. wind ceasing to be noxious ; but one circumstance is never absent, namely, that it is a ground current. That circumstance, therefore, is probably the cause of its injuriousness. This case illustrates :—

A case that exemplifies the canon of agreement.

(1) The Canon of Agreement.

The Canon :

If two or more instances of a phenomenon under investigation have only one other circumstance (antecedent or consequent) in common, that circumstance is probably the cause (or an indispensable condition) or the effect of the phenomenon, or is connected with it by causation.

This rule of proof (so far as it is used to establish direct causation) depends, first, upon observation of an invariable connection between the given phenomenon and one other cir-

Proof depends upon (i) observation of invariable connection and (ii) unconditionality of the causation.

cumstance; and, secondly, upon I. (*a*) and II. (*b*) among the propositions obtained from the unconditionality of causation at the close of the last chapter.

To prove that A is causally related to *p*, suppose two instances of the occurrence of A, an antecedent, and *p*, a consequent, with concomi tant facts or events—and let us represent them thus:

An instance

Antecedents: ABC ADE
Consequents: *p q r* *p s t*;

and suppose further that, in this case, the immediate succession of events can be observed. Then A is probably the cause, or an indispensable condition, of *p*. For, as far as our instances go, A is the invariable antecedent of *p*; and *p* is the invariable consequent of A. But the two instances of A or *p* agree in no other circnmstance. Therefore A is (or completes) the unconditional antecedent of *p*. For B and C are not indispensable conditions of *p*, being absent in the second instance [Rule II. (*b*)]; nor are D and E, being absent in the first instance. Moreover, *q* and *r* are not effects of A, being absent in the second instance [Rule II. (*d*)]; nor are *s* and *t*, being absent in the first instance.

Cogency of the proof depends upon showing unconditionality of the causal sequence.

It should be observed that the cogency of the proof depends entirely upon its tending to show the unconditionality of the sequence A-*p*, or the indispensability of A as a condition of *p*. That *p* follows A, even immediately, is nothing by itself: if a man sits down to study and, on the instant, a hand-organ begins under his window, he must not infer malice in the musician: thousands of things follow one another every moment without traceable connection; and this we call 'accidental.' Even invariable sequence is not enough to prove direct causation; for, in our experience does not night invariably follow day? The proof requires that the instances be such as to show not merely what events *are* in

invariable sequence, but also what *are not.* From among the occasional antecedents of p (or consequents of A) we have to eliminate the accidental ones. And this is done by finding or making 'negative instances' in respect of each of them. Thus the instance $\begin{matrix} A & D & E \\ p & s & t \end{matrix}$ is a negative instance of B and C considered as supposable causes of p (and of q and r as supposable effects of A); for it shows that they are absent that they are absent when p (or A) is present.

And hence the importanec of 'negative instances'

To insist upon the cogency of 'negative instances' was Bacon's great contribution to Inductive Logic. If we neglect them, and merely collect examples of the sequence A-p, this is 'simple enumeration'; and although simple enumeration, when the instances of agreement are numerous enough, may give rise to a strong belief in the connection of phenomena, yet it can never be a methodical or logical proof of causation, since it does not indicate the unconditionalness of the sequence. For simple enumeration of the sequence A-p leaves open the possibility that, besides A, there is always some other antecedent of p, say X; and then X may be the cause of p. To disprove it, we must find, or make, a negative instance of X—where p occurs, but X is absent.

Bacon & 'negative instances'

Plurality of causes renders the Method of Agreement unsatisfactory.

So far as we recognise the possibility of a plurality of causes, this method of Agreement cannot be quite satisfactory. For then, in such instances as the above, although D is absent in the first, and B in the second, it does not follow that they are not the causes of p; for they may be alternative causes: B may have produced p in the first instance, and D in the second; A being in both cases an accidental circumstance in relation to p. To remedy this shortcoming by the method of Agreement itself, the only course is to find more instances of p. We may never find a negative instance of A; and, if not, the probability that A is the cause of p increases with the

number of instances. But if there be no antecedent that we cannot sometimes exclude, yet the collection of instances will probably give at last all the causes of p; and by finding the proportion of instances in which A, B, or X preceeds p, we may estimate the probability of any one of them being the cause of p in any given case of its occurrence.

The Possibility of Plurality of Causes can be excluded by stricter definition of the effect.

But this is not enough. Since there cannot really be vicarious causes, we must define the effect (p) more strictly, and examine the cases to find whether there may not be varieties of p, with each of which one of the apparent causes is correlated: A with p^{1}, B with p^{11}, X with^{111}. Or, again, it may be that none of the recognised antecedents is effective: as we here depend solely on observation, the true conditions may be so recondite and disguished by other phenomena as to have escaped our scrutiny. This may happen even when we suppose that the chief condition has been isolated: the drinking of foul water was long believed to cause dysentery, because it was a frequent antecedent; whilst observation had over looked the bacillus, which was the indispensable condition.

Immediate sequence often eludes observation.

Again, though we have assumed that, in the instances supposed above, immediate sequence is observable, yet in many cases it may not be so, if we rely only on the canon of Agreement; if instances cannot be obtained by experiment and we have to depend on observation. The phenomena may then be so mixed together that A and p seem to be merely concomitant; so that, though connection of some sort may be rendered highly probable, we may not be able to say which is cause and which is effect. We must then try (as Bain says) to trace the expenditure of energy: if p gains when A loses, the course of events is from A to p.

Again, the Method of Agree-

Moreover, where succession cannot be traced, the method of Agreement may point to a connection between two or more facts (perhaps as co-effects of a re-

mote cause) where direct causation seems to be out of the question : *e.g.*, that Negroes, though of diferent tribes, different localities, customs, *etc.*, are prognathous, woolly-haired and dolichocephalic.

ment points to non-casual co-existence event.

The Method of Agreement, then, cannot by itself prove causation. Its chief use (as Mill says) is to suggest hypotheses as to the cause ; which must then be used (if possible) experimentally to try if it produces the given effect. A bacillus, for example, being always found with a certain disease, is probably the chief condition of it: give it to a guinea-pig, and observe whether the disease appears in that animal.

Thus, its chief use is only to suggest hypothesis.

Men often use arguments which, if they knew it, might be shown to conform more or less to this canon ; for they collect many instances to show that two events are connected ; but usually neglect to bring out the negative side of the proof ; so that their arguments only amount to simple enumeration. Thus Ascham in his *Toxophilus*, insisting on the national importance of archery, argues that victory has always depended on superiority in shooting ; and, to prove it, he shows how the Parthians checked the Romans, Sesostris conquered great part of the known world. Tiberius overcame Arminius, the Turks established their empire, and the English defeated the French (with many like examples)—all by superior archery. But having cited these cases to his purpose, he is content ; whereas he might have greatly strengthened his proof by showing how one or the other instance excludes other possible causes of success. Thus : the cause was not discipline, for the Romans were better disciplined than the Parthians ; nor yet the boasted superiority of a northern habitat, for Sesostris issued from the south ; nor better manhood, for here the Germans probably had the advantage of the Romans ; nor superior civilisation, for the Turks were less civilised than most of those they conquered ; nor numbers, nor

Neglect of instances often reduces the argument almost to Simple Enumeration.

even a good cause, for the French were more numerous than the English, and were shamefully attacked by Henry V. on their own soil. Many an argument from simple enumeration may thus be turned into an induction of greater plausibility according to the Canon of Agreement.

Still, in the above case, the effeet (victory) is so vaguely conceived, that a plurality of causes must be allowed for: although, *e.g.*, discipline did not enable the Romans to conquer the Parthians, it may have been their chief advantage over the Germans; and it was certainly important to the English under Henry V. in their war with the French.

Another example to prove the same point:

Here is another argument, somewhat similar to the above, put forward by H. Spencer with a full consciousness of its logical character. States that make war the chief object, he says, assume a certain type of organisation, involving the growth of the warrior class and the treatment of labourers as existing solely to sustain the warriors; the complete subordination of individuals to the will of the despotic soldier-king, their property, liberty and life being at the service of the State; the regimentation of society not only for military but also for civil purposes; the suppression of all private associations, *etc.* This is the case in dahomey and in Russia, and it was so at Sparta, in Egypt, and in the empire of the Yncas. But the similarity of organisation in these States cannot have been due to race, for they are all of different races; nor to size, for some are small, some large; nor to climate or other circumstances of habitat, for here again they differ widely: the one thing they have in common is the military pursose; and this, therefore, must be the cause of their similar organisation. (*Political Institutions.*)

By this method, then, to prove that one thing is causally connected with another, say A with p, we show, first, that in all instances of p, A is present; and, secondly, that any other supposable cause of p may be absent without disturbing p. We next come to a method the use of which greatly strengthens the fore-

going, by showing that where p is absent A is also absent, and (if possible) that A is the only supposable cause that is always absent along with p.

§ 2. The Canon of the Joint Method of Agreement in Presence and in Absence: *If* (1) *two or more instances in which a phenomenon occurs have only one other circumstance (antecedent or consequent) in common, while* (2) *two or more instances in which it does not occur (though in important points they resemble the former set of instances) have nothing else in common save the absence of that circumstance—the circumstance in which alone the two sets of instances differ throughout (being present in the first set and absent in the second) is probably the effect, or the cause, or an indispensable condition of the phenomenon.* *The Canon*

The first clause of this Canon is the same as that of the method of Agreement, and its significance depends upon the same propositions concerning causation. The second clause, relating to instances in which the phenomenon is absent, depends for its probative force upon Prop. II. (a), and I. (b): its function is to exclude certain circumstances (whose nature or manner of occurrence gives them some claim to consideration) from the list of possible causes (or effects) of the phenomenon investigated. It might have been better to state this second clause separately as the Canon of the Method of Exclusions. *Three clauses of the Canon*

To prove that A is causally related to p, let the two sets of instances be represented as follows: *Elucidation of the Canon*

Instances of Presence.	Instances of Absence.
A B C	C H F
p q r	r x v
A D E	B D K
p s t	q y s
A F G	E G M
p u v	t f u

Then A is probably the cause or a condition of p, or p is dependent upon A: first, by the Canon of Agreement in Pre-

sence, as represented by the first set of instances; and, secondly, by Agreement in absence in the second set of instances. For there we see that C, H, F, B, D, K, E, G, M occur without the phenomenon p, and therefore (by Prop. II (a)), are not its cause, or not the whole cause, unless they have been counteracted (which is a point for further investigation). We also see that r, v, q, s, t, u occur without A, and therefore are not the effects of A. And, further, if the negative instances represent all possible cases, we see that (according to Prop. I. (b)). A is the cause of p, because it cannot be omitted without the cessation of p. The inference that A and p are cause and effect, suggested by their being present throughout the first set of instances, is therefore strengthened by their being both absent throughout the second set.

This method so, as it depends upon observation, cannot prove direct causation. But, if the negative set is exhaustive it excludes any possibility of the Plurality of causes.

So far as this Double Method, like the Single Method of Agreement, relies on observation, sequence may not be perceptible in the instances observed, and then, direct causation cannot be proved by it, but only the probability of causal connection; and, again, the real cause, though present, may be so obscure as to evade observation. It has, however, one peculiar advantage, namely, that if the second list of instances (in which the phenomenon and its supposed antecedent are both absent) can be made exhaustive, it precludes any hypothesis of a plurality of causes; since all possible antecedents will have been included in this list without producing the phenomenon. Thus, in the above symbolic example, taking the first set of instances, the supposition is left open that B, C, D, E, F, G, may, at one time or another, have been a condition of p; but, in the second list, these antecedents all occur, here or there, without producing p, and therefore (unless counteracted somehow) cannot be a condition of p. A, then, stands out as the one thing that is present whenever p is present, and absent whenever p is absent.

Tyndall's use of this Method.

Stated in this abstract way, the Double Method may seem very elaborate and difficult; yet, in fact, its use may be very simple. Tyndall, to prove that dispersed light in the air is due to motes, showed by a number of cases (1) that any gas containing motes is luminous; (2) that air in which the motes had been destroyed by heat, and any gas so prepared as to exclude motes, are not luminous. All the instances are of gases, and the result is: motes—luminosity; no motes—no luminosity. Darwin, to show that cross-fertilisation is favourable to flowers, placed a net about 100 flower-heads, and left 100 others of the same varieties exposed to the bees: the former bore no seed, the latter nearly 3,000. We must assume that, in Darwin's judgment, the net did not screen the flowers from light and heat sufficiently to affect the result.

Further illustrations of the use of the same Method.

There are instructive applications of this Double Method in Wallace's *Darwinism*. In chap. viii., on *Colour in Animals*, he observes, that the usefulness of their coloration to animals is shown by the fact that, "as a rule, colour and marking are constant in each species of wild animal, while, in almost every domesticated animal, there arises great variability. We see this in our horses and cattle, our dogs and cats, our pigeons and poultry. Now the essential difference between the conditions of life of domesticated and wild animals is, that the former are protected by man, while the latter have to protect themselves." Wild animals protect themselves by acquiring qualities adapted to their mode of life; and coloration is a very important one, its chief, though not its only use, being concealment. Hence a useful coloration having been established in any species, individuals that occasionally may vary from it, will generally perish; whilst among domestic animals, variation of colour or marking is subject to no check except the taste of owners. We have, then, two lists of instances; first, innumerable species of wild animals in which the coloration is constant

and which depend upon their own qualities for existence; secondly, several species of domestic animals in which the coloration is *not* constant, and which do *not* depend upon their own qualities for existence. In the former list two circumstances are present together (under all sorts of conditions) ; in the latter they are absent together. The argument may be further strengthened by adding a third list, parallel to the first, comprising domestic animals in which coloration is approximately constant, but where (as we know) it is made a condition of existence by owners, who only breed from those specimens that come up to a certain standard of coloration.

Wallace goes on to discuss the colouring of arctic animals. In the arctic regions, he says, some animals are wholly white all the year round, such as the polar bear, the American polar hare, the snowy owl and the Greenland falcon : these live amidst almost perpetual snow. Others, that live where the snow melts in summer, only turn white in winter, such as the arctic hare, the arctic fox, the ermine and the ptarmigan. In all these cases the white colouring is useful, concealing the herbivores from their enemies, and also the carnivores in approaching their prey ; this usefulness, therefore, is a condition of the white colouring. Two other explanations have, however, been suggested : first, that the prevalent white of the arctic regions directly colours the animals, either by some photographic or chemical action on the skin, or by a reflex action through vision (as in the chameleon) ; secondly, that a white skin checks radiation and keeps the animals warm. But there are some exceptions to the rule of white colouring in arctic animals which refute these hypotheses, and confirm the author's. The sable remains brown throughout the winter ; but it frequents trees, with whose bark its calour assimilates. The musk-sheep is brown and conspicuous ; but it is gregarious, and its safety depends upon its ability to recognise its kind and keep with the herd. The raven is always black ; but it fears no enemy and feeds on carrion, and therefore does not need concealment for either defence

or attack. The colour of the sable, then, though not white, serves, for concealment; the colour of the musk-sheep serves a purpose more important than concealment; the raven needs no concealment. There are thus two sets of instances:—in one set the animals are white (*a*) all the year, (*b*) in winter; and white conceals them (*a*) all the year, (*b*) in winter; in the other set, the animals are *not* white, and to them either whiteness would *not* give concealment, or concealment would *not* be advantageous. And this second list refutes the rival hopotheses: for the sable, the musk-sheep and the raven are as much exposed to the glare of the snow, and to the cold, as the other animals are.

§ 3. THE CANON OF DIFFERENCE: *If an instance in which a phenomenon occurs, and an instance in which it does not occur, have every other circumstance in common save one, that one (whether consequent or antecedent) occuring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable condition of the phenomenon.*

The Canon

This follows from Props. I (*a*) and (*b*), in chapter xv. §7. To prove that A is a condition of *p*, let two instances, such as the Canon requires, be represented thus:

A B C	B C
p q r	*q r*

Then A is the cause or a condition of *p*. For, in the first instance, A being introduced (without further change), *p* arises (Prop. I. (*a*); and, in the second instance, A having been removed (without other change), *p* disappears (Prop. I. (*b*)). Similarly we may prove, by the same instances, that *p* is the effect of A.

The order of the phenomena and the immediacy of their connection is a matter for observation, aided by whatever instruments and methods of inspection and measurement may be available.

As to the invariability of the connection, it may of course be tested by collecting more instances or making more experiments; but it has been maintained, that a single perfect experiment according to this method is sufficient to

A single experiment according to this method can prove causation for it establishes the unconditionality of connection once for all.

prove causation, and therefore implies invariability (since causation is uniform), though no other instances should ever be obtainable ; because it establishes once for all the unconditionality of the connection $\frac{A\ B\ C}{p\ q\ r}$. Now, formally this is true; but in any actual investigation how shall we decide what is a satisfactory or perfect experiment? Such an experiment requires that in the negative instance $\frac{B\ C}{q\ r}$, BC shall be the least assemblage of conditions necessary to co-operate with A in producing p ; and that it is so cannot be ascertained without either general prior knowledge of the nature of the case or special experiments for the purpose. So that invariability will not really be inferred from a single experiment ; besides that every prudent inquirer repeats his experiments, if only to guard against his own liability to error.

Plurality of causes does not affect this method.

The supposed plurality of causes does not affect the method of Difference. In the above symbolic case, A is clearly *one* cause (or condition) of p, whatever other causes may be possible ; whereas with the Single Method of Agreement, it remained doubtful (admitting a plurality of causes) whether A, in spite of being always present with p, was ever a cause or condition of it.

This Method is often the implicit basis of ordinary judgments.

This method of Difference without our being distinctly aware of it, is oftener than any other the basis of ordinary judgments. That the sun gives light and heat, that food nourishes and fire burns, that a stone breaks a window or kills a bird, that the turning of a tap permits or checks the flow of water or of gas, and thousands of other propositions are known to betrue by rough but often emphatic applications of this method in common experience.

This method can be applied either (1) by observation, or (2) by experiment.

The method of Difference may be applied either (1) by observation, on finding two instances (distinct assemblages of conditions) differing only in one phenomenon together with its antecedent or consequent; or (2) by experiment, and then, either (*a*) by preparing two instances that may be compared side by side, or (*b*) by taking certain coditions, and then introducing (or subtracting) some agent, supposed to be the cause, to see what happens: in the latter case the "two instances" are the same assemblage of conditions considered before and, again, after the introduction of the agent. As an example of (*a*) there is an experiment to show that radium gives off heat: take two glass tubes, in one put some chloride of radium, in both thermometers, and close them with cotton-wool. Soon the thermometer in the tube along with radium reads $5\cdot4^{\circ}$ F. higher than the other one. The tube without the radium, whose temperature remains unaltered, is called the "control" experiment. Most experiments are of the type (*b*); and since the Canon, which describes two co-existing instances, does not readily apply to this type, an alternative version may be offered: *Any agent whose introduction into known circumstances (without further change) is immediately followed by a definite phenomenon is a condition of the occurence of that phenomenon.*

Examples of these different applications.

Knowledge of all other conditions are necessary for the application of this method.

The words *into known circumstances* are necessary to emphasise what is required by this Method, namely, that the two instances differ in only one thing; for this cannot be ascertained unless all the other conditions are known; and this further implies that they have been prepared. It is, therefore, not true (as Sigwart asserts) that this method determines only one condition of a phenomenon, and that it is then necessary to inquire into the other conditions. If they were not known they must be investigated; but then the experiment would not have been made upon this method.

Practically, experiments have to be made in all degrees of imperfection, and the less perfect they are, that is, the less the circumstances are known beforehand, the more remains to be done. A common imperfection is delay, or the occurrence of a latent period between the introduction of an agent and the manifestation of its effects; it cannot then be the unconditional cause; though it may be an indispensable remote condition of whatever change occurs. If, feeling out of sorts, you take a drug and some time afterwards feel better, it is not clear on this ground alone that the drug was the cause of recovery, for other curative processes may have been active meanwhile—food, or sleep, or exercise.

Various illustrations of the application of the method:

Any book of Physics or of Chemistry will furnish scores of examples of the method of Difference: such as Galileo's experiment to show that air has weight, by first weighing a vessel filled with ordinary air, and then filling it with condensed air and weighing it again; when the increased weight can only be due to the greater quantity of air conditioned. The melting-point of solids is determined by heating them until they do melt (as silver at 1000° C., gold at 1250°, platinum at 2000°); for the only difference between bodies at the time of melting and just before is the addition of so much heat. Similarly with the boiling-point of liquids. That the transmission of sound depends upon the continuity of an elastic ponderable medium, is proved by letting a clock strike in a vacuum (under a glass from which the air has been withdrawn by an air-pump), and standing upon a non-elastic pedestal: when the clock may be seen to strike, but makes only such a faint sound as may be due to the imperfections of the vacuum and the pedestal.

The experiments by which the chemical analysis or synthesis of various forms of matter is demonstrated are simple or compound applications of this method of Difference, together with the quantitative mark of causation (that cause and effect are equal); since the bodies resulting from an analysis are equal in

weight to the body analysed and the body resulting from a synthesis is equal in weight to the bodies synthesised. That an electric current resolves water into oxygen and hydrogen may be proved by inserting the poles of a galvanic battery in a vessel of water; when this one change is followed by another, the rise of bubbles from each pole and the very gradual decrease of the water. If the bubbles are caught in receivers placed over them, it can be shown that the joint weight of the two bodies of gas thus formed is equal to the weight of the water that has disappeared; and that the gases are respectively oxygen and hydrogen may then be shown by proving that they have the properties of those gases according to further experiments by the method of Difference; as (*e.g.*,) that one of them is oxygen because it supports combustion, *etc.*

When water was first decomposed by the electric current, there appeared not only oxygen and hydrogen, but also an acid and an alkali. These products were afterwards traced to impurities of the water and of the operator's hands. Mill observes that in any experiment the effect, or part of it, may be due, not to the supposed agent, but to the means employed in introducing it. We should know not only the other conditions of an experiment, but that the agent or change introduced is nothing else than what it is supposed to be.

In the more complex sciences the method of Difference is less easily applicable, because of the greater difficulty of being sure that only one circumstance at a time has altered; still, it is frequently used. Thus, if by dividing a certain nerve certain muscles are paralysed, it is shown that normally that nerve controls those muscles. That the sense of smell in flies and cockroaches is connected with the antennæ has been shown by cutting them off: whereupon the insects can no longer find carrion. In his work on *Earthworms*, Darwin shows that, though sensitive to mechanical tremors, they are deaf (or, at least, not sensitive to sonorous vibrations transmitted through the air), by the following experiment. He placed a pot containing a worm that had

come to the surface, as usual at night, upon a table, whilst close by a piano was violently played ; but the worm took no notice of the noise. He then placed the pot upon the piano, whilst it was being played, when the worm, probably feeling mechanical vibrations, hastily slid back into its burrow.

When, instead of altering one circumstance in an instance (which we have done our best not otherwise to disturb) and then watching what follows, we try to find two ready-made instances of a phenomenon, which only differ in one other circumstance, it is, of course, still more difficult to be sure that there is only one other circumstance in which they differ. It may be worth while, however, to look for such instances. Thus, that the temperature of ocean currents influences the climate of the shores they wash, seems to be shown by the fact that the average temperature of Newfoundland is lower than that of the Norwegian coast some 15° farther north. Both regions have great continents at their back ; and as the mountains of Norway are higher and capped with perennial snow, we might expect a colder climate there : but the shore of Norway is visited by the Gulf Stream, whilst the shore of Newfoundland is traversed by a cold current from Greenland. Again, when in 1841 the railway from Rouen to Paris was being built, gangs of English and gangs of French workmen were employed upon it, and the English got through about one-third more work man than the French. It was suspected that this difference was due to one other difference, namely, that the English fed better, preferring beef to thin soup. Now, Logically, it might have been objected that the evidence was unsatisfactory, seeing that the men differed in other things besides diet—in 'race' (say), which explains so much and so easily. But the Frenchmen, having been induced to try the same diet as the English, were, in a few days, able to do as much work : so that the "two instances" were better than they looked. It often happens that evidence, though logically questionable, is good when used by experts, whose familiarity with the subject makes it good.

§ 4. THE CANON OF CONCOMITANT VARIATIONS :

The Canon of Concomitant Variations.

Whatever phenomenon varies in any manner whenever another phenomenon (consequent or antecedent) varies in some particular manner [no other change having concurred] is either the cause or effect of that phenomenon [or is connected with it through some fact of causation].

The Canon

This method, in fact, is a special case either of Agreement or of Difference, as the case may be.

This is not an entirely fresh method, but may be regarded as a special case either of Agreement or of Difference, to prove the cause or effect, not of a phenomenon as a whole, but of some increment of it (positive or negative). There are certain forces, such as gravitation, heat, friction, that can never be eliminated altogether, and therefore can only be studied in their degrees. To such phenomena the method of difference cannot be applied, because there are no negative instances. But we may obtain negative instances of a given quantity of such a phenomenon (say, heat), and may apply the method of Difference to that quantity. Thus, if the heat of a body increases 10 degrees, from 60 to 70, the former temperature of 60 was a negative instance in respect of those 10 degrees ; and if only one other circumstance (say, friction) has altered at the same time, that circumstance (if an antecedent) is the cause. Accordingly, if in the above Canon we insert, after 'particular manner', "[no other change having concurred]" it is a statement of the method of Difference as applicable to the increment of a phenomenon instead of to the phenomenon as a whole ; and we may then omit the last cause—"[or is connected, *etc.*]." For these words are inserted to provide for the case of co-effects of a common cause (such as the flash and report of a gun) ; but if no other change (such as the discharge of a gun) has concurred with the variations of two phenomena, there cannot have been a common cause, and they are therefore cause and effect.

If, on the other hand, we omit the cause "[no other change having concurred,]" the Canon is a statement of the method of

This method applies only to quantitative variations.

Agreement as applicable to the increment of a phenomenon instead of to the phenomenon as a whole; and it is then subject to the imperfections of that method; that is to say, it leaves open the possibilities, than an inquirer may overlook a plurality of causes; or may mistake a connection of two phenomena, which (like the flash and report of a gun) are co-effects of a common cause, for a direct relation of cause and effect.

It may occur to the reader that we ought also to distinguish Qualitative and Quantitative Variations as two orders of phenomena to which the present method is applicable. But, in fact, Qualitative Variations may be adequately dealt with by the foregoing methods of Agreement, double Agreement, and Difference; because a change of quality or property entirely gets rid of the former phase of that quality, or substitutes one for another; as when the ptarmigan changes from brown to white in winter, or as when stag grows and sheds its antlers with the course of the seasons. The peculiar use of the method of Variations, however, is to formulate the conditions of proof in respect of those causes or effects which cannot be entirely got rid of, but can be obtained only in greater or less amount; and such phenomena are, of course, quantitative.

Even when there are two parallel series of phenomena, the one quantitative and the other qualitative—like the rate of air-vibration and the pitch of sound, or the rate of ether-vibration and the colour-series of the spectrum—the method of Variations is not applicable. For (1) two such series cannot be said to vary together, since the qualitative variations are heterogeneous: 512 : 576 is a definite ratio; but the corresponding notes, C, D, in the treble clef, present only a difference. Hence (2) the correspondence of each note with each number is a distinct fact. Each octave

This method is not applicable even to two parallel series of which one is qualitative and the other quantitative.

even is a distinct fact; there is a difference between C 64 and C 128 that could never have been anticipated without the appropriate experience. There is, therefore, no such law of these parallel series as there is for temterature and change of volume (say) in mercury. Similar remarks apply to the physical and sensitive light-series.

A symbolic Example of Agreement in Variation.

We may illustrate the two cases of the method thus (putting a dash against any letter A′ or *p*′, to signify an increase or decrease of the phenomenon the letter stands for): Agreement in Variations (other changes being admissible)—

A B C	A′ D E	A″ F G
p q r	*p*′ *s t*	*p*″ *u v*

Here the accompanying phenomena (*B C q r*, *D E s t*, *F G u v*) change from time to time, and the one thing in which the instances agree throughout is that any increase of A (A′ or A″) is followed or accompanied by an increase of *p* (*p*′ p″): whence it is argued that A is the cause of *p*, according to Prop. III. (*a*) (ch. xv. § 7). Still, it is supposable that, in the second instance, D or E may be the cause of the increment of *p*; and that, in the third instance, F or G may be its cause: though the probability of such vicarious causation decreases rapidly with the increase of instances in which A and *p* vary together. And, since an actual investigation of this type must rely on observation, it is further possible that some undiscovered cause, X, is the real determinant of both A and *p* and of their concomitant variations.

An example of the misuse of this method

Professor Ferri, in his *Criminal Sociology*, observes: "I have shown that in France there is a manifest correspondence of increase and decrease between the number of homicides, assaults and malicious wounding, and the more or less abundant vintage, especially in the years of extraordinary variations, whether of failure of the vintage (1853-5, 1859, 1867, 1873, 1878-80), attended by a remarkable diminution of crime (assaults

and wounding), or of abundant vintages 1850, 1856-8, 1862-3, 1865, 1868, 1874-5), attended by an increase of crime" (p. 117, Eng. trans.). And earlier he had remarked that such crimes also "in their oscillations from month to month display a characteristic increase during the vintage periods, from June to December, notwithstanding the constant diminution of other offences" (p. 77). This is necessarily an appeal to the canon of Concomitant Variations, because France is never without her annual vintage, nor yet without her annual statistics of crime. Still, it is an argument whose cogency is only that of Agreement, showing that probably the abuse of the vintage is a cause of crimes of violence, but leaving open the supposition, that some other circumstance or circumstances, arising or varying from year to year, may determine the increase or decrease of crime; or that there is some unconsidered agent which affects both the vintage and crimes of violence. French sunshine, it might be urged, whilst it matures the generous grape, also excites a morbid fermentation in the human mind.

A symbolic Example of Difference in Variation.

Difference in Variations may be symbolically represented thus (no other change having concurred):

$$\begin{array}{lll} A\;B & A'\;B & A''\;B \\ p\;\;q, & p'\;\;q, & p''\;\;q. \end{array}$$

Here the accompanying phenomena are always the same $\frac{B}{q}$; and the only point in which the successive instances differ is in the increments of A (A', A'') followed by corresponding increments of p (p', p''); hence the increment of A is the cause of the increment of p.

Application of this in exact sciences.

For examples of the application of this method, the reader should refer to some work of exact science. He will find in Deschanel's *Natural Philosophy*, c. 32, an account of some experiments by which the connection between heat and mechanical work has been established. It is there shown that "whenever

work is performed by the agency of heat" [as in driving an engine], "an amount of heat disappears equivalent to the work performed; and whenever mechanical work is spent in generating heat" [as in rubbing two sticks together], "the heat generated is equivalent to the work thus spent." And an experiment of Joule's is described, which consisted in fixing a rod with paddles in a vessel of water, and making it revolve and agitate the water by means of a string wound round the rod, passed over a pulley and attached to a weight that was allowed to fall. The descent of the weight was measured by a graduated rule, and the rise of the water's temperature by a thermometer. It was found that the heat communicated to the water by the agitation amounted to one pound-degree Fahrenheit for every 772 foot-pounds of work" expended by the falling weight. As no other material change seems to take place during such an experiment, it shows that the progressive expenditure of mechanical energy is the cause of the progressive heating of the water.

Thermometer as an example of this method.

The thermometer itself illustrates this method. It has been found that the application of heat to mercury expands it according to a law; and hence the volume of the mercury, measured by a graduated index, is used to indicate the temperature of the air, water, animal body, *etc.*, in which the thermometer is immersed, or with which it is brought in contact. In such cases, if no other change has taken place, the heat of the air, water, or body is the cause of the rise of the mercury in its tube. If some other substance (say spirit) be substituted for mercury in constructing a thermometer, it serves the same purpose, provided the index be graduated according to the law of the expansion of that substance by heat, as experimentally determined.

Absence of concomitant

Instances of phenomena that do not vary together indicate the exclusion of a supposed cause [by Prop. III. (c)]. The stature of the human race has been supposed to depend on temperature; but there is no

variation indicates exclusion of a supposed cause.

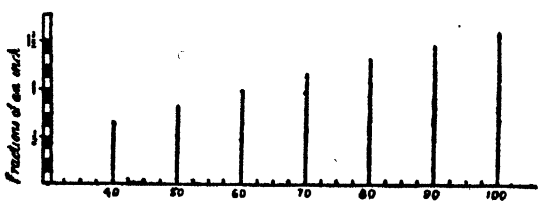
correspondence. The "not varying together," however, must not be confused with "varying inversely," which when regular indicates a true concomitance. It is often a matter of convenience whether we regard concomitant phenomena as varying directly or inversely. It is usual to say—'the greater the friction the less the speed'; but it is really more intelligible to say—'the greater the friction the more rapidly molar is converted into molecular motion.'

Graphic presentation of this method

The Graphic Method exhibits Concomitant Variations to the eye, and is extensively used in physical and statistical inquiries. Along a horizontal line (the abscissa) is measured one of the conditions (or agents) with which the inquiry is concerned, called the Variable; and along perpendiculars (ordinates) is measured some phenomenon to be compared with it, called the Variant.

Elucidation, through example, of this Graphic Method

Thus, the expansion of a liquid by heat may be represented by measuring degrees of temperature along the horizontal, and the expansion of a column of the liquids in units of length along the perpendicular.



Degrees of Temperature

FIG. 9.

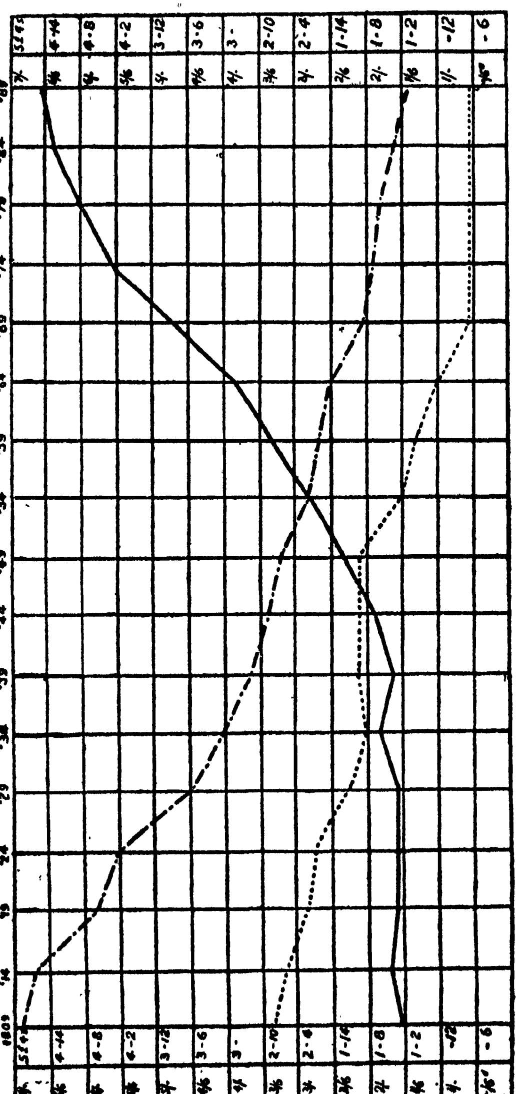
In the next diagram (Fig. 10), reduced from one given by Mr. C. H. Denyer in an article on the Price of Tea (*Economic Journal*, No. 9), the condition measured horizontally is Time;

and, vertically, three variants are measured simultaneously, so that their relations to one another from time to time may be seen at a glance. From this it is evident that, as the duty on tea falls, the price of tea falls, whilst the consumption of tea rises; and, in spite of some irregularity of correspondence in the courses of the three phenomena, their general causal connection can hardly be mistaken. However, the causal connection may also be inferred by general reasoning; the statistical Induction can be confirmed by a Deduction; thus illustrating the combined method of proof to be discussed in the next chapter. Without such confirmation the proof by Concomitant Variations would not be complete; because, from the complexity of the circumstances, social statistics can only yield evidence according to the method of Agreement in Variations. For, besides the agents that are measured, there may always be some other important influence at work. During the last fifty years, for example, crime has decreased whilst education has increased: true; but at the same time wages have risen and many other things have happened.

It will be noticed that in the diagram p. 98 the three lines, especially those of Price and Consumption (which may be considered *natural* resultants, in contrast with the arbitrary fixation of a Tax), do not depart widely from regular curves; and accordingly, assuming the causes at work to vary continuously during the intervals between points of measurement, curves may be substituted. In fact, a curve often represents the course of a phenomenon more truthfully than can be done by a line that zigzags along the exact measurements; because it is less influenced by temporary and extraordinary causes that may obscure the operation of those that are being investigated. On the other hand, the abrupt deviations of a punctilious zigzag may have their own logical value, as will appear in the next section.

In working with the Method of Variations one must allow for the occurrence in a series of 'critical points,' at which sudden and sometimes heterogeneous changes may take place.

Diagram showing (1)———the average Price of Tea (in bond), but with duty added per lb.: (2)..........the rate of Duty; (3)—·—·—the consumption per head, from 1809 to 1889.



One horizontal space=5 years. One vertical space—6 pence, or 6 ounces.

FIG. 10.

'very substance exists at different temperatures in three tates, gaseous, liquid, solid; and when the hange takes place, from one state to another, he series of variations is broken. Water, *e.g.*, ɔllows the general law that cooling is accom-ıanied by decrease of volume between 212° and ;9° F.: but above 212°, undergoes a sudden ex-ıansion in becoming a gas; and below 39° begins to expand, ıntil at 32° the expansion is considerable on its ıecoming solid. This illustrates a common ex-ıerience that concomitant variations are most egular in the 'median range,' and are apt to be-ome irregular at the extremities of the series, vhere new conditions begin to operate.

'Critical points' of abrupt and even heterogeneous changes.

Hence variations are most regular in the 'range'.

The Canon of Variations, again, deals not with sudden irrup-ions of a cause, force or agent, but with some increase or de-rease of an agent already present, and a corres-ponding increase or decrease of some other pheno-nenon—say an increase of tax and a rise of price. But there are cases in which the energy of a cause not immediately discharged and dissipated. 'Thilst a tax of 6d. per lb. on tea raises the price ɛr lb. by about 6d., however long it lasts, the ɔntinuous application of friction to a body may ;adually raise its temparature to the point of ɔmbustion; because heat is received faster than it is radiated, ıd therefore accumulates. Such cases are treated by Mill ıder the title of 'progressive effects' (*Logic*: B. III., c. 15): gives as an example of it the acceleration of falling bodies. he storage of effects is a fact of the utmost importance in all partments of nature, and is especially interesting in Biology d Sociology, where it is met with as heredity, experience, dition. Evolution of species of plants and animals would (so as we know) be impossible, if the changes (however caused) t adapt some individuals better than others to the coditions

The 'critical points' are points of sudden changes, not of sudden irruptions of cause.

of life were not inherited by, and accumulated in, their posterity. The eyes in the peacock's tail are supposed to have reached their present perfection gradually, through various stages that may be illustrated by the ocelli in the wings of the Argus pheasant and other genera of *Phasianidæ*. Similarly the progress of societies would be impossible without tradition, whereby the improvements made in any generation may be passed on to the next, and the experience of mankind may be gradually accumulated in various forms of culture. The earliest remains of culture are flint implements and weapons ; in which we can trace the effect of tradition in the lives of our remote forefathers, as they slowly through thousands of years learnt to improve the chipping of flints, until the first rudely shaped lumps gave place to works of unmistakable design, and these to the beautiful weapons contemporary with the Bronze Age.

Cases in which the energy of a cause is not immediately discharged : 'Progressive effects'.

The Method of Gradations, the arranging of any phenomena to be studied in series, according to the degree in which some character is exhibited is perhaps, the most definite device in the Art of Discovery. (Bain : *Induction*, c. 6, and App. II.). If the causes are unknown it is likely to suggest hypotheses: and if the causes are partly known, variation in the character of the series is likely to indicate a corresponding variation of the conditions.

The Method of Gradations and the art of Discovery.

§ 5. The Canon of Residues.

Subduct from any phenomenon such part as previous inductions have shown to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

The Canon of Residues.

The phenomenon is here assumed to be an effect : a similar Canon may be framed for residuary causes.

This also is not a fresh method, but a special case of the method of Difference. For if we suppose the phenomenon

This method

Also is only a special case of the method of Difference.

to be $p\ q\ r$, and the antecedent to be A B C, and that we already know B and C to have (either severally or together) the consequents $q\ r$, in which their efficacy is exhausted; we may regard $\begin{matrix}B & C\\ q & r\end{matrix}$ as an instance of the absence of p obtained deductively from the whole phenomenon $\begin{matrix}A & B & C\\ p & q & r\end{matrix}$ by our knowledge of the laws of B and C; so that $\begin{matrix}A & B & C\\ p & q & r\end{matrix}$ is an instance of the presence of p, differing otherwise from $\begin{matrix}B & C\\ q & r\end{matrix}$ in nothing except that A is also present. By the Canon of Difference, therefore A is the cause of p. Or, again, when phenomena thus treated are strictly quantitative, the method may be based on Prop. III. (b), ch. xv. § 7.

Deduction involved in this method.

Of course, if A can be obtained apart from B C and directly experimented with so as to produce p, so much the better; and this may often be done; but the special value of the method of Residues appears, when some complex phenomenon has been for the most part accounted for by known causes, whilst there remains some excess, or shortcoming, deviation from the result which those causes alone would lead us to expect, and this residuary fact has to be explained in relation to the whole. Here the negative instance is constituted by deduction, showing what would happen but for the interference of some unknown cause which is to be investigated; and this prominence of the deductive process has led some writers to class the method as deductive. But we have seen that all the Canons involve deduction; and, considering how much in every experiment is assumed as already known (what circumstances are 'material', and when conditions may be called 'the same'), the wonder is that no one has insisted upon regarding every method as con-

cerned with residues. In fact, as scientific explanation progresses, the phenomena that may be considered as residuary become more numerous and the importance of this method increases.

Various examples of the working of this method.

Examples : The recorded dates of ancient eclipses having been found to differ from those assigned by calculation, it appears that the average length of a day has in the meanwhile increased. This is a residuary phenomenon not accounted for by the causes formerly recognised as determining the rotation of the earth on its axis; and it may be explained by the consideration that the friction of the tides reduces the rate of the earth's rotation, and thereby lengthens the day. Astronomy abounds in example of the method of Residues, of which the discovery of Neptune is the most famous.

Capillarity seems to be a striking exception to the principle that water (or any liquid) 'finds its level that being the condition of equilibrium ; yet capillarity proves to be only a refined case of equilibrium when account is taken of the forces of adhesion exerted by different kinds of bodies in contact.

"Many of the new elements of Chemistry," says Herschel, "have been detected in the investigation of residual phenomena." Thus, Lord Rayleigh and Sir W. Ramsay found that nitrogen from the atmosphere was slightly heavier than nitrogen got from chemical sources ; and, seeking the cause of this difference, discovered argon.

The economist shows that when a country imports goods the chief means of paying for them is to export other goods. If this were all, imports and exports would be of equal value : yet the United Kingdom imports about £400,000,000 annually, and exports about £300,000,000. Here, then, is a residuary phenomenon of £100,000,000 to be accounted for. But foreign countries owe us about £50,000,000 for the use of shipping, and £70,000,000 as interest on the capital we have lent them, and £15,000,000 in commission upon business transacted for them. These sums added together amount to £135,000,000 ; and that

is £35,000,000 too much. Thus another residuary phenomenon emerges; for whilst foreigners seem to owe us £435,000,000 they only send us £400,000,000 of imports. These £35,000,000 are accounted for by the annual investment of our capital abroad, in return for which no immediate payment is due; and, these being omitted, exports and imports balance. Since this was written the figures of our foreign trade have greatly risen; but the character of the explanation remains the same.

When, in pursuing the method of Variations, the phenomena compared do not always correspond in their fluctuations, the irregular movements of that phenomenon which we regard as the effect may often be explained by treating them as residuary phenomena, and then seeking for exceptional causes, whose temporary interference has obscured the influence of the general cause. Thus, returning to the diagram of the Price of Tea in §4, it is clear that generally the price falls as the duty falls; but in Mr. Denyer's more minutely wrought diagram, from which this is reduced, it may be seen that in 1840 the price of tea rose from *3s. 9d.* to *4s. 9d.* without any increase of duty. This, however, is readily explained by the Chinese War of that year, which checked the supply. Again, from 1869 to 1889 the duty was constant, whilst the price of tea fell as much as *8d.* per *lb.*; but this residuary phenomenon is explained by the prodigiously increased production of tea during that period in India and Ceylon.

The above examples of the method of Residues are all quantitative; but the method is often employed where exact estimates are unobtainable. Thus Darwin, having found certain modifications of animals in form, coloration and habits, that were not clearly derivable from their struggle for existence in relation to other species or to external conditions, suggested that they were due to Sexual Selection.

The 'vestiges' and 'survivals' so common in Biology and Sociology are residuary phenomena. It is a general inference from the doctrine of Natural Selection that every organ of a

plant, animal, or society is in some way useful to it. There occur, however, organs that have at present no assignable utility, are at least wasteful, and sometimes even injurious. And the explanation is that formerly they were useful; but that, their uses having lapsed, they are now retained by the force of heredity or tradition. Either they are not injurious enough to be eliminated by natural selection; or they are correlated with other organs, whose utility outweighs their disutility.

SUMMARY WITH SUPPLEMENTARY NOTES

To ascertain the causal connection between two events certain methods are used. These methods are called the inductive or experimental methods. They are called inductive methods as they are the devices for establishing inductive conclusions through the determination of causal connection. And, they are called experimental method or methods of observation in as much they state the characteristics of such observation and experiment as justify us in drawing conclusions about causal connections. They are also called by some logicians, the methods of elimination. By elimination is meant "the successive exclusion of the various circumstances which are found to accompany a phenomena in a given instance in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon" (Mill). This elimination is effected by varying the circumstances i.e., by placing one and the same event in a variety of circumstances. Now, elimination is primarily negative in its nature. It excludes the irrelevant. It is positive only by implication. And some logicians call these methods, methods of elimination in as much as these methods according to them cannot conclusively establish the causal connection. They, that is to say, can only eliminate more or less successfully the non-causes and hence are methods of elimination. There are other logicians who hold that these methods are not merely weapons of elimination, for they have a positive

function viz., the determination of the unconditional or the true cause.

Now, these methods are deduced from the law of causation. From the definition of cause as given in ch. XV. it follows that (1) that which (without further change) is followed by a given event is its cause, (2) that which is not so followed is not the cause; (3) that which cannot be left out without impairing a phenomenon is a condition of it; (4) that which can be left out is not a condition of it, (5) when a cause (or effect) increases or decreases, so does its effect (or cause), (6) if two phenomena having the other marks of cause and effect, seem unequal, the less contains an unexplored factor, (7) if an antecedent and consequent do not increase or decrease correspondingly, they are not cause and effect, so far as they vary. In short (*i*) if an antecedent can be eliminated without a particular consequent being also eliminated, is not the cause i.e., the unconditional antecedent of the consequent, (*ii*) if an antecedent cannot be eliminated without affecting a particular consequent is the cause or a part of the cause of the consequent, and (*iii*) if an antecedent cannot increase or decrease without a corresponding variation in a particular consequent is a condition of the conseqnent. These deduced principles are called the principles of elimination, and the inductive methods are based on them. So it is also said that these methods make the definition explicit. The five methods are the Method of Agreement, the Joint Method, the Method of Difference, the Method of Concomitant Variations, and the Method of Residues. The first three are sometimes called Qualitative Methods and the two last Quantitative; and although this grouping is not correct, as the Method of Difference is often used quantitatively, yet it draws attention to an important distinction between a mere description of conditions and determination by exact measurement.

THE METHOD OF AGREEMENT

Its Canon (as given by Mill): If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.

This method as Mill says is used when experimentation is impossible ; and the rule of proof depends, first, upon observation of an invariable connection between the given phenomenon and one other circumstance ; and secondly upon the following propositions or principles of elimination as obtained from the unconditionality of causation :—(1) that which is followed by a given event is its cause and (2) that which cannot be left out is not a condition of it. The cogency of the proof depends entirely upon its tending to show the unconditionality of the sequence A—*p* or the indispensability of A as a condition of *p*. Invariable sequence is not enough to prove direct causation. Negative instances are necessary for this purpose. The use of the negative instances distinguishes the method of agreement from simple enumeration. Although simple enumeration, when the instances of agreement are numerous enough, may give rise to a strong belief in the connection of phenomena, yet it can never be a methodical or logical proof of causation since it does not indicate the unconditionalness of the sequence. For, simple enumeration of the sequence A—*p*, leaves open the possibility that besides A there is always some other antecedent of *p*, say X ; and then X may be the cause of *p*. To disprove it we must find or make a negative instance of X—where *p* occurs, but X is absent.

ITS DEFECTS :—(*i*) So far as we recognise the possibility of a plurality of causes this method cannot be quite satisfactory. The liability of the method to be frustrated by the plurality of causes is called by Mill its characteristic imperfection. This defect can to a certain extent be remedied (1) by the multiplication of instances, (2) by the application of the joint method, and (3) by means of calculation of probability and elimination of chance.

(*ii*) Immediate sequence in many cases may not be observation, if we rely on the canon of Agreement. That is, if instances cannot be obtained by experiment and we have to depend on observation, we cannot ascertain the cause and the effect. The phenomena may be so mixed together that A and *p* seem to be merely concomitant ; so that though connection of some sort may be rendered highly probable, we may not be able to say which is cause and which is effect.

(*iii*) This method fails to distinguish between cases of coexistence from cases of causation.

(*iv*) It is liable to be frustrated by the operation of hidden causes.

ITS ADVANTAGES : (*i*) The method of Agreement cannot by itself prove causation. Its chief use is to suggest hypothesis as to the cause; which must then be used, if possible, experimentally to try if it produces the given effect.

(*ii*) As a method of observation it can be applied to ascertain the effect as well as the cause of a given event.

(*iii*) Where experiment is not possible, this method can be used.

(*iv*) Where succession cannot be traced, this method may point to a connection between two or more facts where direct causation seems to be out of question.

THE JOINT METHOD OF AGREEMENT IN PRESENCE AND IN ABSENCE

ITS CANON. (Mill's) If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect or the cause or an indispensable part of the cause of the phenomenon.

This method is, as Mill says, "an improvement upon the common Method of Agreement". The first clause of its canon is the same as that of the method of Agreement, and its significance depends upon the same propositions concerning causation. The second clause relating to instances in which the phenomenon is absent, depends for its probative force upon the following propositions concerning causation : (1) That which is not followed by an event is not its cause, and (2) that which cannot be left out without impairing a phenomenon is a condition of it. Its function is to exclude certain circumstances from the list of possible causes or effects of the phenomenon investigated. Like the method of Agreement, it also pre-eminently is a method of observation.

ITS DISADVANTAGES : So far as this Double Method, like the Single Method of Agreement, relies on observation, sequence may not be perceptible in the instances observed and then direct causation cannot be proved by it and only the probability of

causal connection; and again, the real cause though present may be so obscure as to *evade* observation.

NOTE: [The Double Method of Agreement "consists in a double employment of the Method of Agreement, each proof being independent of the other, and corroborating it. But it is not equivalent to a proof by the direct method of Difference. For the requisitions of the Method of Difference are not satisfied unless we can be quite sure either that the instances affirmative of *a* agree in no antecedent whatever but A or that the instances negative of *a* agree in nothing but the negative of A. Now, if it were possible, which it never is, to have this assurance, we should not need the joint Method, for either of the two sets of instances separately would then be sufficient to prove causation. This indirect method, therefore, can only be regarded as a great extension and improvement of the Method of Agreement, but not as participating in the more cogent nature of the Method of Difference" (Mill).]

ITS ADVANTAGE: It has one peculiar advantage, namely, that if the second list of instances can be made exhaustive, it precludes any hypothesis of a plurality of causes; since all possible antecedents will have been included in this list without producing the phenomenon. Stated in an abstract manner the Double Method may seem very elaborate and difficult, yet in fact its use is very simple.

NOTE: [The Double Method is not affected by the characteristic imperfection of the single method. "For, in the joint Method, it is supposed not noly that the instances in which *a* is, agree only in containing *A*, but also that the instances in which *a* is not, agree only in not containing *A*. Now, if this be so, *A* must be not only the cause of *a* but the only possible cause: for if there were another as, for example, *B* then in the instances in which *a* is not *B* must have been absent as well as *A*, and it would not be true that these instances agree *only* in not containing A. This, therefore, constitutes an immense advantage of the joint Method over the simple method of Agreement. It may seem indeed that the advantage does not belong so much to the joint method as to one of its two premises, (if they may be so called) the negative premise. The Method of Agreement, when applied to negative instances or those in which a phenomenon does *not* take place, is certainly free from the characteristic imperfection which affects it in the affirmative case. The negative premise, it might therefore, be supposed could be worked as a

simple case of the method of Agreement, without requiring an affirmative premise to be joined with it. But though this is true in principle, it is generally altogether impossible to work the Method of Agreement by negative instances without possitive ones: it is so much more difficult to exhaust the field of negation than that of affirmation. For instance, let the question be what is the cause of the transparency of bodies; with what prospect of success could we set ourselves to inquire directly in what the multifarious substances which are *not* transparent agree? But we might hope much sooner to seize some point of resemblance among the comparatively few and definite species of objects which *are* trnsparent; and this being attained we should quite naturally be put upon examining whether the *absence* of this one circumstance be not precisely the point in which all opaque circumstances will be found to resemble." (Mill).]

THE METHOD OF DIFFERENCE.

Its canon (Mill). If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which along the two instances differ, is the effect or the cause or an indispensable part of the cause of the phenomenon.

This method follows from the following propositions concerning causation: (*i*) that which is followed by a given event is its cause; and (*ii*) that which cannot be left out without impairing a phenomenon is a condition of it. In other words, "the Method of Agreement stands on the ground that whatever can be eliminated is not connected with the phenomenon by any law. The Method of Difference has for its foundation that whatever cannot be eliminated is connected with the phenomena by a law." (Mill).

While the method of agreement "is more especially the resource employed where experimentation is impossible" the method of Difference is "more particularly a method of artificial experiment." The nature of the instances demanded by this method is more strictly defined than in the method of Agreement. The two instances that this method requires, must be exactly similar in all circumstances except one. And as mere observation can-

not collect such instances ; i.e., as in the spontaneous operations of nature such instances seldom occur, the instances required by this method can be collected by means of experiment. So though the method of Difference may be applied either (1) by observation on finding two instances (direct assemblages of conditions) differing only in one phenomenon together with its antecedent or consequent or (2) by experiment and then either *(a)* by preparing two instances that may be compared side by side or *(b)* by taking certain coditions, and then introducing (or subtracting) some agent, supposed to be the cause, to see what happens, yet the method requires that the two instances differ in only one thing which cannot be assertained unless all the other conditions are known and this further implies that the instances are collected by means of experiment. Indeed when this method is not used under laboratory conditions it becomes the fertile source of the fallacy known as *post hoc ergo propter hoc.*

Its disadvantages : (1) Being a method of experiment it cannot proceed from effects to causes.

(2) It cannot successfully deal with the plurality of causes.* Though this method is largely free from this defect, yet it does not enable us to deal completely with the plurality of causes. For it can at least show that *A* is *a* cause of *p* but it can never show that it is the cause of *p*.

*["Nor does the Method of Difference which is undoubtedly the most cogent of the methods, avoid the difficulty of a plurality of cause. It is true that if in a situation ABC—X the withdrawal of A is followed by the withdrawal of A, then it can be asserted that in *this* situation A is the cause of X. There may, however, be another situation DBC—X such that the withdrawal of D is followed by the withdrawal of X and in that case D is the cause of X. The possibility of a plurality of causes can be avoided only by increasing the negative analogy." (Stebbing.)]

(3) Intermixture of effects or the composition of causes frustrate this method. We cannot say that A is the cause of *p* for it may be that B or C is the cause of *p*, but it can produce it only when it is aided by A. A thus may be a condition of *p*, or merely the medium through which the cause of *p* operates.

(4) "There are, however, many cases in which though our power of prducing the phenomenon is complete, the method of Difference either cannot be made available at all or not without a previous employment of the Method of Agreement. This

occurs when the agency by which we can produce the phenomenon is not that of one single antecedent, but a combination of antecedents, which we have no power of separating from each other and exhibiting apart. For instance, suppose the subject of inquiry to be the cause of the double refraction of light. We can produce this phenomenon at pleasure, by employing anyone of the many substances which are known to refract light in that peculiar manner. But, if taking one of those substances, as Iceland Spar, for example, we wish to determine on which of the properties of Iceland Spar this remarkable phonomenon depends. We can make no use for that purpose of the Method of Difference; for we cannot find another substance precisely resembling Iceland Spar except in some one property. The only mode, therefore, of prosecuting this inquiry is that afforded by the Method of Agreement; by which, in fact, though a comparison of all the known substances which have the property of doubly refracting light, it was ascertained that they agree in the circumstance of being crystalline circumstances; and though the converse does not hold, though all crystalline substances have not the property of double refraction, it was concluded with reason, that there is a real connection between these two properties; that either crystalline structure or the cause which give rise to that structure, is one of the conditions of double refraction." (Mill).

Its Advantages: (*i*) When used under laboratory conditions, this method can establish a causal connection. That is, a single perfect experiment according to this method is sufficient to prove causation.

(*ii*) The supposed plurality of causes does not affect this method.

(*iii*) This method without our being distinctly aware of it, is oftener than any other the basis of ordinary judgment.

The Method of Agreement and the Method of Difference: "By the method of Difference alone that we can ever, in the way of direct experience, arrive with certainty at causes. The Method of Agreement leads only to laws of phenomena, that is to uniformities, which either are not laws of causation or in which the question of causation must for the present remain undecided. The Method of Agreement is chiefly to be resorted to as a means of suggesting applications of the Method of Difference or as an inferior resource in case the Method of Difference

is impracticable. And hence, it is the Method of Agreement, though applicable in principle to either case, is more emphatically the method of investigation on those subjects where artificial experimentation is impossible; while in phenomena which we can produce at pleasure, the Method of Difference generally affords a more efficacious process, which will ascertain causes as well as mere laws." (Mill).

THE METHOD OF CONCOMITANT VARIATIONS

Its canon (after Mill). Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner is either a cause or an effect of that phenomenon or is connected with it through some fact of causation.

This is not an entirely fresh method, but may be regarded as a special case either of the Method of Agreement or of Difference. The equality of cause and effect is the foundation of this method. The application of this method is often illustrated by what is called the Graphic method. The method of Correlation used in Psychology and similar science is also a form of this method.

Its disadvantages: *(i)* This method cannot deal with qualitative variations. It can be used only in cases of quantitative variations. Even when there are two parallel series of phenomena, the one quantitative and the other qualitative—like the rate of air vibration and the pitch of sound—the method of variations is not applicable.

(ii) It is not an entirely fresh method, and so is subject to the imperfections of those methods of which it is a special case.

(iii) It cannot with certainty be always extended to cases beyond the limits of our actual observation. For it may be the cases, that quantitative change *i.e.*, difference in degree may initiate some qualitative change and thereby render this method quite powerless. Thus water which contracts as temperature falls becomes ice at the freezing point and begins to expand even though temperature steadily decreases.

Its uses: (1) There are certain forces, such as gravitation, heat, friction, that can never be eliminated altogether and therefore can only be studied in their degrees. To such phenomena

the Method of Difference cannot be applied, because there are no negative instances. But we may obtain negative instances of a given quantity of such a phenomenon and may apply the method of difference to that quantity. Thus the peculiar use of the method of variations is to formulate conditions of proof in respect of those causes or effects which cannot be entirely got rid of, but can be obtained only in greater or less amount; and such phenomena are, of course, quantitative.

(2) So long as the quantitative relation between the cause and the effect is not ascertained, the scientific inquiry, when the quantity of the cause and the effect can be measured, cannot be complete. That is why the more a science makes progress the more it becomes mathematical. Consequently even when the cause or the effect of an event is ascertained by the application of the other methods, the application of this method is necessary. Thus, this method plays an important part in scientific inquiry.

THE METHOD OF RESIDUES

Its canon (after Mill): Subduct from any phenomenon such part as is known by previous inductions to be the effect of a certain antecedents, and the residue of the phenomena is the effect of the remaining antecedents.

This also is not a fresh method but a special case of the method of difference. The difference between the two methods lies precisely in the fact that the instance in which the circumstance in which alone the two instances differ does not occur is obtained in the method of Difference inductively *i.e.*, by experiment while in this method it is obtained deductively *i.e.*, by deduction from previous inductions.

Its special value :[1] The special value of this method appears when some complex phenomenon has been for the most part accounted for by known causes, whilst there remains some excess or shortcoming or deviation from the result which those causes alone would lead us to expect and this residuary fact has to be explained.

[1]Note ? [The method of Residues is one of the most important among our instruments of discovery of all the methods of investigating laws of nature, this is the most fertile in unexpected results—'Mill'.

Almost all the greatest discoveries in Astronomy have resulted from the consideration of residual phenomena of a quantitative or numerical kind.—(Herschel).]

IS THE METHOD DEDUCTIVE ?—Here the negative instance is constituted be deduction, showing what would happen but for the interference of some unknown cause which is to be investigated ; and this prominence of the deductive process has led some writers to class this process as deductive. But all the canons involve deduction in and considering how much in every experiment is assumed as already known, the wonder is that no one has insisted upon regarding every method as concerned with residues. In fact, as scientific explanation progresses the importance of this method increases.

ITS DEFECTS : It is subject to the imperfections of the method of Difference. Moreover, as the negative instance is here obtained by deduction there is the danger of making wrong calculations.

EXERCISES WITH HINTS

1. *How is the method of Residues related to the Method of Difference ?*

Do you consider the method of Residues to be a Deductive Method? Discuss the question thoroughly. (C. U. '53)

[See para 3 and 4 of sec. 5].

2. *Why are the Experimental Methods so called ? Explain and illustrate the chief difficulties which tend to frustrate the Experimental Methods indicating clearly the ways how they are overcome.* (C.U. '40)

[See para 1 of summary—the difficulties that tend to frustrate these methods are the plurality of causes and inter-mixture of effects—the difficulty of plurality of causes can be overcome by the multiplication of instances, the application of the Joint Method and calculation of probability and Elimination of chance ; the difficulty of intermixture of eflects can be overcome by the application of the Method of Residues and the Deductive method.

These methods are called experimental as the analysis involved in these methods is not mental. "No mere contemplation of

the phenomena and the partition of them by the intellect alone will itself accomplish the end. Such a mental partition is an indispensable first step." But "this will not of itself tell on which of the antecedents each consequent is invariably attendant. To determine that point, we must endeavour to effect a separation of the facts from one another, not in our minds only, but in nature." So the scientific inquiry which proceeds by varying the circumstances is so to say physical also and not merely mental. And hence these methods are called experimental methods.

Read also carefully, the sections on the Method of Agreement and the Method of Difference. They are the fundamental Methods and their imperfections are so to say the imperfections of all the methods.]

3. *Explain why the Method of Agreement requires many instances while the Method of Difference is satisfied with one precise experiment.* (C. U. '41)

[This is so, as the principle of elimination employed in the two methods differs.]

4. *Explain the relative advantages and disadvantages of the Method of Agreement and Method of Difference.*

5. *Show in what respect the Method of Residues (a) agrees with and (b) differs from the Method of Difference. Illustrate the working of the former in Natural Science.*

6. *"The Method of Agreement is essentially the method of observation, the method of Difference of Experiment"—Discuss. Illustrate your answer by examples.* (C. U. '42)

[See summary]

7. *"The Method of Concomitant variation is only a modification of the Method of Agreement"—Discuss.* (C. U. '43)

[If by 'only' is meant that it is not a fresh method, the contention is correct. But if by 'only' is meant that it is a modification of the method of agreement *alone* the contention is incorrect, as it is also a modification of the Method of Difference.]

8. *"The chief use of the Method of Agreement is to suggest hypothesis as to the cause"—Discuss.* (C.U. '44).

9. *Explain the method of Concomitant variation and show how it differs from the other methods.* (C.U. '45).

10. *What is the importance of the negative instance in inductive reasoning?* (C.U. '45)

[See paras 5 and 6 of sec. 1].

11. *How does Mill's Method of Agreement differ from simple Enumeration ? Explain with illustration.* (C.U. '46).

[See paras 5, 6 and 12 of sec. 1].

12. *What is Plurality of Causes ? Plurality of Causes vitiates Mill's method of Agreement. Is there any way to eliminate this defect of the Method of Agreement ?* (C.U. '47).

13. *What are the qualitative marks of a cause ? How do you connect the inductive Methods with causation ?*

[See sec. 2 of ch. XIV., sec. 7 of ch. XV and first two paras of the summary of ch. XVI.]

14. *"To vary the circumstances is the fuudamental principle upon which the Experimental Method are bases"—Explain.* (C.U. '48).

[These methods involves analysis and elimination. Now the analysis involved is not merely mental. It is so to say physical also. "In every instance which comes under our observation, there are many antecedents and many consequents. If those antecedents could not be served from one atother except in thought or if those consequents never were found apart, it would be impossible for us to distinguish the real laws or to assign to any cause its effect or to any effect its cause. To do so we must be able to meet with some of the antecedents apart from the rest and observe what follows from them ; or some of the consequents and observe by what they are preceded. We must in short, follow the Baconian rule of *varying the circumstances*. This is, indeed, only the first rule of physical inquiry, and not as some have thought the sole rule ; but it is the foundation of all the rest".]

15. *State and explain the canon of the Joint Method.* (C.U. '48)

16. *State and explain the canon of the Method of Difference. Do you think the conditions which the Method of Difference requires can be fulfilled in any situation of our experience ?* (C.U. '49)

[No, only instances obtained under laboratory conditions can satisfy this method.]

17. **Would you admit Plurality of Causes ? Discuss in brief whether or how far the five Experimental Methods are workable in the case of Plurality of Causes.* (C.U. '50)

18. *Distinguish between the Method of Difference and the Method of Residue. Why do these two methods often yield sure conclusions ?* (C.U. '51).

[Because they rest upon a principle of elimination that is cogent].

19. *Explain and illustrate the Joint Method. What are the special cases for which the Method of Concomitant variation is particularly recommended?* (C.U. '51)

20. *Show which of the experimental methods are based mainly on observation and which on experiment?* (C.U. '52)

[The method of Agreement, the Joint Method, and the method of Concomitant Variation as a modification of the Method of Agreemnt are based on observation. The rest are based on experiment. Thus the methods that rest upon the principle of elimination on which the method of Agreemt rests are methods of observation while the methods that rest upon the principle of elimination on which the method of Difference rests are the methods of Experiment.]

21. *Explain the method of concomitant variations. How is it related to the method of difference?* (M.U. '40)

22. *Show that the Joint Method of Double Agreement and Difference is really a Method of Double Agreement.* (M.U. '41)

[See summary].

23. *What grounds are there for the view that the Method of Difference and the Method of Concomitant variations are not distinct?* (M.U. '41)

24. *Describe the Method of Difference and state what precautions have to be taken in applying this method.* (M.U. '41)

25. *"The Inductive Methods are weapons of Elimination"—Discuss.* (M.U. '41)

26. *Explain the Joint Method of Agreement and Difference. Show by means of an illustration how it overcomes the weakness of the method of Agreement.* (M.U. '44)

27. *"Mill's Inductive Methods are all reducible to one principle, the elimination of the inessential"—Explain and discuss.* (M.U. '43)

28. *Are Mill's 'Experimental Methods' rightly called experimental? Examine.* (M.U. '47)

29. *Explain and illustrate the Method of Residues. What are the limitation of this Method?* (M.U. '48)

30. *What are the advantages of the Method of Difference over the Method of Agreement? What advantage has the method of Agreement over the Method of Difference?* (M.U. '48)

31. *Discuss whether Mill's Inductive Methods are methods of proof or methods of discovery.* (M.U. '49)

["Mill defined Logic as the science of proof. He insisted that it is 'the business of Inductive Logic to provide rules and models to which if inductive arguments conform, those arguments are conclusive and not otherwise.' He adds : 'This is what the Four Methods profess to be.' Thus Mill conceives the Methods as models or Schemata, comparable to the figures of the syllogism, into which inductive investigation may be fitted. The canon in each case is regarded as standing to its method, as for example, the *dictum de omni* stands to the first figure of the syllogism. Owing to the difficulties arising from the plurality of causes Mill considered that only the Method of Difference* was completely cogent but there can be no doubt that, according to Mill, were it not for these difficulties, each Method would have to be regarded as both demonstrative and complete in itself'—(Stebbing). So, "the fact is, the methods have two entirely different functions to perform. In the first place, they afford us a general line of inquiry in scientific investigation. In using them for this purpose, we have to be particularly on our guard against their imperfections and their inherent danger ; and many of these though pointed out by Mill himself are wisely emphasized by Professor Welton, as well as by such well-known logicians as Bradley and Sigwart. In the second place, they form admirable lists of logical proof in scientific investigation, just as the syllogism is a useful testing instrument when doubt has arisen over a praticular deductive inference. The Methods set us on the road 'to discovery' though they do not take us quite to the goal". (Westaway)].

* *(That is, the methods others than the method of difference are methods of discovery.)*

CHAPTER XVII

COMBINATION OF INDUCTION WITH DEDUCTION

§ 1. DEDUCTIVE CHARACTER OF FORMAL INDUCTION: We have now reviewed Mill's five Canons of Inductive Proof. At bottom, as he observes, there are only two, namely, Agreement and Difference: since the Double Method, Variations and Residues are only special forms of the other two. Indeed, in their function of *proof*, they are all reducible to one, namely, Difference; for the cogency of the method of Agreement (as distinguished from a simple enumeration of instances agreeing in the coincidence of a supposed cause and its effect), depends upon the omission, in one instance after another, of all other circumstances; which omission is a point of difference.

In their function of proof, the methods are all reducible to one, namely, Difference.

The Canons are an analysis of the conditions of proving directly (where possible), by means of observation or experiment, any proposition that predicates causation. But if we say 'by means of observation or experiment,' it is not to be understood that these are the only means and that nothing else is involved; for it has been shown that the Law of Causation is itself an indispensable foundation of the evidence. In fact, Inductive Logic may be considered as having a purely formal character. It consists (i) in a statement of the Law of Cause and Effect; (ii) in certain immediate inferences from this Law, expanded into the Canons; (iii) in the syllogistic application of the Canons to special predications of causation by means of minor premises, showing that certain instances satisfy the Canons.

Inductive Logic may be considered as having a purely formal character.

At the risk of some pedantry, we may exhibit the process as follows (*cf.* Prof. Ray's *Logic*: Appendix D):

Whatever relation of events has certain marks is a case of causation;

The relation A : *p* has some or all of these marks (as shown by observation and by the conformity of instances to such or such a Canon):

Therefore, the relation A : *p* a case of causation. Now, the parenthesis, "as shown by the conformity, *etc.*," is an adscititious member of an Epicheirema, which may be stated, as a Prosyllogism, thus:

Form of induction.

If an instance, etc. (Canon of Difference);

The instances $\frac{A\ B\ C}{p\ q\ r}$, $\frac{B\ C}{q\ r}$ are of the kind required:

Therefore, A, present where *p* occurs and absent where it does not occur, is an indispensable antecedent of *p*.

Such is the bare Logic of Induction: so that, strictly speaking, observation or experiment is no part of the Logic, but a means of applying the logic to actual, that is, not merely symbolical, propositions. The formal Logic of Induction is essentially deductive; and it has been much questioned whether any transition from the formal to the material conditions of proof is possible. As long as we are content to illustrate the Canons with symbols, such as A and *p*, all goes well; but can we in any actual investigation show that the relevant facts or 'instances' correspond with those symbols?

Is any transition from the formal to the material conditions of proof possible?

In the first place, as Dr. Venn shows, natural phenomena want the distinctness and capability of isolation that belong to symbols. Secondly, the observing whether instances conform to a Canon, must always be subject at last to the limits of our faculties. How can we ascertain exact equality, immediate sequence? The Canon of Difference, in its experimental application, is usually considered the most cogent sort of proof: yet when can the two sequent instances, before and after the introduction of a certain agent,

Objections:

be said to differ in nothing else? Are not earth and stars always changing position; is not every molecule in the room and apparatus always oscillating? It is true that our senses are now aided by elaborate instruments; but the construction of these depends on scientific theories, which again depend on experiments.

It is right to touch upon this well-known sceptical topic; but to insist much upon it is not a sign of good sense. The works of Herschel, Whewell, and Jevons should be consulted for the various methods of correcting observations, by repeating them, averaging them, verifying one experimental process by another, always refining the methods of exact measurment, multiplying the opportunities of error (that if any exist it may at last show itself), and by other devices of what may be called Material Logic or Methodology. But only direct experience and personal manipulation of scientific processes, can give a just sense of their effectiveness; and to stand by, suggesting academic doubts, is easier and more amusing.

But to insist much upon them is not good sense.

§ 2. Further complication of Deduction with Induction: Still, it is not so much in laws based upon direct observation or experiment, that the material validity of scientific reasoning appears, as in the cumulative evidence that arises from the co-ordination of laws within each science, and the growing harmony and coherence of all sciences. This requires a more elaborate combination of deduction with observation and experiment. During the last three hundred years many departments of science have been reduced under principles of the greatest generality, such as the Conservation of Energy, the Law of Gravitation, the Undulatory theory of Light, the Law of combining Equivalents, and the Theory of Natural Selection; connecting and explaining the less general laws, which, again, are said to connect and explain the facts.

The material validity of scientific reasoning requires an elaborate combination of deduction with observation and experiment.

Meanwhile, those sciences that were the first to make progress have helped to develop others which, like Biology and Sociology, present greater difficulties; and it becomes more and more apparent that the distinctions drawn among sciences are entirely for the convenience of study, and that all sciences tend to merge in one universal Science of Nature. Now, this process of the 'unification of knowledge' is almost another name for deduction; but at the same time it depends for its reality and solidity upon a constant reference to observation and experiment. Only a very inadequate notion of it can be given in the ensuing chapters.

In cases of heteropathic intermixture of effects the inductive canons hold good.

We saw in chap. xiv. § 6, that when two or more agents or forces combine to produce a phenomenon, their effects are intermixed in it, and this in one of two ways according to their nature. In chemical action and in vegetable and animal life, the causal agents concerned are blended in their results in such a way that most of the qualities which they exhibited severally are lost, whilst new qualities appear instead. Thus chlorine (a greenish-yellow gas) and sodium (a metal) unite to form common salt $NaCl$; which is quite unlike either of them: a man eats bread, and it becomes muscle, nerve and bone. In such cases we cannot trace the qualities of the causal agents in the qualities of the effects; given such causes, we can prove experimentally, according to the canons of induction, that they have such effects; but we may not be able in any new case to calculate what the effects will be.

But they do not in cases of homogeneous intermixture of effects,

On the other hand, in Astronomy and Physics, the causes treated of are mechanical; at least, it is the aim of Physics to attain to a mechanical conception of phenomena; so that, in every new combination of forces, the intermixed effect, or resultant, may be calculated beforehand; provided that the forces concerned admit of being quantitatively estimated, and that the conditions of their combination are not so

complex as to baffle the powers of mathematicians. In such cases, when direct observation or experiment is insufficient to resolve an effect into the laws of its conditions, the general method is to calculate what may be expected from a combination of its conditions, as either known or hypothetically assumed, and to compare this anticipation with the actual phenomenon.

the laws of its conditions the general method is to use deduction also.

§ 3. The Direct Deductive (of Physical) Method : This is what Mill calls the Direct Deductive Method ; or, the Physical Method, because it is so much relied on in treating of Light, Heat, Sound, *etc.*; it is also the method of Astronomy and much used in Economics : Deduction leads the way, and its results are tested inductively by experiments or observations. Given any complex mechanical phenomenon, the inquirer considers—(1) what laws already ascertained seem likely to apply to it (in default of known laws, hypotheses are substituted : *cf.* chap. xviii.) ; he then—(2) computes the effect that will follow from these laws in circumstances similar to the case before him ; and (3) he verifies his conclusion by comparing it with the actual phenomenon.

Direct Deductive Method : involves (i) considering laws already, ascertained, of separate causes (ii) computation of these laws into a result.

A simple example of this method is the explanation of the rise of water in the 'common-pump'. We know three laws applicable to this case : (*a*) that the atmosphere weighs upon the water outside the pump with a pressure of 15 lb. to the square inch ; (*b*) that a liquid (and therefore the water) transmits pressure equally in all directions (upwards as well as downwards and side ways) ; and (*c*) that pressure upon a body in any direction, if not counteracted by an opposite pressure, produces motion. Hence, when the rise of the piston of the pump removes the pressure upon the water within the cylinder tending to produce

A simple example :

a vacuum there, this water is pushed up by the pressure of the air upon the water outside the cylinder, and follows the rising piston, until the column of water inside the cylinder exerts a pressure equal to that of the atmosphere upon an equal area. So much for the computation; does it correspond with the fact? It is found that at the sea level water can be pumped to the height of 33 ft.; and that such a column of water has a pressure of 15 lb. to the square inch. We may show further that, at the sea level, spirits of wine may be pumped higher according to its less specific gravity; and that if we attempt to pump water at successive altitudes above the sea level, we can only raise it to less and less heights, corresponding with the lessened atmospheric pressure at those altitudes, where the column of air producing the pressure is shorter. Finally, if we try to work a pump, having first produced a vacuum over the water outside the cylinder, we shall find that the water inside will not rise at all; the piston can be raised, but the water does not follow it. The verification thus shows that the computed effect corresponds with the phenomenon to be explained; that the result does not depend upon the nature of water only, but is true (allowing for differences of specific gravity) of other liquids; that if the pressure of the outside air is diminished, the height of pumping is so too (canon of Variations); and that if that pressure is entirely removed, pumping becomes impossible (canon of Difference).

Other examples:

Any text-book of Astronomy or Physics furnishes numerous illustrations of the deductive method. Take, for example, the first chapter of Deschanel's *Optics*, where are given three methods of determining the velocity of Light. This was first deduced from observation of Jupiter's satellites. The one nearest the planet passes behind it, or into its shadow, and is eclipsed, at intervals of about $42^1/_2$ hours. But it can be shown that, when Jupiter and the earth are nearest together on the same side of the Sun, an eclipse of this satellite is visible from the earth 16

min. 26·6 sec. earlier than when Jupiter and the earth are furthest apart on opposite sides of the Sun : 16 min. 26·6 sec., then, is the time in which light traverses the diameter of the Earth's orbit. Therefore, supposing the Earth's distance from the Sun to be 92 millions of miles, light travels about 186,000 miles a second. Another deduction, agreeing with this, starts from the fact of aberration, or the displacement of the apparent from the actual position of the stars in the direction of the earth's motion. Aberration depends partly on the velocity of light, partly on the velocity of the Earth ; and the latter being known, the former can be computed. Now, these two deductive arguments, verifying each other, have also been verified experimentally. Foucault's experiment to measure the velocity of ligh is too elaborate to be described here : a full account of it will be found in the treatise above cited, § 687.

When the phenomena to be explained are of such a character, so vast in exient, power or duration, that it is impossible, in the actual circumstances of the case, to frame experiments in order to verify a deductive explanation, it may still be possible to reproduce a similar phenomenon upon a smaller scale. Thus Monge's explanation of mirage by the great heat of the desert sand, which makes the lowest stratum of air less dense than those above it, so that rays of light from distant objects are refracted in descending, until they are actually turned upwards again to the eye of the beholders, giving him inverted images of the objects as if they were reflected in water, is manifestly incapable of being verified by experiment in the natural conditions of the phenomenon. But by heating the bottom of "a sheet-iron box, with its ends cut away," the rarefied air at the botton of the box may sometimes be made to yield reflections ; and this shows at least that the supposed cause is a possible one (Deschanel, *Optics*, § 726).

When the deductive calculation admits of no inductive verification under natural conditions, the varification is done by producing a similer phenomena upon a smaller scale.

Similarly as to the vastest of all phenomena, the evolution of the stellar system, and of the solar system as part of it, from an immense cloudlike volume of matter : H. Spencer, in his Essay on *The Nebular Hypothesis*, says, amidst a great array of deductive arguments from mechanical principles, that "this *a priori* reasoning harmonises with the results of experiment. Dr. Plateau has shown that when a mass of fluid is, as far as may be, protected from the action of external forces, it will, if made to rotate with adequate velocity, form detached rings; and that these rings will brake up into spheroids, which turn on their axes in the same direction with the central mass." The theory of the evolution of species of plants and animals by Natural Selection, again, though, of course, it cannot be verified by direct experiment (since experiment implies artificial arrangement), and the process is too slow for observation, is, nevertheless, to some extent confirmed by the practice of gardeners and breeders of animals : since, by taking advantage of accidental variations of form and colour in the planats or animals under their care, and relying on the inheritability of these variations they obtain extensive modifications of the original stocks, and adapt them to the various purposes for which flowers and cereals, poultry, dogs and cattle are domesticated. This shows, at least, that living forms are plastic, and extensively modifiable in a comparatively short time.

Examples :

§ 4. Opportunities of Error in the Physical Method : Suppose, however, that, in verifying a deductive argument, the effect as computed from the laws of the causes assigned, does not correspond with the facts observed : there must then be an error somewhere. If the fact has been accurately observed, the error must lie either in the process of deduction and computation, or else in the premises. As to the process of deduction, it may be very simple and easily revised, as in the above explananion of the common pump ; or it may be

When facts correctly observed do not correspond with deductive calculation, there must be error eithor in deduction or in

the premises.

very involved and comprise long trains of mathematical calculation. If, however, on re-examining the computations, we find them correct, it remains to look for some mistake in the premises.

Mistake in premise may be due to (1) inaccurate ascertainment of the laws or the forces present.

(1) We may not have accurately ascertained the laws, or the modes of operation, or the amounts of the forces present. Thus, the rate at which bodies fall was formerly believed to vary in proportion to their relative weights; and any estimate based upon this belief cannot agree with the facts. Again, the corpuscular theory of light, namely, that the physical cause of light is a stream of fine particles projected in straight lines from the luminous object, though it seemed adequate to the explanation of many optical phenomena, could not be made to agree with the facts of interference and double refraction.

(2) Incorrect conception of the circumstances in which the agents are combined.

(2) The circumstances in which the agents are combined may not have been correctly conceived. When Newton began to inquire whether the attraction of the earth determined the orbit of the moon, he was at first disappointed. "According to Newton's calculations, made at this time," says Whewell, "the moon, by her motion in her orbit, was deflected from the tangent every minute through a space of thirteen feet. But by noticing the space which bodies would fall in one minute at the earth's surface, and supposing this to be diminished in the ratio of this inverse square, it appeared the gravity would, at the moon's orbit, draw a body through more than fifteen feet." In view of this discrepancy he gave up the inquiry for sixteen years, until in 1682, having obtained better data, he successfully renewed it. "He had been mistaken in the magnitude of the earth, and consequently in the distance of the moon, which is determined by measurements of which the earth's radius is the base." It was not, therefore, a mistake as to the law or as to the nature of the

forces concerned (namely, the law of the inverse square and the identity of celestial with terrestrial gravity), but as to the circumstances in which the agents (earth and moon) were combined that prevented his calculations being verified. *(Hist. Ind Sc.*; VII. ii. 3.)

(3) Overlooking of one or more of the agents affecting the result.

(3) One or more of the agents affecting the result may have been overlooked and omitted from the estimate. Thus, an attempt to explain the tides by taking account only of the earth and the moon, will not entirely agree with the facts, since the snn also influences the tides. This illustration, however, shows that when the conclusion of a deductive explanation does not entirely agree with the facts, it is not always to be inferred that the reasoning is, properly speaking, wrong; it may be right as far as it goes, and merely inadequate. Hence *(a)* in such cases an opportunity occurs of applying the Method of Residues, by discovering the agent that must be allowed for in order to complete the explanation. And *(b)* the investigation of a phenomenon is often designedly begun upon an imperfect basis for the sake of simplicity; the result being regarded as a first approximation, to be afterwards corrected by including, one by one, the remaining agents or circumstances affecting the phenomenon, until the theory is complete; that is, until its agreement with the facts is satisfactory.

(4) inclusion of agents of circumstances that do not exist or do not affect the phenomenon.

(4) We may have included among the data of our reasonings agents or circumstances that do not exist or do not affect the phenomenon in question. In the early days of science purely fanciful powers were much relied upon : such as the solid spheres that carried the planets and stars; the influence of the planets upon human destiny; the tendency of everything to seek "its own place" so that fire rises to heaven, and solids fall to the earth; the "plastic virtue" of the soil, which was once thought to have pro-

duced fossils. When, however, such conceptions hindered the progress of explanation, it was not so much by vitiating the deductive method as by putting men off from exact inquiries. More to our present purpose were the supposed cataclysms, or extraordinary convulsions of the earth, a belief in which long hindered the progress of Geology. Again, in Biology, Psychology, and Sociology many explanations have depended upon the doctrine that any improvement of structure or faculty acquired by an individual may be inherited by his descendants: as that, if an animal learns to climb trees, his offspiring have a greater aptitude for that mode of life; that if a man tries to be good, his children find it easier to be virtuous; that if the inhabitants of a district carry on cloth-work, it becomes easier for each successive generation to acquire dexterity in that art. But now the inheritability of powers acquired by the individual through his own efforts, is disputed; and, if the denial be made good, all such explanations as the above must be revised.

If, then, the premises of a deductive argument be vitiated in any of these four ways, its conclusion will fail to agree with the results of observation and experiment, unless, of course, one kind of error happen to be cancelled by another that is 'equal and opposite.' We now come to a variation of the method of combining Induction with Deduction, so important as to require separate treatment.

§ 5. THE INVERSE DEDUCTIVE (OR HISTORICAL) METHOD: The Inverse or Historical Method has of late years become remarkably fruitful. When the forces determining a phenomenon are too numerous or too indefinite, to be combined in a direct deduction, we may begin by collecting an empirical law of the phenomenon (as that 'the democracies of City-States are arbitrary and fickle'), and then endeavour to show by deductions from "the nature of the case," that is, from a consideration of the circumstances and forces known to be operative (of which, in the above ins-

In the inverse deductive method, deduction verifies previous induction.

tance, the most important is sympathetic contagion), that such a law was to be expected. Deduction is thus called in to verify a previous induction; whereas in the 'Physical Method' a deduction was verified by comparing it with an induction or an experiment; hence the method now to be discussed has been named the Inverse Deductive Method.

The results of this method are indefinite.

But although it is true that, in such inquiries as we are now dealing with, induction generally takes the lead; yet I cannot think that the mere order in which the two logical processes occur is the essential distinction between the two ways of combining them. For, in the first place, in investigations of any complexity both induction and deduction recur again and again in whatever order may be most convenient; and, in the second place, the so-called 'inverse order' is sometimes resorted to in Astronomy and Physics. For example, Kepler's Laws were first collected empirically from observations of the planetary motions, and afterwards deduced by Newton from the Law of Gravitation; this, then, was the Inverse Method; but the result is something very different from any that can be obtained by the Historical Method. The essential difference between the Physical and Historical Methods is that, in the former, whether Direct or Inverse, the deductive process, when complete, amounts to exact demonstration; whereas, in the latter, the deductions may consist of qualitative reasonings, and the results are indefinite. They establish—(1) a merely probable connection between the phenomena according to an empirical law (say, between City-democracy and fickle politics); (2) connect this with other historical or social generalisations, by showing that they all alike flow from the same causes, namely, from the nature of races of men under certain social and geographical conditions; and (3) explain why such empirical laws may fail, according to the differences that prevail among races of men and among the conditions under which they live. Thus, seeing how rapidly excitement is propagated by the chatter, grimacing, and gesti-

culation of townsmen, it is probable enough that the democracy of a City-state should be fickle (and arbitrary, because irresponsible). A similar phenomenon of panic, sympathetic hope and despair, is exhibited by every stock-exchange, and is not peculiar to political life. And when political opinion is not manufactured solely in the reverberating furnace of a city, fickleness ceases to characterise democracy; and, in fact, is not found in Switzerland or the United States, nor in France so far as politics depend upon the peasantry.

It is used specially in the historical sciences.

This is called the Historical Method, then, because it is especially useful in explaining the movements of history, and in verifying the genralisations of political and social science. We must not, however, suppose that its use is confined to such studies. Only a ridiculous pedantry would allot to each subject its own method and forbid the use of any other; as if it were not our capital object to establish truth by any means. Wherever the forces determining a phenomenon are too numerous or too indefinite to be combined in a deductive demonstration, there the Historical Method is likely to be useful; and this seems often to be the case in Geology and Biology, as well as in the Science of History, or Sociology, and its various subsidiary studies.

The forces determining historical events are too numerous and too indefinite.

Consider upon what causes historical events depend: the customs, character, and opinions of all the people concerned; the organisation of their government, and the character of their religious institutions; the development of industry among them, of the military art, of fine art, literature and science; their relations, commercial, political and social, with other nations; the physical conditions of climate and geographical position amidst which they live. Hardly an event of importance occurs in any nation that is not, directly or indirectly, influenced by every one of these circumstances, and that does not react upon them.

Now, from the nature of the Canons of direct Induction, a satisfactory employment of them in such a complex and tangled situation as history presents, is rarely possible; for they all require the actual or virtual isolation of the phenomenon under investigation. They also require the greatest attainable immediacy of connection between cause and effect; whereas the causes of social events may accumulate during hundreds of years. In collecting empirical laws from history, therefore, only very rough inductions can be hoped for, and we may have to be content with simple enumeration. Hence the importance of supporting such laws by deduction from the nature of the case, however faint a probability of the asserted connection is thereby raised; and this even if each law is valued merely for its own sake. Still more, if anything worth the name of Historical Science is to be constructed, must a mere collection of such empiricisms fail to content us; and the only way to give them a scientific character is to show deductively their common dependence upon various combinations of the same causes. Yet even those who profess to employ the Historical Method often omit the deductive half of it; and of course 'practical politicians' boast of their entire contenment with what they call 'the facts.'

So here very rough induction can be hoped for, and they ought to be verified deductively.

Sometimes, however, politicians, venturing upon deductive reasoning, have fallen into the opposite error of omitting to test their results by any comparison with the facts: arguing from certain 'Rights of Man,' or 'Interests of Classes,' or 'Laws of Supply and Demand,' that this or that event will happen, or ought to happen, without troubling themselves to observe whether it does happen or ever has happened. This method of Deduction without any empirical verification, is called by Mill the Geometrical; and, plainly, it can be trustworthy only when

The Geometrical method consists in Deduction without empirical verification.

there is no actual conflict of forces to be considered. In pure mathematical reasoning about space, time, and number, provided the premises and the reasoning be correct, verification by a comparison with the facts may be needless, because there is no possibility of counteraction. But when we deal with actual causes, no computation of their effccts can be relied upon without comparing our conclusions with the facts: not even in Astronomy and Physics, least of all in Politics.

How the statesman should proceed.

Burke, then, has well said that "without the guide and light of sound, well-understood principles all our reasoning in politics, as in everything else, would be only a confused jumble of particular facts and details without the means of drawing any sort of theoretical or practical conclusion"; but that, on the other hand, the statesman, who does not take account of circumstances, infinite and infinitely combined, "is not erroneous, but stark mad—he is metaphysically mad" (*On the Petition of the Unitarians*). There is, or ought to be, no logical difference between the evidence required by a statesman and that appealed to by a philosopher; and since, as we have seen, the combination of principles with circumstances cannot, in solving problems of social science, be made with the demonstrative precision that belongs to astronomical and physical investigations, there remains the Historical Method as above described.

Examples of the empirical laws obtained by the application of the historical method.

Examples of the empirical laws from which this method begins abound in histories, newspapers, and political discussions, and are of all shades of truth or half-truth: as that 'History consists in the biographies of great men'; in other words, that the movements of society are due to exceptional personal powers, not to general causes; That at certain epochs great men occur in groups; That every Fine Art passes through periods of development, culmination and decline; That Democracies tend to change into Despotisms; That the possession of power, whether by classes or

despots, corrupts the possessor; That 'the governments most distinguished for sustained vigour and abilities have generally been aristocracies'; That 'revolutions always begin in hunger'; That civilisation is inimical to individuality; That the civilisation of the country proceeds from the town; That 'the movement of progressive societies has hitherto been a movement from *Status* to *Contract* (i.e., from a condition in which the individual's rights and duties depend on his caste, or position in his family as slave, child, or patriarch, to a condition in which his rights and duties are largely determined by the voluntary agreements he enters into)'; and this last is treated by H. Spencer as one aspect of the law first stated by Comte, that the progress of societies is from the military to the industrial state.

The deductive process we may illustrate by Spencer's explanation of the co-existence in the military state of those specific characters, the inductive proof of which furnished an illustration of the method of Agreement (ch. xvi. § 1). The type of the military State involves the growth of the warrior class, and the treatment of labourers as existing solely to support the warriors; the complete subordination of all individuals to the will of the despotic soldier-king, their property, liberty and life being at the service of the State; the regimentation of society, not only for military, but also for civil purposes; the suppression of all private associations, *etc.* Now all these characteristics arise from their utility for the purpose of war, a utility amounting to necessity if war is the State's chief purpose. For every purpose is best served when the whole available force co-operates toward it: other things equal, the bigger the army the better; and to increase it, men must be taken from industry, until only just enough remain to feed and equip the soldiers. As this arrangement is not to everybody's taste, there must be despotic control; and this control is most effective through regimentation by grades of command. Private associations, of course, cannot live openly in such a State, because they may have wills of their own and are convenient for conspiracy. Thus the in-

duction of characteristics is verified by a deducation of them from the nature of the case.

§ 6. Precautions in using the Historical Method: The greater indefiniteness of the Historical compared with the Physical Method, both in its inductions and in its deductions, makes it even more difficult to work with. It wants much sagacity and more impartiality; for the demon of Party is too much with us. Our first care should be to make the empirical law as nearly true as possible, collecting as many as we can of the facts which the law is supposed to generalise, and examining them according to the canons of Induction, with due allowance for the imperfect applicability of those canons to such complex, unwieldy, and indefinite instances. In the examples of such laws given above, it is clear that in some cases no pains have been taken to examine the facts. What is the inductive evidence that Democracies change into Despotisms; that revolutions always begin in hunger; or that civilisation is inimical to individuality? Even Mill's often quoted saying, "that the governments remarkable in history for sustained vigour and ability have generally been aristocracies," is oddly over-stated. For if you turn to the passage (*Rep. Gov. chap.* vi), the next sentence tells you that such governments have alway been aristocracies of public functionaries; and the next sentence but one restricts, apparently, the list of such remarkable governments to two—Rome and Venice. Whence, then, comes the word "generally" into Mill's law?

We should be extremely careful in the application of this method.

As to deducing our empirical law from a consideration of the nature of the case, it is obvious that we ought —(*a*) to take account of all the important conditions; (*b*) to allow weight to them severally in proportion to their importance; and (*c*) not to include in our estimates any condition which we cannot show to be probably present and operative. Thus the Great-Man-Theory of history must surely be admitted to assign

The precautions to be taken.

a real condition of national success. The great man organises, directs, inspires: is that nothing? On the other hand, to recognise no other condition of national success is the manifest frenzy of a mind in the mythopœic age. We must allow the great man his due weight, and then inquire into the general conditions that (*a*) bring him to birth in one nation rather than another, and (*b*) give him his opportunity.

An examination of Mill's application of this method.

Mill's explanation of the success of the aristocratic governments of Rome and Venice is, that they were, in fact, bureaucracies; that is to say, their members were trained in the science and art of administration and command. Here, again, we have, no doubt, a real condition; but is it the only one? The popular mind, which little relishes the scaling down of Mill's original law to those two remote cases, is persuaded that an aristocracy is the depository of hereditary virtue, especially with reference to government, and would at once ascribe to this circumstance the greater part of the success of any aristocratic constitution. Now, if the effects of training are inherited, they must, in an hereditary aristocracy, increase the energy of the cause assigned by Mill; but, if not, such heredity is a condition "not present or not operative." Still, if families are ennobled for their extraordinary natural powers of administration or command (as sometimes happens), it is agreed on all hands that innate qualities are inharitable; at least, if care be taken to intermarry with families similarly distinguished, and if by natural or artificial selection all the failures among the offspring be eliminated. The Spartans had some crude notion of both these precautions; and if such measures had been widely adopted, we might deduce from the doctrine of heredity a probability in favour of Mill's original proposition, and thereby verify it in its generality, if it could be collected from the facts.

The Historical Method may be further illustrated by the course adopted in that branch of Social Science which has been

found susceptible of the most extensive independent development, namely, Economics. First, by way of contrast, I should say that the abstract, or theoritical treatment of Economics follows the Physical Method ; because, as Mill explains, although the phenomena of industry are no doubt influenced, like other social affairs, by all the other circumstances of Society, government, religion, war, art, *etc.* ; yet, where industry is most developed, as in England and the United States, certain special conditions affecting it are so much the most importan that, for the purpose at least of a first outline of the science, they may conveniently be considered as the only ones. These conditions are : (1) the general disposition of men to obtain wealth with as little trouble as possible, and (2) to spend it so as to obtain the greatest satisfaction of their various desires ; (3) the facts that determine population ; and (4) the tendency of extractive industry, when pushed beyond a certain limit without any improvement in the industrial arts, to yield "diminishing returns." From these premises it is easy to infer the general laws of prices, of wages and interest (which are the prices of labour and of the use of capital), and of rent ; and it remains to verify these laws by comparing them with the facts in each case ; and (if they fail to agree with the facts) to amend them, according to the Method of Residues, by taking account of those influential conditions which were omitted from the first draft of the theory.

Its application in Economics

Whilst, however, this is usually the procedure of those inquirers who have done most to give Economics its scientific character, to insist that no other plan shall be adopted would be sheer pedantry; and Dr. Keynes has shown, in his *Scope and Method of Political Economy*, that Mill has himself sometimes solved economic problems by the Historical Method. With an analysis of his treatment of Peasant proprietorship (*Political Economy*, B. II., cc. 7 and 8) we may close this section. Mill first shows inductively, by collecting evidence from Switzerland, Germany, Norway, Belgium, and France (countries differing in

race, government, climate and situation), that peasant proprietors are superhumanly industrious; intelligent cultivators, and generally intelligent men; prudent, temperate, and independent, and that they exercise self-control in avoiding improvident marriages. This group of empirical generalisations as to the character of peasant proprietors he then deduces from the nature of the case: their industry, he says, is a natural consequence of the fact that, however much they produce, it is all their own; they cultivate intelligently, because for generations they have given their whole mind to it; they are generally intelligent men, because the variety of work involved in small farming, requiring foresight and calculation, necessarily promotes intelligence; they are prudent, because they have something to save, and by saving can improve their station and perhaps by more land; they are temperate, because intemperance is incompatible with industry and prudence; they are independent, because secure of the necessaries of life, and from having property to fall back upon; and they avoid improvidence in marriage, because the extent and fertility of their fields is always plainly before them, and therefore how many children they can maintain is easily calculated. The worst of them is that they work too hard and deny themselves too much: but, over the greater part of the world, other peasantry work too hard: though they can scarcely be said to deny themselves too much; since all their labour for others brings them no surplus to squander upon self-indulgence.

The comparative method is used in investigating the history of institutions or the true sense of legends.

§ 7. THE COMPARATIVE METHOD: The foregoing account of the Historical Method is based upon Mill's discussions in B. IV. of his *Logic*, especially cc. 6 to 11. Mill ascribes to Comte the first clear statement of the method; and it is highly scientific, and important in generalising the connections of historical events. But perhaps the expression, 'Historical Method,' is more frequently applied to the Comparative Method, as used in investigating the history of institutions or the true sense of legends.

(1) Suppose we are trying to explain the institution of capital punishment as it now exists in England. (1) We must try to trace the history of it back to the earliest times; for *social custom and tradition is one line of causation.* At present the punishment of death is legally incident only to murder and high treason. But early in the last century malefactors were hung for forgery, sheep-stealing, arson and a long list of other offences down to pocket-picking: earlier still the list included witchcraft and heresy. At present hanging is the only mode of putting a malefactor to death; but formerly the ways of putting to death included also burning, boiling, pressing, beheading, and mixed modes. Before the Restoration, however, the offences punishable with death were far fewer than they afterwards became; and until the twelfth century, the penalty of death might be avoided by paying compensation, the wer-geld.

It assumes that (1) social custom and tradition are caused.

(2) Every change in the history of an institution must be explained by pointing to *the special causes* in operation during the time when the change was in progress. Thus the restriction of the death penalty, in the nineteenth century, to so few offences was due partly to the growth of humane feelings, partly to the belief that the infliction, or threat, of the extreme penalty had failed to enforce the law and had demoralised the administration of Justice. The continual extension of the death penalty throughout the eighteenth century may be attributed to a belief that it was the most effectual means of deterring evil-doers when the means of detecting and apprehending criminals were feeble and ill-organised. The various old brutal ways of execution were adopted sometimes to strike terror, sometimes for vengeance, sometimes from horror of the crime, or even from 'conscientious scruples';—which last were the excuse for preferring the burning of heretics to any sort of bloodshed.

(2) every change in history is to be explained by pointing to the special causes in operation.

(3) Such causes are to be investigated by this method.

(3) The causes of any change in the history of an institution in any country may not be directly discoverable: they must then be investigated by the Comparative Method. Again, the recorded history of a nation, and of all its institutions, followed backwards, comes at last to an end; then the antecedent history must also be supplied by the Comparative Method; whose special use is to indicate the existence of facts for which there is no direct evidence.

Which assumes that where the causes are alike, the effects will be alike, and that similar effects are due to similar causes;

This method rests upon the principle that where the causes are alike the effects will be alike, and that similar effects are traceable to similar causes. Every department of study—Astronomy, Chemistry, Zoology, Sociology—is determined by the fact that the phenomena it investigates have certain common characteristics; and we are apt to infer that any process observed in some of these phenomena, if depending on those common characteristics, will be found in others. For example, the decomposition, or radio-activity, of certain elements prepares one to believe that all elements may exhibit it. Where the properties of an object are known to be closely interdependent, as in the organisation of plants, animals and societies, we are especially justified in inferring from one case to another. The whole animal Kingdom has certain common characters—the metabolic process, dependence upon oxygen, upon vegetable food (ultimately), heredity, etc., and, upon this ground, any process (say, the differentiation of species by Natural Selection) that has been established for some kinds of animal is readily extended to others. If instead of the whole animal Kingdom we take some district of it—Class, Order, Family—our confidence in such inferences increases; because the common characters are more numerous and the conditions of life are more alike; or, in other words, the common causes are more numerous that initiate and control the

development of nearly allied animals. For such reasons a few fragmentary remains of an extinct animal enable the palæontologist to reconstruct with some probability an outline of its appearance, organisation, food, habitat and habits.

and that human nature is approximately uniform.

Applied to History, the Comparative Method rests upon an assumption (which the known facts of (say) 6,000 years amply justify) that human nature, after attaining a recognisable type as *homo sapiens*, is approximately uniform in all countries and in all ages, though more especially where states of culture are similar. Men living in society are actuated by similar motives and reasons in similar ways; they are all dependent upon the supply of food, and therefore on the sun and the seasons and the weather, and upon means of making fire, and so on. Accordingly, they entertain similar beliefs, and develop similar institutions through similar series of changes. Hence, if in one nation some institution has been altered for reasons that we cannot directly discover, whereas we know the reasons why a similar change was adopted elsewhere, we may conjecture with more or less probability, after making allowance for differences in other circumstances, that the motives or causes in the former case were similar to those in the latter, or in any cases that are better known. Or, again, if in one nation we cannot trace an institution beyond a certain point, but can show that elsewhere a similar institution has had such or such an antecedent history, we may venture to reconstruct with more or less probability the earlier history of that institution in the nation we are studying.

Amongst the English and Saxon tribes that settled in Britain, death was the penalty for murder, and the criminal was delivered to the next-of-kin of his victim for execution; he might, however, compound for his crime by paying a certain compensation. Studying the history of other tribes in various parts of the world, we, are able, with much probability, to reconstruct the antecedents of this death-penalty in our own prehistoric

ages, and to trace it to the blood-feud; that is, to a tribal condition in which the next-of-kin of a murdered man was socially and religiously bound to avenge him by slaying the murderer or one of his kindred. This duty of revenge is sometimes (and perhaps was at first everywhere) regarded as necessary to appease the ghost of the victim; sometimes as necessary to compensate the surviving members of his family. In the latter case, it is open to them to accept compensation in money or cattle, *etc.* Whether the kin will be ready to accept compensation must depend upon the value they set upon wealth in comparison with revenge; but for the sake of order and tribal strength, it is the interest of the tribe, or its elders, or chieftain, to encourage or even to enforce such acceptance. It is also their interest to take the questions—whether a crime has been committed, by whom, and what compensation is due—out of the hands of the injured party, and to submit them to some sort of court or judicial authority. At first, following ancient custom as much as possible, the act of requital, or the choice of accepting compensation, is left to the next-of-kin; but with the growth of central power these things are entrusted to ministers of the Government. Then revenge has undergone its full transformation into punishment. Very likely the wrong itself will come to be treated as having been done not to the kinderd of the murdered man, but to the State or the King, as in fact a "breach of the King's peace." This happened in our own history.

(4) the term 'approximately' must not be overlooked.

(4) The Comparative Method assumes that human nature is approximately the same in different countries and ages; but, of course, 'approximately' is an important word. Although there is often a striking and significant resemblance between the beliefs and institutions of widely separated peoples, we expect to draw the most instructive parallels between those who are nearly related by descent, or neighbourhood, or culture. To shed light upon our own manners, we turn first to other Teutons, then to Slavonians and Kelts, or

other Aryans, and so on; and we prefer evidence from Europe to examples from Africa.

(5) As to national culture, that it exhibits certain 'stages' of development is popularly recognised in the distinction drawn between savages, barbarians and civilised folk.

(5) National culture exhibits certain stages.

But the idea remains rather vague; and there is not space here to define it. I refer, therefore, to the classifications of stages of culture given by A. Sutherland, *Origin and Growth of Moral Instinct*, Vol. I, p. 103), and L. T. Hobhouse (*Morals in Evolution* c. 2). That in any 'state of Society', its factor—religion, government, science, *etc.*—are mutually dependent, was a leading doctrine with Comte, adopted by Mill. There must be some truth in it; but in some cases we do not understand social influences sufficiently well to trace the connection of factors; and whilst preferring to look for historical parallels between nations of similar culture, we find many cases in which barbarous or savage customs linger in a civilised country.

(6) It was another favourite doctrine with Comte, also adopted by Mill—that the general state of culture is chiefly determined by the prevailing intellectual condition of a people, especially by the accepted ground of explanation—whether the will of supernatural beings, or occult powers, or physical antecedents: the "law of the three stages," Fetichism, Metaphysics, Positivism. And this also is, at least, so far true, that it is useless to try to interpret the manners and institutions of any nation until we know its predominant beliefs. Magic and animism are beliefs everywhere held by mankind in early stages of culture, and they influence every action of life. But that is not all; these beliefs retain their hold upon great multitudes of civilised men and affect the thoughts of the most enlightened. Whilst the saying 'that human nature is the same in all ages' seems to make no allowance for the fact that, in

(6) The general state of culture is chiefly determined by the prevailing intellectual condition of a people.

some nations, a considerable number of individuals has attained to powers of deliberation, self-control, and exact reasoning, far above the barbarous level, it is yet so far true that, even in civilised countries, masses of people, were it not for the example and instruction of those individuals, would fall back upon magic and animism and the manners that go with such beliefs. The different degrees of enlightenment enjoyed by different classes of the population often enable the less educated to preserve a barbarous custom amidst many civilised characteristics of the national life.

§ 8. Historical Evidence: Historical reasoning must start from, or be verified by, observations. If we are writing the history of our own times, we can observe some of the events for ourselves: if of another time or country, we can observe some of the present conditions of the country, its inhavitants, language, manners, institutions, which are effects of the past and must be traceable to it; we may also be able to observe ancient buildings or their ruins, funerary remains, coins, dating from the very times we are to treat of. Our own observations, of course, are by no means free from error.

Evidences on which history relies.

Our own observation

But even in treating of our own age and country, most of our information must be derived from the testimony of others, who may have made mistakes of observation and further mistakes in reporting their observations and may have intentionally falsified them. Testimony is of two kinds: Oral; and Written, inscribed or printed. In investigating the events of a remote age, nearly all our direct evidence must be some sort of testimony.

Testimony—oral and written

(1) Oral testimony depends upon the character of the witness; and the best witness is not perfectly trustworthy; for he may not have observed accurately, or he may not have reported correctly; especially if some time elapsed between the

Requirements of good oral testimony.

event and his account of it; for no man's memory is perfect. Since witnesses vary widely in capacity and integrity, we must ask concerning any one of them—was he a good judge of what he saw, and of what was really important in the event? Had he good opportunities of knowing the circumstances? Had he any interest in the event—personal or partisan, or patriotic? Such interests would colour his report; and so would the love of telling a dramatic story, if that was a weakness of his. Nay, a love of truth might lead him to modify the report of what he remembered if—as he remembered it—the matter seemed not quite credible. We must also bear in mind that, for want of training, precision in speaking the truth is not understood or appreciated by many honest people even now, still less in unscientific ages.

How oral traditions are formed.

Oral tradition is formed by passing a report from one to another, generation by generation; and it is generally true that such a tradition loses credit at every step, because every narrator has some weakness. However, the value of tradition depends upon the motives people have to report correctly, and on the form of the communication, and on whether monuments survive in connection with the story. Amongst the things best remembered are religious and magic formulæ, heroic poems, lists of ancestors, popular legends about deeply impressive events, such as migrations, conquests, famines, plagues. We are apt now to underrate the value of tradition, because the use of writing has made tradition less important, and therefore less pains are taken to preserve it. In the middle of last century, it was usual (and then quite justifiable) to depreciate oral tradition as nearly worthless; but the spread of the archæological and anthropological research, and the growth of the Comparative Method, have given new significance to legends and traditions which, merely by themselves, could not deserve the slightest confidence.

(2) As to written evidence, contemporary inscriptions—such as are found on rocks and stones and bricks in various parts of

the world, and most abundantly in Egypt and Western Asia—are of the highest value, because least liable to fraudulent abuse; but must be considered with reference to the motives of those who set them forth. Manuscripts and books give rise to many difficulties. We have to consider whether they were originally written by some one contemporary with the events recorded: if so, they have the same value as immediate oral testimony, provided they have not been tampered with since. But if not contemporary records, they may have been derived from other records that were contemporary, or only from oral tradition. In the latter case they are vitiated by the weakness of oral tradition. In the former case, we have to ask what was the trustworthiness of the original records, and how far do the extant writings fairly represent those records?

How to utilise written testimony:

Our answers to these questions will partly depend upon what we know or can discover of the authors of the MSS. or books. Who was the author? If a work bears some man's name, did he really write it? The evidence bearing upon this question is usually divided into internal, external and mixed; but perhaps no evidence is purely internal, if we define it as that which is derived entirely from the work itself. Under the name of internal evidence it is usual to put the language, the style, consistency of ideas; but if we had no grounds of judgment but the book itself, we could not possibly say whether the style was the author's: this requires us to know his other works. Nor could we say whether the language was that of his age, unless we knew other literature of the same age; nor even that different passages seem to be written in the manner of different ages, but for our knowledge of change in other literatures. There must in every case be some external reference. Thus we judge that a work is not by the alleged author, nor contemporary with him, if words are used that only became current at a latter date, or are used in a sense that they only later acquired, or if later

Internal evidence.

writers are imitated, or if events are mentioned that happened later ('anachronism'). Books are sometimes forged outright, that is, are written by one man and deliberately fathered upon another; but sometimes books come to be ascribed to a well-known name, which were written by some one else without fraudulent intent, dramatically or as a rhetorical exercise.

External evidence.

As to external evidence, if from other sources we have some knowledge of the facts described in a given book, and if it presents no serious discrepancies with those facts, this is some confirmation of a claim to contemporaneity. But the chief source of external evidence is other literature, where we may find the book in question referred to or quoted. Such other literature may be by another author, as when Aristotle refers to a dialogue of Plato's or Shakespeare quotes Marlowe; or may be other work of the author himself, as when Aristotle in the *Ethics* refers to his own *Physics*, or Chaucer in *The Canterbury Tales* mentions as his own *The Legend of Good Women*, and in *The Legend* gives a list of other works of his. This kind of argument assumes that the authorship of the work we start from is undisputed; which is practically the case with the *Ethics* and *The Canterbury Tales*.

Other considerations.

But, now, granting that a work is by a good author, or contemporary with the events recorded, or healthily related to others that were contemporary, it remains to consider whether it has been well preserved and is likely to retain its original sense. It is, therefore, desirable to know the history of a book or MS., and through whose hands it has passed. Have there been opportunities of tampering with it; and have there been motives to do so? In reprinting books, but still more in copying MSS., there are opportunities of omitting or interpolating passages, or of otherwise altering the sense. In fact, slight changes are almost sure to be made even without meaning to make them, especially in copying MSS., through the carelessness or ignorance of transcribers. Hence the oldest MS. is reckoned the best.

If a work contains stories that are physically impossible, it shows a defect of judgment in the author, and decreases our confidence in his other statements; but it does not follow that these others are to be rejected. We must try to compare them with other evidence. Even incredible stories are significant: they show what people were capable of believing, and, therefore, under what conditions they reasoned and acted. One cause of the incredibility of popular stories is the fusion of legend with myth. A legend is a traditionary story about something that really happened: it may have been greatly distorted by stupidity, or exaggeration, or dramatisation, or rationalisation, but may still retain a good deal of the original fact. A myth, however, has not necessarily any basis of fact: it may be a sort of primitive philosophy, an hypothesis freely invented to explain some fact in nature, such as eclipses, or to explain some social custom whose origin is forgotten, such as the sacrificing of a ram.

Concluding remarks.

All historical conclusions, then, depend on a sum of convergent and conflicting probabilities in the nature of circumstantial evidence. The best testimony is only highly probable, and it is always incomplete. To complete the picture of any past age there is no resource but the Comparative Method. We use this method without being aware of it, whenever we make the records of the last generation intelligible to ourselves by our own experience. Without it nothing would be intelligible: an ancient coin or weapon would have no meaning, were we not acquainted with the origins and uses of other coins and weapons. Generally, the further we go back in history, the more the evidence needs interpretation and reconstruction, and the more prominent becomes the appeal to the Comparative Method. Our aim is to construct a history of the world, and of the planet as part of the world, and of mankind as part of the life of the planet, in such a way, that every event shall be consistent with, and even required by, the rest according to the principle of Causation.

SUMMARY

Mill formulated five canons of Inductive proof and observed that at bottom there are two, namely, Agreement and Difference. For, the Joint Method is a double employment of the Method of Agreement; the Method of Residues, a peculiar modification of the Method of Difference and the Method of concomitant Variation a modification either of the Method of Agreement or of the Method of Difference. But in their function of *proof*, they are all reducible to one, namely, Difference; for the cogency of the method of agreement, as distinguished from a simple enumeration of instances, depends upon the omission in one instance after another, or all other circumstances. [*See also additional notes 1*].

The methods, it may be said, are in essence deductive. The Law of Causation is itself an indispensable foundation of their evidence. So, Inductive Logic may be considered as having a purely formal character. It consists (1) in a statement of the Law of Cause and effect, (2) in certain immediate inferences from their law expanded into the canons; (3) in the syllogistic application of the canons to special predications of causation by means of minor premises which show that certain instances satisfy the canons.

N.B. [*These are called by courtesy Inductive methods: they are more properly Deductive methods available in Inductive investigation.—(Bain.)*]

Thus, the Formal Logic of Induction is essentially deductive; and it has been much questioned whether any transition from the formal to the material conditions of proof is possible. Dr. Venn has shown that natural phenomena want the distinctness and capability of isolation that belong to symbols. (*See additional notes—2*). Again, observing whether instances conform to a canon must always be subject at last to the limits of our faculties. How can we ascertain exact equality, immediate sequence, perfect induction?

Now it is right to touch upon this sceptical topic, but to insist much upon it is not a sign of good sense. But still, it is not so much in laws based upon direct observation that the ma-

terial validity of scientific reasoning depends. It requires an elaborate combination of deduction with observation and experiment. In chemical action or heteropathic intermixture of effects, given the causal agents we can prove experimentally, according to the canons of Induction that they have such effects. But in cases of homogeneous intermixture of effects direct observation or experiment is insufficient to resolve an effect into laws of its conditions and the general method is to calculate what may be expected from a combination of the conditions as either known or hypothetically assumed and to compare this anticipation with actual phenomena.

This is the Direct Deductive Method. It is primarily used in Physics and so is also called the Physical Method. Here, given any complex mechanical phenomenon the inquirer considers (1) what laws already ascertained seem likely to apply to it (in default of known laws, hypothesis are substituted); he then (2) computes out the effect that will follow from these laws in circumstances similar to the cases before him; and (3) he verifies his conclusion by comparing it with the actual phenomenon. When the facts observed do not correspond with deductive calculation there must be error either in observation of facts or in deduction or in the premises. The inaccuracy in observation of facts can be easily remedied by repeating the observation. Similarly the process of deduction can also be very simply and easily revised. But mistake in premise may not be so easily detected. It may be due to (1) inaccurate ascertainment of the laws or of the forces present, (2) incorrect conception of the circumstances in which the agents are combined (3) overlooking of one or more of the agents affecting the results, and (4) inclusion of agents or circumstances that do not exist or do not affect the phenomenon.

When the forces determining a phenomena are too numerous or too indefinite to be combined in a direct deduction the Inverse of Historical Method is applied. Here we begin by collecting an empirical law of the phenomenon and then endeavour to show by deductions from the "nature of the case," that is from a consideration of the circumstances and forces known to be operative, that such a law was to be expected. In the Physical Method, deduction is verified by induction but here induction is verified by deduction. And the essential differences between the Physical and the Historical Method is that in the former the deductive process, when complete, amounts to exact

demonstration ; whereas in the latter, the deductions may consist of qualitative reasonings and the results are indefinite. So we ought to be very careful in applying this method, and while deducing our empirical law from a consideration of the nature of the case we ought (a) to take account of all the important conditions ; (*b*) to allow weight to them severally in proportion to their importance; and (*c*) not to include in our estimates any condition that we cannot show to be probably present and operative.

The name Historical Method is also used to denote the comparative method that is used in investigating the history of institutions or the true sense of legends.

The third form of the deductive method is the Abstract or Geometrical Method. It is the direct deductive method without inductive verification. It can be applied successfully only in an abstract science like Geometry.

ADDITIONAL NOTES

1. "The Methods might, indeed, all be expressed by the formula AB—XY AC—XZ. Here as B can be removed without affecting X, it is assumed that B and X, are not causally connected ; whilst as the removal of B is attended by the removal of Y, it is assumed that B—Y is a causal sequence. This represents the mode of proceeding by each Method. Each, by removing parts of a complex whole, seeks to establish a relation between the remaining parts. Thus the methods are at bottom, as Mr. Bradley points out all of them Methods of Residues or Methods of Difference".—(Welton Logic II pp. 147-48.).

2. According to Whewell these methods "take for granted the very thing which is most difficult to discover, the reduction, of the phenomena to formulae. When we have any set of complex facts offered to us, and when we would discover the law of nature which governs them, where are we to look for our ABC and abc ? Nature does not present to us the case in this form and how are we to reduce them to this form ? You say *when* we find the combination ABC with abc, and ABD with abd, then we may draw our inference. Granted, but when and where are we to find such combination ?" But it may be said

as against this objection that the "Methods are primarily rules and models, to which if inductive arguments conform, those arguments are conclusive and not otherwise" (*Westaway*). That is, these methods primarily lay down the conditions of proof, or state the ideal to which an inductive investigation must conform.

EXERCISES WITH HINTS

1. *What is the special business of the Deductive Method? What are its different forms?* (C.U. '52)

2. *How do you distinguish between what Mill calls the Geometrical, Physical and Historical Methods?*

3. *The comparative Method is appealed to where direct. What are its different forms?* (C.U. '52)

4. *What methods would you adopt to solve the following problems?*
   - *(a) The cause of a train disaster.*
   - *(b) The outbreak of a riot in a town.*
   - *(c) Fall in customs revenue.* (M.U. '41)

5. *What kind of reasoning is applied by (i) an engineer who designs a new bridge, (ii) a physician when he prescribes a particular medicine to a patient, and (iii) by a legistator who introduces a new law.*

6. *Explain the nature of Mill's Deductive Method, and indicate the aid given to induction by deduction.*

7. *What is the importance of what Mill calls 'The Deductive Method.'*

[This method is the actual method of scientific progress. An inductive science is not merely inductive. It is both deductive and inductive. For "the method of Induction consists throughout in the framing of hypotheses to explain the phenomena given in experience, and the verification of those hypotheses by constant appeal to facts.... This Method is in its essential features, what Mill calls the 'Deductive Method' to which however he gives too limited an application regarding it as applicable only to the more complex phenomena where several causes combine to produce an effect."]

CHAPTER XVIII

HYPOTHESES

§ 1. Hypothesis defined and distinguished from Theory: An Hypothesis, sometimes employed instead of a known law, as a premise in the deductive investigation of nature, is defined by Mill as "any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be or at least is likely to be, true." The deduction of known truths from an hypothesis is its Verification; and when this has been accomplished in a good many cases, and there are no manifest failures, the hypothesis is often called a Theory; though this term is also used for the whole system of laws of a certain class of phenomena, as when Astronomy is called the 'theory of the heavens.' Between hypothesis and theory in the former sense no distinct line can be drawn; for the complete proof of any speculation may take a long time, and meanwhile the gradually accumulating evidence produces in different minds very different degrees of satisfaction; so that the sanguine begin to talk of 'the theory', whilst the circumspect continue to call it 'the hypothesis'.

Mill's definition of hypothesis.

Hypothesis and Theory

An Hypothesis may be made concerning (1) an Agent, such as the ether; or (2) a Collocation, such as the plan of our solar system—whether geocentric or heliocentric; or (3) a Law of an agent's operation, as that light is transmitted by a wave-motion of such lengths or of such rates of vibration.

Hypothesis concerning (1) Agent, (2) Collocation (3) Law of agent's operation.

The received explanation of light involves both an agent, the ether, as an all-pervading elastic fluid, and also the law of its operation, as transmitting light in waves of definite form and length, with definite velocity. The agreement between the calculated results of this complex hypothesis and the observed phenomena of light is the chief part of the verification ; which has now been so successfully accomplished that we generally hear of the 'Undulatory Theory.' Sometimes a new agent only is proposed ; as the planet Neptune was at first assumed to exist in order to account for perturbations in the movements of Uranus, influencing it according to the already established law of gravitation. Sometimes the agents are known, and only the law of their operation is hypothetical, as was at first the case with the law of gravitation itself. For the agents, namely, Earth, falling bodies on the Earth, Moon, Sun, and planets were manifest ; and the hypothesis was that their motions might be due to their attracting one another with a force inversely proportional to the squares of the distances between them. In the Ptolemaic Astronomy, again, there was an hypothesis as to the collocation of the heavenly bodies (namely, that our Earth was the centre of the universe, and that Moon, Sun, planets and stars revolved around her) : in the early form of the system there was also an hypothesis concerning agents upon which this arrangement depended (namely, the crystalline spheres in which the heavenly bodies were fixed, though these were afterwards declared to be imaginary) ; and an hypothesis concerning the law of operation (namely, that circular motion is the most perfect and eternal, and therefore proper to celestial things).

Examples

Hypotheses are by no means confined to the physical sciences: we all make them freely in private life. In searching for anything, we guess where it may be before going to look for it : the search for the North Pole was likewise guided by hypotheses how best to get there. In estimating the characters or explaining

Hypotheses are freely made in private life.

the conduct of acquaintances or of public men, we frame hypotheses as to their dispositions and principles. 'That we should not impute motives' is a peculiarly absurd maxim, as there is no other way of understanding human life. To impute bad motives, indeed, when good are just as probable, is to be wanting in the scientific spirit, which views every subject in 'a dry light.' Nor can we help 'judging others by ourselves'; for self-knowledge is the only possible starting point when we set out to interpret the lives of others. But to understand the manifold combinations of which the elements of character are susceptible; and how these are determined by the breeding of race or family under various conditions, and again by the circumstances of each man's life, demands an extraordinary union of sympathetic imagination with scientific habits of thought.

Historians also make them.

Such should be the equipment of the historian, who pursues the same method of hypothesis when he attempts to explain (say) the states of parties upon the Exclusion Bill, or the the policy of Louis XI. Problems such as the former of these are the easier; because, amidst the compromises of a party, personal peculiarities obliterate one another, and expose a simpler scheme of human nature with fewer fig-leaves. Much more hazardous hypotheses are necessary in interpreting the customs of savages, and the feelings of all sorts of animals.

Literary criticism abound with hypotheses:

Literary criticisms, again abound with hypotheses: *e.g.*, as to the composition of the Homeric poems, the order of the Platonic dialogues, the authorship of the Cædmonic poems, or the Ossianic, or of the letters of Junius.

To think is to make and verify hypothesis.

Thus the method of our everyday thoughts is identical with that of our most refined speculations; and in every case we have to find whether the hypothesis accounts for the facts.

§ 2. AN HYPOTHESIS MUST BE VERIFIABLE: It follows from the definition of an hypothesis that none is of any use that does not admit of verification (proof or disproof), by

An hypothesis must admit of verification.

comparing the results that may be deduced from it with facts or laws. If so framed as to elude every attempt to test it by facts, it can never be proved by them nor add anything to our understanding of them.

Explanation with examples.

Suppose that a conjurer asserts that his table is controlled by the spirit of your deceased relative, and makes it rap out an account of some adventure that could not easily have been within a stranger's knowledge. So far good. Then, trying again, the table raps out some blunder about your family which the deceased relative could not have committed; but the conjurer explains that 'a lying spirit' sometimes possesses the table. This amendment of the hypothesis makes it equally compatible with success and with failure. To pass from small things to great, not dissimilar was the case of the Ptolemaic Astronomy: by successive modifications, its hypothesis was made to correspond with accumulating observations of the celestial motions so ingeniously that, until the telescope was invented, it may be said to have been unverifiable. Consider, again, the sociological hypothesis, that civil order was at first founded on a Contract which remains binding upon all mankind: this is reconcilable with the most opposite institutions. For we have no record of such an event: and if the institutions of one State (say the British) include ceremonies, such as the coronation oath and oath of allegiance, which may be remnants of an original contract, they may nevertheless be of comparatively recent origin; whereas if the institutions of another State (say the Russian) contain nothing that admits of similar interpretation, yet traces of the contract once existing may long since have been obliterated. Moreover, the actual contents of the contract not having been preserved, every adherent of this hypothesis supplies them at his own discretion, 'according to the dictates of Reason'; and so one derives from it the duty of passive obedience, and another with equal cogency establishes the right of rebellion.

To be verifiable an hypothesis must be definite:

To be verifiable, then, an hypothesis must be definite; if somewhat vague in its first conception (which is reasonably to be expected), it must be made definite in order to be put to the proof. But, except this condition of verifiability, and definiteness for the sake of verifiability, without which a proposition does not deserve the name of an hypothesis, it seems inadvisable to lay down rules for a 'legitimate' hypothesis. The epithet is misleading. It suggests that the Logician makes rules for scientific inquirers; whereas his business is to discover the principles which they, in fact, employ in what are acknowledged to be their most successful investigations. If he did make rules for them, and they treated him seriously, they might be discouraged in the exercise of that liberty of hypothesising which is the condition of all originality; whilst if they paid no attention to him, he must suffer some loss of dignity. Again, to say that a 'legitimate hypothesis' must explain all the facts; at least in the department for which it is invented, is decidedly discouraging. No doubt it may be expected to do this in the long run when (if ever) it is completely established; but this may take a long time: is it meanwhile illegitimate? Or can this adjective be applied to Newton's corpuscular theory of light, even though it has failed to explain all the facts?

Legitimate hypothesis—what it exactly means.

§ 3. Proof of Hypothesis: Given a verifiable hypothesis, however, what constitutes proof or disproof?

MUST AN HYPOTHETICAL AGENT BE DIRECTLY OBSERVABLE?

It is desirable that the hypothetical agent be directly observable.

(1) *If a new agent be proposed, it is desirable that we should be able directly to observe it, or at least to obtain some evidence of its existence of a different kind from the very facts which it has been invented to explain.* Thus, in the discovery of Neptune, after the existence of such a planet outside the orbit of Uranus had been conjectured (to accout for the movements of

the latter), the place in the heavens which such a body should occupy at a certain time was calculated, and there by means of the telescope it was actually seen.

Vera Causa:

In case they are not directly observable they have yet to be (a) in the image of things already perceived, i.e., thought of as though they are seen or felt and (b) capable of connecting and explaining perceptions, so that the evidence of their existence be of a different kind from the very facts which they are intended to explain. Such Hypothetical agents are called VERA CAUSA.

Agents, however, are assumed and reasoned upon very successfully which, by their nature, never can be objects of perception: such are the atoms of Chemistry and the ether of Optics. But the severer methodologists regard them with suspicion: Mill was never completely convinced about the ether; the defining of which has been found very difficult. He was willing, however, to make the most of the evidence that has been adduced as indicating a certain property of it distinct from those by which it transmits radiation, namely, mechanical inertia, whereby it has been supposed to retard the career of the heavenly bodies, as shown especially by the history of Encke's comet. This comet returned sooner than it should, as calculated from the usual data; the difference was ascribed to the influence of a resisting medium in reducing the extent of its orbit; and such a medium may be the ether. If this conjecture (now of less credit) should gain acceptance, the ether might be regarded as a *vera causa* (that is, a condition whose existence may be proved independently of the phenomena it was intended to explain), in spite of its being excluded by its nature from the sphere of direct perception. However, science is not a way of perceiving things, but essentially a way of thinking about them. It starts, indeed, from perception and returns to it, and its thinking is controlled by the analogies of perception. Atoms and ether are thought about as if they could be seen or felt, not as noumena; and if still successful in

connecting and explaining perceptions, and free from contradiction, they will stand as hypotheses on that ground.

On the other hand, a great many agents, once assumed in order to explain phenomena, have since been explained away. Of course, a *fact* can never be 'explained away': the phrase is properly applicable to the fate of erroneous hypotheses, when, not only are they disproved, but others are established in their places. Of the Aristotelian spheres, which were supposed to support and translate Sun, Moon and planets, no trace has ever been found: they would have been very much in the way of the comets. Phlogiston, again, an agent much in favour with the earlier Chemists, was found, Whewell tells us, when their theories were tested by exact weighing, to be not merely non-existent but a minus quantity; that is to say, it required the assumption of its absolute lightness "so that it diminished the weight of the compounds into which it entered." These agents, then, the spheres and phlogiston, have been explained away and instead of them we have the laws of motion and oxygen.

A FACT *cannot, be explained away'.*

AN HYPOTHESIS MUST BE ADEQUATE TO ITS PRETENSION:

(2) *Whether the hypothetical agent be perceptible or not, it cannot be established as a cause, nor can a supposed law of such an agent be accepted as sufficient to the given inquiry, unless it is adequate to account for the effects which it is called upon to explain, at least so far as it pretends to explain them.* The general truth of this is sufficiently obvious, since to explain the facts is the purpose of an hypothesis; and we have seen that Newton gave up his hypothesis that the Moon was a falling body, as long as he was unable to show that the amount of its deflection from a tangent (or fall) in a given time, was exactly what it should be, if the Moon was controlled by the same force as falling bodies on the Earth.

It must be adequate to its pretensions.

It is important to observe the limitation to this canon. In

the first place, it says that, unless adequate to explain the facts in question, an hypothesis cannot be *'established'*; but, for all that, such an hypothesis may be a very promising one, not to be hastily rejected, since it may take a very long time fully to verify an hypothesis. Some facts may not be obtainable that are necessary to show the connection of others: as, for example, the hypothesis that all species of animals have arisen from earlier ones by some process of gradual change, can be only imperfectly verified by collecting the fossil remains of extinct species, because immense depths and expanses of fossiliferous strata have been destroyed. Or, again, the general state of culture may be such as to prevent men from tracing the consequences of an hypothesis; for which reason, apparently, the doctrine that the Sun is the centre of our planetary system remained a discreed hopothesis for 2000 years. This should instruct us not to regard an hypothesis as necessarily erroneous or illegitimate merely because we cannot yet see how it works out: but neither can we in such a case regard it as established, unless we take somebody's word for it.

If not so, it is not established, yet it may be very promising.

Secondly, the canon says that an hypothesis is not established, unless it accounts for the phenomena *so far as it professes to*. But it implies a complete misunderstanding to assail a doctrine for not explaining what lies beyond its scope. Thus, it is no objection to a theory of the origin of species, that it does not explain the origin of life: it does not profess to. For the same reason, it is no objection to the theory of Natural Selection, that it does not account for the variations which selection presupposes. But such objections might be perfectly fair against a general doctrine of Evolution.

It is no objection to a hypothesis that it does not explain what lies beyond its scope.

An interesting case in Wallace's *Darwinism* (chap. x.) will illustrate the importance of attending to the exact conditions of an hypothesis. He says that in those groups of "birds that need

protection from enemies," "when the male is brightly coloured and the female sits exposed on the nest, she is always less brilliant and generally of quite sober and protective hues"; and his hypothesis is, that these sober hues have been acquired or preserved by Natural Selection, because it is important to the family that the sitting bird should be inconspicuous. Now to this it might be objected that in some birds both sexes are brilliant or conspicuous; but the answer is that the female of such species *does not sit exposed on the nest*; for the nests are either domed over, or made in a hole; so that the sitting bird does not need protective colouring. If it be objected, again, that some sober-coloured birds build domed nests, it may be replied that the proposition 'All conspicuously coloured birds are concealed in the nest,' is not to be converted simply into 'All birds that sit concealed in the nest are conspicuously coloured.' In the cases alleged the domed nests are a protection against the weather, and the sober colouring is a general protection to the bird, which inhabits an open country. It may be urged, however, that jays, crows, and magpies are conspicuous birds, and yet build open nests: but these are aggressive birds, *not needing protection from enemies*. Finally, there are cases, it must be confessed, in which the female is more brilliant than the male, in which yet have open nests. Yes: but *then the male sits upon the eggs*, and the female is stronger and more pugnacious!

An illustration from Wallace.

Exceptio Probat Regulam:

Thus every objection is shown to imply some inattention to the conditions of the hypothesis; and in each case it may be said, *exceptio probat regulam*—the exception *tests* the rule. (Of course, the usual translation "proves the rule," in the restricted modern sense of "prove," is absurd). That is to say, it appears on examination: (1) that the alleged exception is not really one, and (2) that it stands in such relation to the rule as to con-

The rôle of exceptions.

firm it. For to all the above objections it is replied that, granting the phenomenon in question (special protective colouring for the female) to be absent, the alleged cause (need of protection) is also absent; so that the proof is, by means of the objections, extended, from being one by the method of Agreement, into one by the Double Method.

Inadequacy, in the sense of failure to explain the whole, is not a reason for entirely rejecting hypothesis.

Thirdly, an hypothesis originally intended to account for the whole of a phenomenon and failing to do so, though it cannot be established in that sense may nevertheless contain an essential part of the explanation. The Neptunian Hypothesis in Geology, was an attempt to explain the formation of the Earth's outer crust, as having been deposited from an universal ocean of mud. In the progress of the science other causes, seismic, fluvial and atmospheric, have been found necessary in order to complete the theory of the history of the Earth's crust: but it remains true that the stratified rocks, and some that have lost their stratified character, were originally deposited under water. Inadequacy, therefore, is not a reason for entirely rejecting an hypothesis or treating it as illegitimate.

Every Competing Hypothesis must be Excluded:

Agreement with facts is necessary, but it does not exclude the possibility of vicarious causes.

(3) Granting that the hypothetical cause is real and adequate, the investigation is not complete. Agreement with the facts is a very persuasive circumstance, the more so the more extensive the agreement, especially if no exceptions are known. Still, if this is all that can be said in favour of an hypothesis, it amounts to proof at most by the method of Agreement; it does not exclude the possibility of vicarious causes; and if the hypothesis proposes a new agent that cannot be directly observed, an equally plausible hypothesis about another imagined agent may perhaps be invented.

According to Whewell, it is a strong mark of the truth of an hypothesis when it agrees with distinct inductions concerning different classes of facts, and he calls this the 'Consilience of Inductions,' because they jump together in the unity of the hypothesis. It is particularly convincing when this consilience takes place easily and naturally without necessitating the mending and tinkering of the hypothesis; and he cites the Theory of Gravitation and the Undulatory Theory of Light as the most conspicuous examples of such ever-victorious hypotheses. Thus, gravitation explains the fall of bodies on the Earth, and the orbits of the planets and their satellites; it applies to the tides, the comets, the double stars, and gives consistency to the Nebular Hypothesis, whence flow important geological inferences; and all this without any need of amendment.

Whewell holds that consilience of Inductions is a strong mark of the truth of an hypothesis.

Nevertheless, Mill, with his regorous sense of duty, points out, that an induction is merely a proposition concerning many facts, and that a consilience of inductions is merely a multiplication of the facts explained; and that, therefore, if the proof is merely Agreement in each case, there can be no more in the totality: the possibility of vicarious causes is not precluded; and the hypothesis may, after all, describe an accidental circumstance.

But Mill holds that it also does not preclude the possibility of vicarious causes.

Whewell also laid great stresss upon prediction as a mark of a true hypothesis. Thus, Astronomers predict eclipses, occultations, transits, long beforehand with the greatest precision; and the prediction of the place of Neptune by sheer force of deduction is one of the most astonishing things in the history of science. Yet Mill persisted in showing that a predicted fact is only another fact, and that it is really not very extraordinary that an hypothesis, that happens

According to whewell prediction is a mark of a true hypothesis.

to agree with many known facts, should also agree with some still undiscovered. Certainly, there seems to be some illusion in the common belief in the probative force of prediction. Prediction surprises us, puts us off our guard, and renders persuasion easy; in this it resembles the force of an epigram in rhetoric. But cases can be produced in which erroneous hypotheses have led to prediction; and Whewell himself produces them. Thus, he says that the Ptolemaic theory was confirmed by its predicting eclipses and other celestial phenomena, and by leading to the construction of Tables in which the places of the heavenly bodies were given at every moment of time. Similarly, both Newton's theory of light and the chemical doctrine of phlogiston led to predictions which came true.

But Mill pointed out that this is not this case.

What sound method demands in the proof of an hypothesis, then, *is not merely that it be shown to agree with the facts, but that every other hypothesis be excluded.* This, to be sure, may be beyond our power; there may in some cases be no such negative proof except the exhaustion of human ingenuity in the course of time. The present theory of colour has in its favour the failure of Newton's corpuscular hypothesis and of Goethe's anti-mathematical hypothesis; but the field of conjecture remains open. On the other hand, Newton's proof that the solar system is controlled by a central force, was supported by the demonstration that a force having any other direction could not have results agreeing with Kepler's second law of the planetary motions, namely, that, as a planet moves in its orbit, the areas described by a line drawn from the Sun to the planet are proportional to the times occupied in the planet's motion. When a planet is nearest to the Sun, the area described by such a line is least for any given distance traversed by the planet; and then the planet moves fastest: when the planet is furthest from the Sun, the area described by such a line is greatest for an equal distance traversed; and then the planet moves slowest. This

For proof, every other hypothesis is to be excluded.

law may be deduced from the hypothesis of a central force, but not from any other; the proof, therefore, as Mill says, satisfies the method of Difference.

Apparently, to such completeness of demonstration certain conditions are necessary: the possibilities must lie between alternatives, such as A or not-A, or amongst some definite list of cases that may be exhausted, such as equal, greater or less. He whose hypothesis cannot be brought to such a definite issue, must try to refute whatever other hypotheses are offered, and naturally he will attack first the strongest rivals. With this object in view he looks about for a "crucial instance," that is, an observation or experiment that stands like a cross (sign-post) at the parting of the ways to guide us into the right way, or, in plain words, an instance that can be explained by one hypothesis but not by another. Thus the phases of Venus, similar to those of the Moon, but concurring with great changes of apparent size, presented, when discovered by Galileo, a crucial instance in favour of the Copernican hypothesis, as against the Ptolemaic, so far at least as to prove that Venus revolved around the Sun inside the orbit of the Earth. Foucault's experiment determining the velocity of Light (cited in the last chapter) was at first intended as an *experimentum crucis* to decide between the corpuscular and undulatory theories; and answered this purpose, by showing that the velocity of a beam passed through water was less than it should be by the former, but in agreement with the latter doctrine (Deschanel: § 813).

Crucial instance.

Experimentum crucis.

Perhaps experiments of this decisive character are commonest in Chemistry: chemical tests, says Herschel, "are almost universally crucial experiments." The following is abridged from Playfair (*Encycl. Met.*, *Diss.* III.): The Chemists of the eighteenth century observed that metals were rendered heavier by calcination; and there were two ways of accounting for this: either something had been added in the process, though what,

they could not imagine ; or, something had been driven off that was in its nature light, namely, phlogiston. To decide between these hypotheses, Lavoisier hermetically sealed some tin in a glass retort, and weighed the whole. He then heated it ; and when the tin was calcined, weighed the whole again, and found it the same as before. No substance, therefore, either light or heavy, had escaped. Further, when the retort was cooled and opened, the air rushed in, showing that some of the air formerly within had disappeared or lost its elasticity. On weighing the whole again, its weight was now found to have increased by ten grains; so that ten grains of air had entered when it was opened. The calcined tin was then weighed separately, and proved to be exactly ten grains heavier than when it was placed in the retort ; showing that the ten grains of air that had disappeared had combined with the metal during calcination. This experiment, then, decided against phlogiston, and led to an analysis of common air confirming Priestley's discovery of oxygen.

Example :

HYPOTHESES MUST AGREE WITH THE LAWS OF NATURE :

(4) *An hyposhesis must agree with the rest of the laws of Nature ; and, if not itself af the highest generality, must be derivable from primary laws* (chap. xix. § 1). Gravitation and the diffusion of heat, light and sound from a centre, all follow the 'law of the inverse square,' and agree with the relation of the radius of a sphere to its surface. Any one who should think that he had discovered a new central force would naturally begin to investigate it on the hypothesis that it conformed to the same law as gravitation or light. A Chemist again, who should believe himself to have discovered a new element, would expect it to fill one of the vacant places in the Periodic Table. Conformity, in such cases, is strong confirmation and disagreement is an occasion of misgivings.

Conformity with the laws of nature is strong confirmation and disagreement is an occasion of misgivings.

A narrower hypothesis, as 'that the toad's ugliness is protective,' would be supported by the general theory of protective colouring and figure, and by the still more general theory of Natural Selection, if facts could be adduced to show that the toad's appearance does really deter its enemies. Such an hypothesis resembles an Empirical Law in its need of derivation (chap. xix. §§ 1, 2). If underivable from, or irreconcilable with, known laws, it is a mere conjecture or prejudice. The absolute leviation of phlogiston, in contrast with the gravitation of all other forms of matter, discredited that supposed agent. That Macpherson should have found the Ossianic poems extant in the Gaelic memory, was contrary to the nature or oral tradition; except where tradition is organised, as it was for ages among the Brahmins. The suggestion that xanthodhroid Aryans were "bleached" by exposure during the glacial period, does not agree with Wallace's doctrine concerning the coloration of Arctic animals. That our forefathers being predatory, like bears, white variations amongst them were then selected by the advantage of concealment, is a more plausible hypothesis.

Examples

Although, then, the consilience of Inductions or Hypothesis is not a sufficient proof of their truth, it is still a condition of it; nonconsilience is a suspicious circumstance, and resilence (so to speak), or mutual repugnance, is fatal to one or all.

Concluding remarks.

§ 4. Hypothesis necessary in scientific investigation: We have now seen that a scientific hypothesis, to deserve the name, must be verifiable and therefore definite; and that to establish itself as a true theory, it must present some symptom of reality, and be adequate and exclusive and in harmony with the system of experience. Thus guarded, hypotheses seem harmless enough; but some people have a strong prejudice against them, as against a tribe of savages without government, or laws, or any decent regard for vested

Bacon and Newton's disparaged hypotheses

interests. It is well known, too, that Bacon and Newton disparaged them. But Bacon, in his examples of an investigation according to his own method, is obliged, after a preliminary classification of facts, to resort to an hypothesis, calling it *permissio intellectus*, *interpretatio inchoata* or *vindemiatio prima*. And Newton when he said *hypotheses non fingo*, meant that he did not deal in fictions, or lay stress upon supposed forces (such as 'attraction'), that add nothing to the law of the facts. Hypotheses are essential aids to discovery: speaking generally, deliberate investigation depends wholly upon the use of them.

Real meaning of this hypothesis is essential.

It is true that we may sometimes observe a train of events that chances to pass before us, when either we are idle or engaged with some other inquiry, and so obtain a new glimpse of the course of nature; or we may try experiments haphazard, and watch the results. But, even in these cases, before our new notions can be considered knowledge, they must be definitely framed in hypotheses and reobserved or experimented upon, with whatever calculations or precautions may be necessary to ensure accuracy or isolation. As a rule, when inquiring deliberately into the cause of an event, whether in nature or in history, we first reflect upon the circumstances of the case and compare it with similar ones previously investigated and so are guided by a preconception more or less definite of 'what to look for,' what the cause is likely to be, that is by an hypothesis. Then, if our preconception is justified, or something which we observe leads to a new hypothesis, either we look for other instances to satisfy the canons of Agreement: or (if the matter admits of experiment) we endeavour, under known conditions according to the canon of Difference, to reproduce the event by means of that which our hypothesis assigns as the cause; or we draw remote inferences from our hypothesis, and try to test these by the Inductive Canons.

Where there are no definitely framed hypotheses and scientific testing of them, there is no science.

If we argue from an hypothesis and express ourselves formally, it will usually appear as the major premise; but this is not always the case. In extending ascertained laws to fresh cases, the minor premise may be an hypothesis, as in testing the chemical constitution of any doubtful substance, such as a piece of ore. Some solution or preparation, A, is generally made which (it is known) will, on the introduction of a certain agent, B, give a reaction, C, if the preparation contains a given substance, X. The major premise is the law of reaction—

So sciences have a deductive character; and usually the major premise, but sometimes the minor premise is the hypothesis.

Whenever A is X, if treated with B it is C.

The minor premise is an hypothesis that the preparation contains X. An experiment then treats A with B. If C result, a probability is raised in favour of the hypothesis that A is X; or a certainty, if we know that C results on that condition only.

So important are hypotheses to science, that Whewell insists that they have often been extremely valuable even though erroneous. Of the Ptolemaic system he says, "We can hardly imagine that Astronomy could, in its outset, have made so great a progress under any other form." It served to connect men's thoughts on the subject and to sustain their interest in working it out; by successive corrections "to save appearances," it attained at last to a descriptive sort of truth, which was of great practical utility; it also occasioned the invention of technical terms, and, in general digested the whole body of observations and prepared them for assimilation by a better hypothesis in the fulness of time. Whewell even defends the maxim that "Nature abhors a vacuum," as having formerly served to connect many facts that differ widely in their first aspect. "And in reality is it not true," he asks, "that nature *does* abhor a vacuum, and does all she can to avoid it?" Let no forlorn cause despair of a champion! Yet

Hypotheses are so important to science that they have been valuable even though erroneous.

no one has accused Whewell of Quixotry; and the sense of his position is that the human mind is a rather feeble affairs, that can hardly begin to think except with blunders.

The law of natural selection is applicable to hypotheses also.

The progress of science may be plausibly attributed to a process of Natural Selection; hypotheses are produced in abundance and variety, and those unfit to bear verification are destroyed, until only, the fittest survive. Wallace, a practical naturalist, if there ever was one, as well as an eminent theorist, takes the same view as Whewell of such inadequate conjectures. Of 'Lemuria,' an hypothetical continent in the Indian Ocean, once supposed to be traceable in the islands of Madagascar, Seychelles, and Mauritius, its surviving fragments, and named from the Lemurs, its characteristic denizens, he says (*Island Life*, chap. xix.) that it was "essentially a provisional hypothesis, very useful in calling attention to a remarkable series of problems in geographical distribution [of plants and animals], but not affording the true solution of those problems." We see, then, that 'provisional hypotheses,' or 'working hypotheses,' though erroneous, may be very useful or (as Whewell says) necessary.

To be prolific of hypotheses is the first attribute of scientific genius.

Hence, to be prolific of hypotheses is the first attribute of scientific genius; the first, because without it no progress whatever can be made. And some men seem to have a marked felicity, a sort of instinctive judgment even in their guesses, as if their heads were made according to Nature. But others among the greatest, like Kepler, guess often and are often wrong before the hit upon the truth, and themselves, like Nature, destroy many vain shoots and seedlings of science for one that they find fit to live. If this is how the mind work in scientific inquiry (as it certainly is, wtth most men, in poetry, in fine art, and in the scheming of business), it is useless to complain. We should rather recognise a place for fools' hypotheses, as Darwin did for "fools' experiments." But

to complete the scientific character, there must be great patience, accuracy, and impartiality in examining and testing these conjectures, as well as great ingenuity in devising experiments to that end. The want of these qualities leads to crude work and public failure and brings hypotheses into derision. Not partially and hastily to believe in one's own guesses, nor petulantly or timidly to reject them, but to consider the matter, to suspend judgment, is the moral lesson of science: difficult, distasteful, and rarely mastered.

Reasoning limited by definition implies hypothesis.

§ 5. The Method of Abstractions: The word 'hypothesis is often used also for the scientific device of treating an Abstraction as, for the purposes of argument, equivalent to the concrete facts. Thus, in Geometry, a line is treated as having no breadth; in Mechanics, a bar may be supposed absolutely rigid, or a machine to work without friction; in Economics, man is sometimes regarded as actuated solely by love of gain and dislike of exertion. The results reached by such reasoning may be made applicable to the concrete facts, if allowance be made for the omitted circumstances or properties, in the several cases of lines, bars, and men; but otherwise all conclusions from abstract terms are limited by their definitions. Abstract reasoning, then (that is, reasoning limited by definitions), is often said to imply 'the hypothesis' that things exist as their names are defined, having no properties but those enumerated in their definitions. This seems, however, a needless and confusing extension of the term; for an hypothesis proposes an agent, collocation, or law hitherto unknown; whereas abstract reasoning proposes to exclude from consideration a good deal that is well known. There seems no reason why the latter device should not be plainly called an Abstraction.

It is necessary to science.

Such abstractions are necessary to science, for no object is comprehensible by us in all its properties at once. But if we forget the limitations of our abstract data, we are liable to make strange blunders by

mistaking the character of the results: treating the results as simply true of actual things, instead of as true of actual things only so far as they are represented by the abstractions. In addressing abstract reasoning, therefore, to those who are unfamiliar with scientific methods, pains should be taken to make it clear what the abstractions are, what are the consequent limitations upon the argument and its conclusions, and what corrections and allowances are necessary in order to turn the conclusions into an adequate account of the concrete facts. The greater the number, variety, and subtlety of the properties possessed by any object (such as human nature), the greater are the qualifications required in the conclusions of abstract reasoning, before they can hold true of such an object in practical affairs.

Method of Limits:

Closely allied to this method of Abstraction is the Mathematical Method of Limits. In his *History of Scientific Ideas* (B. II. c. 12), Whewell says: "The *Idea of a Limit* supplies a new mode of establishing mathematical truths. Thus with regard to the length of any portion of a curve, a problem which we have just mentioned; a curve is not made up of straight lines, and therefore we cannot by means of any of the doctrines of elementary geometry measure the length of any curve. But we may make up a figure nearly resembling any curve by putting together many short straight lines, just as a polygonal building of very many sides may nearly resemble a circular room. And in order to approach nearer and nearer to a curve, we may make the sides more and more small, more and more numerous. We may then possibly find some mode of measurement, some relation of these small lines to other lines, which is not disturbed by the multiplication of the sides, however far it be carried. And thus we may do what is equivalent to measuring the curve itself; for by multiplying the sides we may approach more and more closely to the curve till no appreciable difference remains. The curve line is the *Limit* of the polygon; and in this process we proceed on the *Axiom* that 'What is true up to the Limit is true at the Limit,'"

What Whewell calls the Axiom here, others might call an Hypothesis; but perhaps it is properly a Postulate. And it is just the obverse of the Postulate implied in the Method of Abstractions, namely, that 'What is true of the Abstraction is true of concrete cases the more nearly they approach.' What is true of the 'Economic Man' is truer of a broker than of a farmer, of a farmer than of a labourer, of a labourer than of the artist of romance. Hence the Abstraction may be called a Limit or limiting case, in the sense that it stands to concrete individuals, as a curve does to the figures made up "by putting together many short straight lines." Correspondingly, the Proper Name may be called the Limit of the class-name; since its attributes are infinite, whereas any name whose attributes are less than infinite stands for a possible class. In short, for logical purposes, a Limit may be defined and any extreme case to which actual examples may approach without ever reaching it. And in this sense 'Method of Limits' might be used as a term including the Method of Abstractions; though it would be better to speak of them generically as 'Methods of Approximation.'

Methods of Approximation

We may also notice the Assumptions (as they may be called) that are sometimes employed to facilitate in an investigation, because some definite ground must be taken and nothing better can be thought of: as in estimating national wealth, that furniture is half the value of the houses.

It is easy to conceive of an objector urging that such devices as the above are merely ways of avoiding the actual problems, and that they display more cunning than skill. But science, like good sense, puts up with the best that can be had; and, like prudence, does not reject the half-loaf. The position, that a conceivable case that can be dealt with may, under certain conditions, be substituted for one that is unworkable, is a touchstone of intelligence. To stand out for ideals that are known to be impossible, is only an excuse for doing nothing at all.

In another sense, again, the whole of science is sometimes

said to be hypothetical, because it takes for granted the Uniformity of Nature; for this, in its various aspects, can only be directly ascertained by us as far as our experience extends; whereas the whole value of the principle of Uniformity consists in its furnishing a formula for the extension of our other beliefs beyond our actual experience. Transcendentalists, indeed, call it a form of Reason, just because it is presupposed in all knowledge; and they and the Empiricists agree that to adduce material evidence for it, in its full extent, is impossible. If, then, material evidence is demanded by any one, he cannot regard the conclusions of Mathematics and Physical Science as depending on what is itself unproved; he must, with Mill, regard these conclusions as drawn "not from but according to" the axioms of Equality and Causation. That is to say, if the axioms are true, the conclusions are; the material evidence for both the axioms and the conclusions being the same, namely, uncontradicted experience. Now when we say, 'If Nature is uniform, science is true,' the hypothetical character of science appears in the form of the statement. Nevertheless, it seems undesirable to call our confidence in Nature's uniformity an 'hypothesis': it is incongruous to use the same term for our tentative conjectures and for our most indispensable beliefs. 'The Universal Postulate is a better term for the principle which, in some form or other, every generalisation takes for granted.

The sense in which all knowledge is hypothetical.

We are now sometimes told that, instead of the determinism and continuity of phenomena hitherto assumed by science, we should recognise indeterminism and continuity of phenomena hitherto assumed by science, we should recognise indeterminism and discontinuity. But it will be time enough to fall in with this doctrine when its advocates produce a new Logic of Induction, and explain the use of the method of Difference and of control experiments according to the new postulates.

SUMMARY

An hypothesis is a verifiable guess work. It is a work of imagination that admits either of proof or disproof. When verification is accomplished in good many cases and there are no manifest failures, the hypothesis is often called a theory. But between hypothesis and theory no distinct line can be drawn.

An Hypothesis may be made concerning (1) an Agent, such as the ether, or (2) a collocation—such as the plan of our solar system or (3) a Law of all agents' operation as that light is transmitted by a wave-motion of such lengths or of such rates of vibration. *(see also supplementary note* I).

Hypotheses are by no means confined to the physical sciences : we all make them freely in private life. In searching for anything we guess where it may be before going to look for it. *(see also supplementary note* 2).

In estimating the characters of explaining the conduct of acquaintances or of public men we frame hypothesis as to their dispositions and principles. Thus a historian cannot proceed if he does not frame hypotheses. Literary criticisms abound with hypotheses. The method of our everyday thoughts is identical with that of our most refined speculations ; and in every case we have to find whether the hypothesis accounts for facts. An hypothesis that does not admit of verification (proof or disproof) is of no use. If an hypothesis is such that it eludes every attempts to test it by facts, it can have no place in science. Now, to be verifiable, an hypothesis must be definite ; if somewhat vague in its first conception, it must be made definite in order to be put to the proof. And except this conditions of verifiability, and definiteness for the sake of verifiability, there are no rules for the legitimacy of an hypothesis.

N.B. (Some logicians do not recognise the distinction between the conditions of the legitimacy of an hypothesis and those of its proof—and append a long list of the so-called marks of a legitimate hypothesis which are really speaking nothing but statements of what constitutes its proof or disproof).

The proof or disproof of a verifiable hypothesis depends on the following :

(1) If a new agent be proposed, it is desirable that we should be able directly to observe it or at least to obtain some evidence of its existence of a different kind from the very facts which it has been invented to explain. Now, there may be agents which never can be objects of perception : such are the atoms of chemistry and the ether of optics. But still they are to be accepted as they are *vera causa*, (*see also supplementary note* 3) i.e., are conditions whose existence may be proved independently of the phenomena they were intended to explain. Science, in other words, is not a way of perceiving things but essentially a way of thinking about them. But it starts from perception and returns to it, and its thinking is controlled by the analogies of perception. Atoms and ether are thought about as if they could be seen or felt, not as noumena, and if still successful in connecting and explaining perceptions, and free contradiction, they will stand as hypothesis. (2) Whether the hypothetical agent be perceptible or not it cannot be established as a cause, nor can a supposed law of such an agent be accepted as sufficient to the given inquiry unless it is adequate to account for the effects which it is called upon to explain, at least so far as it pretends to explain them. (*see also supplementary note 4*). The hypothesis in other words must be adequate to explain the phenomena under investigation. But there are limitations to this canon. For, it may take a long time fully to verify an hypothesis, and hence if the hypothesis is promising but not adequate it should not be rejected. Again, we should not expect it to account for what is beyond its scope. It is no objection to a theory of the origin of species, that it does not explain the origin of life : it does not profess to. Moreover, what may appear as an exception may turn out on examination to be not really so, it may stand in such relation to the rule as to confirm it. Finally, an hypothesis originally intended to account for the whole of a phenomenon and failing to do so may contain an essential part of explanation. So inadequacy is not a reason for entirely rejecting an hypothesis or treating it as illegitimate. (3) Even when the hypothetical cause is real and adequate the investigation is not complete. The hypothesis that must admit of deductive development should agree with facts. (*Agreement with facts is the sole and sufficient test of a true hypothesis—Jevons.*) But this agreement with facts does not exclude the possibility of vicarious causes. (*see also supplementary note 5*). Whewell supposed that it is a strong mark of the truth of an hypothesis when it agrees with distinct

inductions concerning different classes of facts, and he calls this the consilience of Inductions. But Mill pointed out that a consilience of inductions is merely a multiplication of the facts explained; and therefore the proof is merely Agreement in each case and the possibility of vicarious causes is not precluded. So the consilience of inductions or Hypotheses is not a sufficient proof of their truth, though non-consilience is a suspicious circumstance and resilience or mutual repugnance is fatal to one or all. Whewell supposed that prediction is a mark of true hypothesis. Yet Mill persisted in showing that a predicted fact is only another fact and that it is not very extraordinary that an hypothesis should also agree with some still undiscovered fact. Again, there are cases of erroneous hypotheses leading to prediction.

So what sound method demands in the proof of an hypothesis is not merely that it be shown to agree with the facts but that every other hypothesis be excluded. This seems to be beyond our power. To completeness of demonstration certain conditions are necessary: the possibilities must be between alternatives, such as *A* or *not-A*. He whose hypothesis cannot be brought to such a definite issue must try to refute whatever other hypotheses are offered. (But the refutation of such hypotheses is not the refutation of all hypotheses.) So he will find out a crucial instance i.e., an instance that can be explained by his hypothesis and not by the others. History of science affords numerous examples of such testing of hypothesis.

N.B. (The crucial instance may be obtained by pure observation or by experiment. When it is collected by experiment we have an example of what is called *experimentum crucis* or crucial experiment). *(see also supplementary note—5).*

(4) An hypothesis must agree with the rest of the laws of Nature, and if not itself of the highest generality, must be derived from primary laws. But too much stress ought not to be laid on this condition. (*see also supplementary note—6*).

Hypotheses are necessary for science ; but some people have a strong prejudice against them. *Bacon* disparaged them. According to him an hypothesis was nothing but a rash and premature work of imagination pretending to anticipate nature. Newton also said he did not make hypotheses. (*see also supplementary note —7*). But hypotheses are essential aids to discovery: deliberate investigation depends wholly upon the use of them. (*see also supplementary note 8*). So important are hypotheses to science

that Whewell insists that they have often been extremely valuable even though erroneous ; and the progress of science may be plausibly attributed to a process of natural selection—hypotheses are produced in abundance, and those unfit to bear verification are destroyed until only the fittest survive.

Thus Hypothesis are necessary to Induction ; but we should not forget that a mere hypothesis is not an induction. To be elevated to the rank of induction it must be verified. In a sense the whole of science is hypothetical because it takes for granted the uniformity of Nature. Still, it seems undesirable to call our confidence in Nature's uniformity an hypothesis. The universal postulate is a better term—and the whole of science is not hypothetical in the ordinary sense of the name hypothetical.

SUPPLEMENTARY NOTES

Kinds of Hypothesis

Criticism of Read's view.

1. Read's way of classifying hypotheses is not above criticism. For, rarely an hypothesis is concerned solely with an agent or a collocation or a law. An hypothesis may be concerned with both an agent and its law or relation with other agents. Thus the other hypothesis and the undulatory hypothesis go together. Similarly the Copernican or the Ptolemaic hypothesis is a hypothesis concerning law also.

Welton's view :

Some logicians, e. g., Welton, classify hypothesis into two groups viz., concerning causes and concerning laws. The hypotheses concerning causes are called explanatory hypotheses whereas the hypotheses concerning laws are called descriptive hypotheses. But this classification also is subject to similar criticisms.

Stebbing's view :

Dr. Stebbing has classified hypotheses in a novel manner. Here the principle of classification is different. Here, that is to say, the kind of an hypothesis is ascertained by taking into account the motive of the scientist who makes the hypothesis. Thus viewed, i.e., when the kind of an hypothesis is determined by taking into account the purpose of the hypothesis, hypotheses are found to

be of three main kinds—explanatory, descriptive and analogical. "The simplest kind of hypotheses fall into the first class. These hypotheses are intended to explain, i. e., to account for the occurrence of a certain fact by the interpolation of facts that might have been observed under suitable conditions. The 're-construction' of a crime in a court of law is an instance of this kind of hypothesis. There is another type of explanatory hypothesis in which the interpolated elements are non-observable relations between the occurrences to be connected. The Newtonian hypothesis of attraction affords a well-known example." "The function of a descriptive hypothesis is to symbolize the ordered connexion of the facts. The hypothesis of the ether as a frictionless fluid and as a completely elastic solid must be regarded as descriptive. This hypothesis had undoubted value in guiding observation and in suggesting experiments. The Rutheford-Bohr theory of the planetary atom would seem to be likewise a descriptive hypothesis." "The essential characteristic of descriptive hypotheses is that they are not put forward as generalisations from experience; they are not anticipations of natural laws awaiting confirmation. On the contray, they are descriptions that serve the functions of models enabling the scientist to understand the mode of connection between the facts for which he is trying to account. Such hypotheses must be regarded as being essentially provisional and temporary." "The descriptive hypothesis may develop into an *analogical* hypothesis. By an analogical hypothesis we mean an hypothesis that what is true of one set of phenomenon may be true of another set owing to the fact that the two sets have in common certain formal principles." Maxwell's electro-magnetic theory of light is a well-kown example.

Explanatory Hypothesis:

Descriptive Hypothesis:

Analogical Hypothesis:

There is a kind of explanatory hypothesis that is called *ad hoc* hypothesis. When there is a single discrepancy between the theory and a sensible fact, an *ad hoc* hypothesis is introduced without rejecting the theory. "It will be remembered that an irregularity was observed in the orbit of *Uranus* as calculated in accordance with Newton's theory. This irregularity constituted a discrepant fact that must be accounted for. There were two ways of avoiding this discrepancy. The first was to reject the

Ad hoc Hypothesis:

theory (viz. Newton's law) with which the fact was discrepant. The second was to introduce an *ad hoc* hypothesis which would account for the irregularity in such a way as to remove the discrepancy. The Newtonian theory was so well attested that it was then unthinkable that it should be abandoned. It is well known that the second alternative was adopted. But the hypothesis that these disturbances were due to another planet was an *ad hoc* hypothesis."

2. Scientific observation which is not a mere looking at nature, but interrogating her is impossible without an hypothesis. So it is often said that perception involves hypothesis. That is, the scientist who observes nature, by asking proper questions to nature frames hypotheses and observes.

Vera Causa

3. "It is never allowable in a scientific hypothesis to assume a cause but only to ascribe an assumed law to a known cause? I do not assert this. I only say, that in the latter case alone can the hypothesis be received as true merely because it explains the phenomena. In the former case it may be very useful by suggesting a line of investigation which may possibly terminate in obtaining real proof. But for this purpose it is indispensable that the cause suggested by the hypothesis should be in its own nature susceptible of being proved by other evidence. This seems to be the philosophical import of Newton's maxim that the cause assigned for any phenomenon must not only be such as, if admitted, would explain the phenomenon, but must also be a *vera causa*" (Mill).

Mill's view:

4. The consequences that are deduced must be tested by appeal to observed facts. In so far as there is conflict the hypothesis must be modified or must be held *provisionally*. *Jevons* undoubtedly overstates the position when he asserts: 'A single absolute conflict between fact and hypothesis, is fatal to the hypothesis; *falsa in uno falsa in omnibus*. When a deduced consequence is found to be true the hypothesis is so far verified. If a careful experiment shows that a deduced consequence is false, then, it follows logically that the hypothesis as it stands cannot be true. But it does not follow that it is entirely wrong. Sometimes the negative experiment itself affords a suggestion as to what kind of modification is required. —(Stebbing)

5. "No experiment nor series of experiments can suffice to

establish an hypothesis beyond the reach of doubt. The final test is to be found only in the comprehensiveness of the system into which a given hypothesis fits. Verification does not amount to proof. To suppose otherwise is to commit the fallacy of the consequent.—Stebbing.

ON CONDITION (4)

6. This condition "has an illusory appearance of precision. In his discussion of this condition Jevons makes qualifications which seem to reduce it to the caution 'Don't make wild guesses.' Provided that there is a well established law of nature concerning a certain region of fact, it would be foolish to entertain an hypothesis directly contrary to such a law *unless a certain region of other facts requires such an hypothesis.* But to assert that an hypothesis cannot be a good hypothesis if it run counter to some other hypothesis is to confound hypothesis with fact and thus to mistake the function of hypothesis in scientific development."—Stebbing.

Stebbing's view :

7. In the passage in which *hypotheses non fingo* (I do not make hypothesis) occurs, Newton "is clearly speaking of the *cause* of the properties of gravity. He rejects two classes of hypotheses namely (i) metaphysical hypotheses involving occult qualities and (ii) physical hypotheses involving mechanical qualities. They are rejected on the ground that they cannot be inferred from the phenomena. It is not the business of 'experimental philosophy' to introduce unobservable causes. Whatever is not 'deduced from phenomena' is an hypothesis and may not be entertained by the physicist. The word 'deduced' requires some explanation. Newton is not using deduction in the precise sense of 'proof.' He is insisting upon the empirical basis of science. There is, however, *nothing in this passage* to suggest that hypotheses are inadmissible, provided that they be not put forward as demonstrations, nor that experimental investigation may not be aided by such hypotheses"—(Stebbing.)

Newton on importance of hypothesis :

8. Unless the role of hypothesis be grasped the structure of scientific method cannot be understood. It was contended by the great German scientist *Ostwald* that hypotheses were a hindrance to science. This view canont however be maintained. Ostwald seems to have thought of hypothesis as essentially a

picture or model or image containing elements not given in the original observations but imported by the Scientist, and therefore not necessarily in accord with what is given. Now......a descriptive hypothesis may well contain such elements and yet the hypothesis may be of the utmost value as a guide to experiment. Such hypotheses are psychologically valuable in the art of discovery. ......In the perfected scientific method hppothesis remains as the 'anticipation of nature' which conditions the process of experimental inquiry"—(Stebbing.)

EXERCISES WITH HINTS

1. *Define hypothesis. What are the conditions of a valid hypothesis ?*

[See secs. 1, 2 and 3]

2. *Is hypothesis an essential factor in Inductive investigation ? When is a hypothesis said to be raised to the position of induction.*

[See sec. 4 and Summary]

3. *What are the different forms of hypotheses ? Explain and illustrate.*

[See paras 2 and 3 of sec. 1 and note 1 to summary]

4. *What are the conditions of a legitimate hypothesis ?*

[See secs. 2 and 3]

5. *'Neither experiment nor observation is possible without hypothesis'—Do you agree ?*

[See supplementary note 2]

(The scientist is not a person who just observes: he observes with a purpose, namely, the purpose of discovering relevant data. He observes in the light of a theory about the facts. The theory may be vague ; nevertheless it directs his observation. The dangers to which expectant observation is liable does not rob it of its value—Stebbing.)

6. *To mistake verification of hypothesis for proof is to commit the fallacy of the consequent—Explain.*

[See supplementary note 5]

7. *How do you distinguish between hypothesis and theory, law and fact.*

[Sec sec. 1]

8. *Write notes on: crucial experiment, working hypothesis, vera causa, representative fiction and consilience of inductions.*

[Working hypothesis—when an hypothesis is such that it recognised to be inadequate and yet accepted for want of a better one it is called a wrong hypothesis.

Representative fiction :—Those hypothetical agents which admit of no direct proof but are recognised for their suitability to express phenomena are called by Bain representative fictions.]

9. *What is a postulate? Is the principle of uniformity of nature a postulate?*

[See last but one para sec. 5]

10. *What is a crucial instance? What is its value in inductive inquiry?*

11. *Describe and illustrate the method of hypothesis.*

12. *How is a hypothesis suggested?*

[It is often said that hypotheses ere suggested by the simple conversion of an A proposition, induction by simple enumeration and analogy. But we must note that "no precise rules can be laid down for the formation of hypotheses. They arise out of the context of our experience. Every hypothesis springs from the union of knowledge and sagacity. Neither alone is enough. Knowledge of facts may sometimes be acquired by patient observation and diligent labour; sagacity is a gift from heaven. Finally only a mind stored with the relevant knowledge and trained in the methods of science is capable of devising hypotheses that are both fruitful and comprehensive"—Stebbing.]

N.B. (That no rules can be formulated for the suggestion of an hypothesis has been expressed admirably by Singer in his History of science as folllows: 'How slight a thing determines a discovery—an accident, a suggestion, an analogy, a phrase, a word, even word misunderstood.")

13. *What is an ad hoc hypothesis?*

[See last para of supplementary note 1]

14. *Are all sciences hypothetical?*

[See last but one para sec. 5]

CHAPTER XIX

LAWS CLASSIFIED ; EXPLANATION ; CO-EXISTENCE ; ANALOGY

§ 1. AXIOMS ; PRIMARY LAWS ; SECONDARY LAWS, DERIVATIVE OR EMPIRICAL ; FACTS : Laws are classified according to their degrees of generality, as higher and lower, though the grades may not be decisively distinguishable.

Classification of Laws

First, there are Axioms or Principles, that is real, universal, self-evident propositions. They are—real propositions ; not, like 'The whole is greater than any of its parts,' merely definitions, or implied in definitions. (2) They are regarded as universally true of phenomena, as far as the form of their expression extends ; that is, for example, Axioms concerning quantity are true of everything that is considered in its quantitative aspect. (3) They are self-evident ; that is, each rests upon its own evidence (whatever that may be) ; they cannot be derived from one another, nor from any more general law. Some, indeed, are more general than others : the Logical Principle of Contradiction, 'if A is B, it is not not-B,' is true of qualities as well as of quantities ; whereas the Axioms of Mathematics apply only to quantities. The Mnthematical Axioms, again, apply to time, space, mental phenomena, and matter and energy ; whereas the Law of Causation is only true of concrete events in the redistribution of matter and energy : such, at least, is the strict limit of Causation, if we identify it with the Conservation of Energy ; although our imperfect knowledge of life and mind often drives us to speak of feelings, ideas, volitions, as causes. Still, the Law of Causation cannot be derived from the Mathematical Axioms, nor these from the Logical. The

(i) Axioms or principles—real universal, self-evident propositions

kind of evidence upon which Axioms rest, or whether any evidence can be given for them, is (as before observed) a question for Metaphysics, not for Logic. Axioms are the upward limit of Logic, which, like all the special sciences, necessarily takes them for granted,, as the starting point of all deduction and the goal of all generalisation.

(ii) Primary laws—of less generality than axioms and subject to the conditions of methodical proof.

Next to axioms, come Primary Laws of Nature: these are of less generality than the Axioms, and are subject to the conditions of methodical proof; being universally true only of certain forces or properties of matter, or of nature under certain conditions; so that proof of them by logical or mathematical reasoning is expected, because they depend upon the Axioms for their formal evidence. Such are the law of gravitation, in Astronomy; the law of definite proportions, in Chemistry; the law of heredity, in Biology; and in Psychology, the law of relativity.

(iii) Secondary Laws—of less generality than primary laws and are of two kinds—(1) derivative and (2) empirical.

Then, there are Secondary Laws, of still less generality, resulting from a combination of conditions of forces in given circumstances, and therefore conceivably derivable from the laws of those conditions or forces, if we can discover them and compute their united effects. Accordingly, Secondary Laws are either—(1) Derivative, having been analysed into, and deduced from, Primary Laws; or (2) Empirical, those that have not yet been deduced (though from their comparatively special and complex character, it seems probable they may be, given sufficient time and ingenuity), and that meanwhile rest upon some unsatisfactory sort of indication by Agreement or Simple Enumeration.

Laws established by the canon of Difference.

Whether laws proved only by the canon of Difference are to be considered Empirical, is perhaps a question: their proof derives them from

the principle of Causation ; but, being of narrow scope, some more special account of them seems requisite in relation to the Primary Laws before we can call them Derivative in the technical sense.

Many secondary laws are partially or imperfectly derivative.

Many Secondary Laws, again, are partially or imperfectly Derivative; we can give general reasons for them, without being able to determine theoretically the precise relations of the phenomena they describe. Meteorologists can explain the general conditions of all sorts of weather, but have made little progress toward predicting the actual course of it (at least, for our island) : Geologists know the general causes of mountain ranges, but not why they rise just where we find them : Economists explain the general course of a commercial crisis, but not why the great crisis recurred at intervals of about ten years.

Derivative laws make up the body of exact sciences while empirical laws are the undigested material of science.

Derivative Laws make up the body of the exact sciences, having been assimilated and organised : whilst Empirical Laws are the undigested materials of science. The theorems of Euclid are good examples of derivative laws in Mathematics; in Astronomy, Kepler's laws and the laws of the tides ; in Physics, the laws of shadows, of perspective, of harmony ; in Biology, the law of protective coloration : in Economics, the laws of prices, wages, interest, and rent.

Examples of empirical laws.

Empirical Laws are such as Bode's law of the planetary distances; the laws of the expansion of different bodies by heat, and formulae expressing the electrical conductivity of each substance as a function of the temperature. Strictly speaking, I suppose, all the laws of chemical combination are empirical: the law of definite propositions is verifiable in all cases that have been examined, except for variations that may be ascribed to errors of experiment. Much the same is true in Biology; most of the second-

ary laws are empirical, except so far as structures or functions may be regarded as specialised cases in Physics or Chemistry and deducible from these sciences. The theory of Natural Selection, however, has been the means of rendering many laws, that were once wholly empirical, at least partially derivative; namely, the laws of the geographical distribution of plants and animals, and of their adaptation in organisation, form and colour, habits and instincts, to their various conditions of life. The laws that remain empirical in Biology are of all degrees of generality from that of the tendency to variation in size and in every other character shown by every species (though as to the reason of this there are promising hypotheses), down to such curious cases as that the colour of roses and carnations never varies into blue, that scarlet flowers are never sweet-scented, that bullfinches fed on hemp-seed turn black, that the young of white, yellow and dun pigeons are born almost naked (whilst others have plenty of down); and so on. The derivation of empirical laws is the greater part of the explanation of Nature (§§ 5, 6).

How some of them become derivative.

A 'Fact,' in the common use of the word, is a particular observation: it is the material of science in its rawest state. As perceived by a mind, it is, of course, never absolutely particular: for we cannot perceive anything without classing it, more or less definitely, with things already known to us; nor describe it without using connotative terms which imply a classification of the things denoted. Still, we may consider an observation as particular, in comparison with a law that includes it with numerous others in one general proposition. To turn an observation into an experiment, or (where experiment is impracticable) to repeat it with all possible precautions and exactness, and to describe it as to the duration, quantity, quality and order of occurrence of its phenomena, is the first stage of scientific manufacture. Then comes the formulation of an empirical law; and

Fact and empirical laws.

lastly, if possible, deduction or derivation, either from higher laws previously ascertained, or from an hypothesis. However, as a word is used in various senses, we often speak of laws as 'facts': we say the law of gravitation is a fact, meaning that it is real, or verifiable by observations or experiments.

Another sense of the word 'fact'.

§ 2. SECONDARY LAWS EITHER INVARIABLE OR APPROXIMATE GENERALISATIONS: Secondary Laws may also be classified according to their constancy into—(1) the Invariable (as far as experience reaches), and (2) Approximate Generalisations in the form—Most X's are Y. Of the invariable we have given examples above. The following are approximate generalisations: Most comets go round the Sun from East to West; Most metals are solid at ordinary temperatures; Most marsupials are Australasian; Most arctic animals are white in winter; Most cases of plague are fatal; Most men think first of their own interests. Some of these laws are empirical, as that 'Most metal are solid at ordinary temperatures': at present no reason can be given for this; nor do we know why most cases of plague are fatal. Others, however, are at least partially derivative, as that 'Most arctic animals are white'; for this seems to be due to the advantage of concealment in the snow; whether, as with the bear, the better to surprise its prey, or, with the hare, to escape the notice of its enemies.

Secondary Laws—another classification—(1) invariable (2) approximate generalisation.

But the scientific treatment of such a proposition requires that we should also explain the exceptions: if 'Most are,' this implies that 'Some are not'; why not, then? Now, if we can give reasons for all the exceptions, the approximate generalisation may be converted into an universal one, thus: 'All arctic animals are white, unless (like the raven) they need no concealment either to prey or to escape; or unless mutual recognition is more important to them than concealment (as with the musk-sheep).' The same end of universal statement may be gained

by including the conditions on which the phenomenon depends, thus : 'All arctic animals to whom concealment is of the utmost utility are white.'

When statistics are obtainable, it is proper to convert an approximate generalisation into a proportional statement of the fact, thus : instead of 'Most attacks of plauge are fatal,' we might find that in a certain country 70 per cent. were so. Then, if we found, that in another country the percentage of deaths was 60, in another 40, we might discover, in the different conditions of these countries, a clue to the high rate of mortality from this disease. Even if the proportion of cases in which two facts are connected does not amount to 'Most,' yet, if any definite percentage is obtainable, the propostion has a higher scientific value than a vague 'Some' : as if we know that 2 per cent. of the deaths in England are due to suicide, this may be compared with the rates of suicide in other countries ; from which perhaps inferences may be drawn as to the causes of suicide.

In one department of life, namely Politics, there is a special advantage in true approximate generalisations amounting to 'Most cases.' The citizens of any State are so various in character, enlightenment, and conditions of life, that we can expect to find few propositions universally true of them : so that propositions true of the majority must be trusted as the bases of legislation. If most men are deterred from crime by fear of punishment ; if most men will idle if they can obtain support without industry ; if most jurymen will refuse to convict of a crime for which the prescribed penalties seem to them too severe ; these are most useful truths, though there should be numerous exceptions to them all.

§ 3. SECONDARY LAWS TRUSTWORTHY ONLY IN 'ADJACENT CASES' : Secondary Laws can only be trusted in 'Adjacent Cases' ; that is where the circumstances are similar to those in which the laws are known to be true.

Secondary Laws can be trusted only in adjacent cases.

A Derivative Law will be true wherever the

forces concerned exist in the combinations upon which the law depends, if there are no counteracting conditions. That water can be pumped to about 33 feet at the sea-level, is a derivative law on this planet: is it true in Mars? That depends on whether there are in Mars bodies of a liquid similar to our water; whether there is an atmosphere there, and how great its pressure is; which will vary with its height and density. If there is no atmosphere, there can be no pumping; or if there is an atmosphere of less pressure than ours, water such as ours can only be pumped to a less height than 33 feet. Again, we know that there are arctic regions in Mars; if there are also arctic animals, are they white? That may depend upon whether there are any beasts of prey. If not, concealment seems to be of no use.

Where a derivative law can be trusted.

When an empirical law can be trusted.

An empirical Law, being one whose conditions we do not know, the extent of its prevalence is still less ascertainable. Where it has not been actually observed to be true, we cannot trust it unless the circumstances, on the whole, resemble so closely those amongst which it has been observed, that the unknown causes, whatever they may be, are likely to prevail there. And, even then, we cannot have much confidence in it; for there may be unknown circumstances which entirely frustrate the effect. The first naturalist who travelled (say) from Singapore east-ward by Sumatra and Jave, or Borneo, and found the mammalia there similar to those of Asia, may naturally have expected the same thing in Celebes and Papua; but, if so, he was entirely disappointed; for in Papua the mammalia are marsupials like those of Australia. Thus his empirical law, 'The mammalia of the Eastern Archipelago are Asiatic,' would have failed for no apparent reason. According to Mr. Wallace, there is a reason for it, though such as could only be discovered by extensive researches; namely, that the sea is deep between Borneo and Celebes, so that they must have been separated for

many ages; whereas it is shallow from Borneo westward to Asia, and also southward from Papua to Australia; so that these regions, respectively, may have been recently united: and the true law is that similar mammalia belong to those tracts which at comparatively recent dates have formed parts of the same continents (unless they are the remains of a former much wider distribution).

A considerable lapse of time may make an empirical law no longer trustworthy.

A considerable lapse of time may make an empirical law no longer trustworthy; for the forces from whose combination it resulted may have ceased to operate, or to operate in the same combination; and since we do not know what those forces were, even the knowledge that great changes have taken place in the meantime cannot enable us, after an interval, to judge whether or not the law still holds true. New stars shine in the sky and go out; species of plants and animals become extinct; diseases die out and fresh ones afflict mankind: all these things doubtless have their causes, but if we do not know what they are, we have no measure of the effects, and cannot tell when or where they will happen.

The nature of the laws of concomitant variations.

Laws of Concomitant Variations may hold good only within certain limits. That bodies contract as the temperature falls, is not true of water below 39°F. In Psychology, Weber's Law is only true within the median range of sensation-intensities, not for very faint, nor for very strong, stimuli. In such cases the failure of the laws may depend upon something imperfectly understood in the collocation: as to water, on its molecular constitution; as to sensation, upon the structure of the nervous system.

Another classification—secon-

§ 4. Secondary Laws of Succession or of Co-existence: Secondary Laws, again, are either of Succession or of Co-existence.

Those of succession are either—(1) of direct

dary laws, of succession and of co-existence.

causation, as that 'Water quenches fire,' or (more strictly) that 'Evaporation reduces temperature'; or (2) of the effect of a remote cause, as 'Bad harvests tend to raise the price of bread'; or (3) of the joint effects of the same cause, as that

Kinds of Laws of succession.

'Night follows day' (from the revolution of the earth), or the course of the seasons (from the inclination of the earth's axis).

Laws of Co-existence are of several classes. (1) One has the generality of a primary law, though it is proved only by Agreement, namely, 'All gravitating bodies are inert'.

Kinds of laws of co-existence —(1) relating to the fundamental properties of bodies.

Others, though less general than this, are of very extensive range, as that 'All gases that are not decomposed by rise of temperature have the same rate of expansion'; and, in Botany that 'All monocotyledonous plants are endogenous.' These laws of Co-existence are concerned with fundamental properties of bodies.

(2) Next come laws of the Co-existence of those properties which are comprised in the definitions of Natural kinds. Mill distinguished between (a) classes of things that

(2) relation to natural kinds.

agree among themselves and differ from others only in one or a few attributes (such as 'red things,' 'musical notes,' 'carnivorous animals,' 'soldiers'), and (b) classes of things that agree among themselves and differ from others in a multitude of characters: and the latter he calls Natural Kinds. These comprise the chemical elements and their pure compounds (such as water, alcohol, rock-salt), and the species of plants and animals. Clearly, each of these is constituted by the co-existence or co-inherence of a multitude of properties, some of which are selected as the basis of their definitions. Thus, Gold is a metal of high specific gravity, atomic weight 197·2, high melting point, low chemical affinities, great ductility, yellow colour, *etc.*: a Horse has 'a vertebral column, mammæ, a placental embryo, four legs, a single

well-developed toe in each foot provided with a hoof, a bushy tail, and callosities on the inner sides of both the fore and the hind legs' (Huxley).

Should organic species be called natural kinds?

Since Darwinism has obtained general acceptance, some Logicians have doubted the propriety of calling the organic species 'Kinds,' on the ground that they are not, as to definiteness and permanence, on a par with the chemical elements or such compounds as water and rock-salt; that they vary extensively, and that it is only by the loss of former generations of animals that we are able to distinguish species at all. But to this it may be replied that species are often approximately constant for immense periods of time, and may be called permanent in comparison with human generations; and that, although the leading principles of Logic are perhaps eternal truths, yet upon a detail such as this, the science may condescend to recognise a distinction if it is good for (say) only 100,000 years. That if former generations of plants and animals were not lost, all distinctions of species would disappear, may be true; but they are lost—for the most part beyond hope of recovery; and accordingly the distinction of species is still recognised; although there are cases, chiefly at the lower stages of organisation, in which so many varieties occur as to make adjacent species almost or quite indistinguishable. So far as species are recognised, then, they present a complex co-inherence of qualities, which is, in one aspect, a logical problem; and, in another, a logical datum; and coming more naturally under the head of Natural Kinds than any other, they must be mentioned in this place.

(3) Relating to certain coincidences of qualities not essential to any kind.

(3) There are, again, certain coincidences of qualities not essential to any kind, and sometimes prevailing amongst many different kinds: such as 'Insects of nauseous taste have vivid (warning) colours'; 'White tom-cats with blue eyes are deaf'; 'White spots and patches, when they appear in domestic animals, are most frequent on the left side.'

(4) relating to constancy of relative position.

(4) Finally, there may be constancy of relative position, as of sides and angles in Geometry; and also among concrete things (at least for long periods of time), as of the planetary orbits, the apparent positions of fixed stars in the sky, the distribution of land and water on the globe, opposite seasons in opposite hemispheres.

All co-existences except the geometrical present the problem of deriving them from causation.

All these cases of Co-existence (except the geometrical) present the problem of deriving them from Causation; for there is no general Law of Co-existence from which they can be derived; and, indeed, if we conceive of the external world as a perpetual redistribution of matter and energy, it follows that the whole state of Nature at any instant, and therefore every co-existence included in it, is due to causation issuing from some earlier distribution of matter and energy. Hence, indeed, it is not likely that the problems of co-existence as a whole will ever be solved, since the original distribution of matter is, of course, unknown. Still, starting with any given state of nature, we may hope to explain some of the co-existences in any subsequent state. We do not, indeed, know why heavy bodies are always inert, nor why the chemical elements are what they are; but it is known that "the properties of the elements are functions of their atomic weight," which (though, at present, only an empirical law) may be a clue to some deeper explanation. As to plants and animals, we know the conditions of their generation, and can trace a connection between most of their characterstics and the conditions of their life: as that the teeth and stomach of animals vary with their food, and that their colour generally varies with their habitat.

Geometrical co-existence is

Geometrical Co-existence, when it is not a matter of definition (as 'a square is a rectangle with four equal sides'), is deduced from the definitions and axioms: as when it is shown that in triangles the

deduced from definition and axioms.

greater side is opposite the greater angle. The deductions of theorems or secondary laws, in Geometry is a type of what is desirable in the Physical Sciences: the demonstration, namely, that all the connections of phenomena, whether successive or co-existent, are consequences of the redistribution of matter and energy according to the principle of Causation.

Coincidences of co-existence may sometimes be deduced and sometimes not.

Coincidences of Co-existence (Group (3)) may sometimes be deduced and sometimes not. That 'nauseous insects have vivid colouration' comes under the general law of 'protective colouration'; as they are easily recognised and therefore avoided by insectivorous birds and other animals. But why white tom-cats with blue-eyes should be deaf, is (I believe) unknown. When Co-existences cannot be derived from causation, they can only be proved by collecting examples and trusting vaguely to the Uniformity of Nature. If no exceptions are found, we have an empirical law of considerable probability within the range of our exploration. If exceptions occur, we have at most an approximate generalisation, as that 'Most metals are whitish,' or 'Most domestic cats are tabbies' (but this probably is the ancestral colouring). We may then resort to statistics for greater definiteness, and find that in Hampshire (say) 90 *per cent* of the domestic cats are tabby.

The nature of scientific explanation.

§ 5. Explanation consists in tracing resemblance, especially of Causation: Scientific Explanation consists in discovering, deducing, and assimilating the laws of phenomena; it is the analysis of that Heracleitan 'flux' which so many philosophers have regarded as intractable to human inquiry. In the ordinary use of the word, 'explanation' means the satisfying a man's understanding; and what may serve this purpose depends partly upon the natural soundness of his understanding, and partly on his education; but it is always at last an appeal

to the primary functions of cognitions, discrimination and assimilation.

So long as curiosity is not excited explanation is not demanded.

Generally, what we are accustomed to seems to need no explanation, unless our curiosity is particularly directed to it. That boys climb trees and throw stones, and that men go fox-hunting, may easily pass for matters of course. If any one is so exactting as to ask the reason, there is a ready answer in the 'need of exercise'. But this will not explain the peculiar zest of those exercises, which is something quite different from our feelings whilst swinging dumb-bells or tramping the highway. Others, more sophisticated, tell us that the civilised individual retains in his nature the instincts of his remote ancestors, and that these assert themselves at stages of his growth corresponding with ancestral periods of culture or savagery : so that if we delight to climb trees, throw stones, and hunt, it is because our forefathers once lived in trees, had no missiles but stones, and depended for a livelihood upon killing something. To some of us, again, this seems an explanation ; to others it merely gives annoyance, as a superfluous hypothesis, the fruit of a wanton imagination and too much leisure.

What we are not accustomed immediately demands explanation.

However, what we are not accustomed to immediately excites curiosity. If it were exceptional to climb trees, throw stones, ride after foxes, however did such things would be viewed with suspicion. An eclipse, a shooting star, a solitary boulder on the heath, a strange animal, or a Chinaman in the street, calls for explanation ; and among some nations, eclipses have been explained by supposing a dragon to devour the sun or moon; solitary boulders, as the missiles of a giant; and so on. Such explanations, plainly, are attempts to regard rare phenomena as similar to others that are better known; a snake having been seen to swallow a rabbit, a bigger one may swallow the sun : a giant is supposed to bear much the same relation to a boulder as a boy does to half a

brick. When any very common thing seems to need no explanation, it is because the several instances of its occurence are a sufficient basis of assimilation to satisfy most of us. Still, if a reason for such a thing be demanded, the commonest answer has the same implication, namely, that assimilation or classification is a sufficient reason for it. Thus, if climbing trees is referred to the need of exercise, it is assimilated to running, rowing, *etc.*; if the customs of a savage tribe are referred to the command of its gods, they are assimilated to those things that are done at the command of chieftains.

Explanation and classification:

Explanation in Mathematics.

Explanation, then, is a kind of classification; it is the finding of resemblance between the phenomenon in question and other phenomena. In Mathematics, the explanation of a theorem is the same as its proof, and consists in showing that it repeats, under different conditions, the definitions and axioms already assumed and the theorems already demonstrated. In Logic, the major premise of every syllogism is an explanation of the conclusion; for the minor premise asserts that the conclusion is an example of the major premise.

Logic and the concrete sciences.

Explanation by quantitative laws

In Concrete Sciences, to discover the cause of a phenomenon, or to derive an empirical law from laws of causation, is to explain it; because a cause is an invariable antecedent, and therefore reminds us of, or enables us to conceive, an indefinite number of cases similar to the present one wherever the cause exists. It classifies the present case with other instances of causation, or brings it under the universal law; and, as we have seen that the discovery of the laws of nature is essentially the discovery of causes, the discovery and derivation of laws is scientific explanation.

The discovery of quantitative laws is especially satisfactory, because it not only explains why an event happens at all, but

why it happens just in this direction, degree, or amount; and not only is the given relation of cause and effect definitely assimilated to other causal instances, but the effect is identified with the cause as the same matter and energy redistributed; wherefore, whether the conservation of matter and energy be universally true or not, it must still be an universal postulate of scientific explanation.

and by an empirical law of coexistence.

The mere discovery of an empirical law of coexistence, as that 'white tom-cats with blue eyes are deaf,' is indeed something better than an isolated fact: every general proposition relieves the mind of a load of facts; and, for many people, to be able to say —'It is always so'—may be enough; but for scientific explanation we require to know the reason of it, that is, the cause. Still, it asked to explain an axiom, we can only say, 'It is always so:' though it is some relief to point out particular instances of its relation, or to exhibit the similarity of its form to that of other axioms—as of the *Dictum* to the axiom of equality.

The modes of scientific explanation (1) Analysis.

§ 6. THREE MODES OF EXPLANATION: There are three modes of scientific Explanation: First, the analysis of a phenomenon into the laws of its causes and the concurrence of those causes.

The pumping of water implies (1) pressure of the air, (2) distribution of pressure in a liquid, (3) that motion takes the direction of least resistance. Similarly, that thunder follows forked lightning, and that the report of a gun follows the flash, are resolvable into (1) the discharge of electricity, or the explosion of gunpowder; (2) distance of the observer from the event; (3) that light travels faster than sound. The planetary orbits are analysable into the tendency of planets to fall into the sun, and their tendency to travel in a straight line. When this conception is helped out by swinging a ball round by a string, and then letting it go, to show what

Examples.

would happen to the earth if gravitation ceased, we see how the recognition of resemblance lies at the bottom of explanation.

(ii) Concatenation.

Secondly, the discovery of steps of causation between a cause and its remote effects; the interpolation and concatenation of causes.

examples.

The maxim 'No cats no clover' is explained by assigning the intermediate steps in the following series; that the fructification of red clover depends on the visits of humble-bees, who distribute the pollen in seeking honey; that if field-mice are numerous they destroy the humble bees' nests; and that (owls and weasels being exterminated by gamekeepers) the destruction of field-mice depends upon the supply of cats; which, therefore, are a remote condition of the clover crop. Again, the communication of thought by speech is an example of something so common that it seems to need no explanation; yet to explain it is a long story. A thought in one man's mind is the remote cause of a similar thought in another's: here we have (1) a thought associated with mental words; (2) a connection between these thoughts and some tracts of the brain; (3) a connection between these tracts of the brain and the muscles of the larynx, the tongue and the lips; (4) movements of the chest, larynx and mouth, propelling and modifying waves of air; (5) the impinging of these air-waves upon another man's ear, and by a complex mechanism exciting the aural nerve; (6) the transfer of this excitation to certain tracts of his brain; (7) a connection there with sounds of words and their associated thoughts. If one of these links fail, there is no communication.

Thirdly, the subsumption of several laws under one more general expression.

and (iii) subsumption.

The tendency of bodies to fall to the earth and the tendency of the earth itself (with the other planets) to fall into the sun, are subsumed under the general law that "All matter gravitates.' The same

law subsumes the movements of the tide. By means of the notion of specific gravity, it includes 'levitation,' or the actual rising of some bodies, as of corks in water, of balloons, or flames in the air: the fact being that these things do not tend to rise, but to fall like everything else; only as the water or air weighs more in proportion to its volume than corks or balloons, the latter are pushed up.

example

This process of subsumption bears the same relation to secondary laws, that these do to particular facts. The generalisation of many particular facts (that is, a statement of that in which they agree) is a law; and the generalisation of these laws (that is, again, a statement of that in which they agree) is a higher law; and this process, upwards or downwards, is characteristic of scientific progress. The perfecting of any science consists in comprehending more and more of the facts within its province, and in showing that they all exemplify a smaller and smaller number of principles, which express their most profound resemblances.

What the secondary laws are to particular facts, that subsumption is to them.

These three modes of explanation (analysis, interpolation, subsumption) all consist in generalising or assimilating the phenomena. The pressure of the air, of a liquid, and motion in the direction of least resistance, are all commoner facts than pumping; that light travels faster than sound is a commoner fact than a thunderstorm or gun-firing. Each of the laws—'Cats kill mice,' 'Mice destroy humble-bees' nests,' 'Humble-bees fructify red clover'—is wider and expresses the resemblance of more numerous cases than the law that 'Clover depends on cats'; because each of them is less subject to further conditions. Similarly, every step in the communication of thought by language is less conditional, and therefore more general, than the completion of the process.

All these modes consist in generalising or assimilating the phenomena.

In all the above cases, again, each law into which the phenomenon (whether pumping or conversation) is resolved, sug-

gests a host of parallel cases: as the modifying of air-waves by the larynx and lips suggests the various devices by which the strings and orifices of musical instruments modify the character of notes.

Subsumption proves the existence of similarity between things where it was formerly not observed.

Subsumption consists entirely in proving the existence of an essential similarity between things where it was formerly not observed: as that the gyrations of the moon, the fall of apples, and the floatation of bubbles are all examples of gravitation: or that the purifying of the blood by breathing, the burning of a candle, and the rusting of iron are all cases of oxidation: or that the colouring of the underside of a red-admiral's wings, the spots of the giraffe, the shape and attitude of a stick-caterpillar, the immobility of a bird on its nest, and countless other cases, though superficially so different, agree in this, that they conceal and hereby protect the organism.

Scientific explanation demands likeness in respect of causation.

Not any sort of likeness, however, suffices for scientific explanation; the only satisfactory explanation of concrete things or events, is to discover their likeness to others in respect of Causation. Hence attempts to help the understanding by familiar comparisons are often worse than useless. Any of the above examples will show that the first result of explanation is not to make a phenomenon seem familiar, but to put (as the saying is) quite a new face upon it.' When, indeed, we have thought it over in all its newly discovered relations, we feel more at home with it than ever; and this is one source of our satisfaction in explaining things; and hence to substitute immediate familiarisation for radical explanation, is the easily besetting sin of human understanding: the most plausible of fallacies, the most attractive, the most difficult to avoid even when we are on our guard against it.

§ 7. Limits of Explanation: The explanation of Nature (if it be admitted to consist in generalisation, or the discovery

of resemblance amidst differences) can never be completed. For —(1) there are (as Mill says) facts, namely, fundamental states or processes of consciousness, which are distinct; in other words, they do not resemble one another and therefore cannot be generalised or subsumed under one explanation. Colour, heat, smell, sound, touch, pleasure and pain, are so different that there is one group of conditions to be sought for each; and the laws of these conditions cannot be subsumed under a more general one without leaving out the very facts to be explained. A general condition of sensation, such as the stimulating of the sensory organs of a living animal, gives no account of the *special* characters of colour, smell, *etc.*; which are, however, the phenomena in question; and each of them has its own law. Nay, each distinct sensation-quality, of degree, must have its own law; for in each ultimate difference there is something that cannot be assimilated. Such differences amount, according to experimental Psychologists, to more than 50,000. Moreover, a neural process can never explain a conscious process in the way of cause and effect; for there is no equivalence between them, and one can never absorb the other.

Limits of explanation

(i) Fundamental states of consciousness cannot be explained.

(2) When physical science is treated objectively (that is with as little reference as possible to the fact that all phenomena are only known in relation to the human mind), colour, heat, smell, sound (considered as sensations) are neglected and attention is fixed upon certain of their conditions: extension, figured resistance, weight, motion, with their derivatives, density, elasticity, *etc.* These are called the Primary Qualities of Matter; and it is assumed that they belong to matter by itself, whether we look on or not: whilst colour, heat, sound, *etc.*, are called Secondary Qualities, as depending entirely upon the reaction of some conscious animal. By physical science the world is considered in the abstract, as a perpetual re-

(ii) Primary qualities of matter cannot be explained.

distribution of matter and energy; and the distracting multiplicity of sensations seems to be got rid of.

But, not to dwell upon the difficulty of reducing the activities of life and chemistry to mechanical principles—even if this were done, complete explanation could not be attained. For—*(a)* as explanation is the discovery of causes, we no sooner succeed in assigning the causes of the present state of the world than we have to inquire into the causes of those causes, and again the still earlier causes, and so on to infinity. But, this being impossible, we must be content, wherever we stop, to contemplate the uncaused, that is, the unexplained; and then all that follows is only relatively explained.

Besides this difficulty, however, there is another that prevents the perfecting of any theory of the abstract material world, namely *(b)*, that it involves more than one first principle. For we have seen that the Uniformity of Nature is not really a principle, but a merely nominal generalisation, since it cannot be definitely stated; and, therefore, the principles of Contradiction, Mediate Equality, and Causation remain incapable of subsumption; nor can any one of them be reduced to another: so that they remain unexplained.

(iii) Particular facts can never be fully explained.

(3) Another limit to explanation lies in the infinite character of every particular fact; so that we may know the laws of many of its properties and yet come far short of understanding it as a whole. A lump of sandstone in the road: we may know a good deal about its specific gravity, temperature, chemical composition, geological conditions; but if we inquire the causes of the particular modifications it exhibits of these properties, and further why it is just so big, containing so many molecules, neither more nor less, disposed in just such relations to one another as to give it this particular figure, why it lies exactly there rather than a yard off, and so forth, we shall get no explanation of all this. The causes determining each particular phenomenon are infinite, and can

never be computed; and, therefore, it can never be fully explained.

§ 8. ANALOGY: Analogy is used in two senses: (1) for the resemblance of relations between terms that have little or no resemblance— as *The wind drives the clouds as a shephered drives his sheep*—where wind and shepherd, clouds and sheep are totally unlike. Such analogies are a favourite figure in poetry and rhetoric, but cannot prove anything. For valid reasoning there must be parallel cases, according to substance and attribute, or cause and effect, or proportion; *e.g. As cattle and deer are to herbivorousness, so are camels*; *As bodies near the earth fall toward it, so does the moon: As 2 is to 3 so is 4 to 6.*

Analogy, as not a form of proof.

(2) Analogy was discussed in Logic a kind of probable proof based upon imperfect similarity (as the best that can be discovered) between the *data* of comparison and the subject of our inference. Like Deduction and Induction, it assumes that things which are alike in some respects are also alike in others; but it differs from them in not appealing to a definite general law assigning the essential points of resemblance upon which the argument relies. In Deductive proof, this is done by the major premise of every syllogism: if the major says that 'All fat men are humorists,' and we can establish the minor, 'X is a fat man,' we have secured the essential resemblance that carries the conclusion. In induction, the Law of Causation and its representatives, the Canons, served the same purpose, specifying the essential marks of a cause. But, in Analogy, the resemblance relied on cannot be stated categorically.

Definition of analogy as a form of proof.

Its relation to deduction and induction.

If we argue that Mars is inhabited because it resembles the datum, our Earth, (1) in being a planet, (2) neither too hot nor too cold for life, (3) having an atmosphere, (4) land and water, *etc.*, we are not prepared to say that 'All planets having these characteristics are

Illustration:

inhabited.' It is, therefore, not a deduction; and since we do not know the original causes of life on the Earth, we certainly cannot show by induction that adequate causes exist in Mars. We rely then, upon some such vague notion of Uniformity as that 'Things alike in some points are alike in others'; which, plainly, is either false or nugatory. But if the linear markings upon the surface of Mars indicate a system of canals, the inference that he has intelligent inhabitants is no longer analogical, since canals can have no other cause.

Value of it and on what it depends.

The cogency of any proof depends upon the *character* and *definiteness* of the likeness which one phenomena bears to another; but Analogy trusts to the general *quantity* of likeness between them, in ignorance of what may be the really important likeness. If, having tried with a stone, an apple, a bullet, *etc.*, we find that they all break an ordinary window, and thence infer that a cricket ball will do so, we do not reason by analogy, but make instinctively a deductive extension of an induction, merely omitting the explicit generalisation, 'All missiles of a certain weight, size and solidity break windows.' But if, knowing nothing of snakes except that the viper is venomous, a child runs away from a grass-snake, he argues by analogy; and, though his conduct is prudentially justifiable, his inference is wrong: for there is no law that 'All snakes are venomous', but only that those are venomous that have a certain structure of fang; a point which he did not stay to examine.

Its uses.

The discovery of an analogy, then, may suggest hypotheses; it states a problem—to find the causes of the analogy; and thus it may lead to scientific proof; but merely analogical argument is only probable in various degrees. (1) The greater the number and importance of the points of agreement, the more probable is the inference. (2) The greater the number and importance of the points of difference, the less probable is the inference. (3) The greater the number of unknown properties in the subject of our argu-

ment, the less the value of any inference from those that we do know. Of course the number of unknown properties can itself be estimated only by analogy. In the case of Mars, they are *probably very* numerous; and, apart from the evidence of canals, the prevalent assumption that there are *intelligent beings* in that planet, seems to rest less upon probability than on a curiously imaginative extension of the gregarious sentiment, the chilly discomfort of mankind at the thought of being alone in the universe, and a hope that there may be conversable and 'clubable' souls nearer than the Dog-star.

SUMMARY

Laws *(see also supplementary note* 1) are classified according to their degrees of generality into three classes viz. (i) Axioms (ii) Primary Laws and (iii) Secondary Laws. Axioms or principles are real, universal and self-evident propositions. They are the most general laws. In their own sphere there are no laws more general than they. And so they cannot be proved. They are, thus, the upward limit of Logic which like all the special sciences, necessarily takes them for granted, as the starting point of all deduction and the goal of all generalisation. The laws of Thought, the law of Causation etc. are examples of Axioms. Next to axioms come Primary laws of nature : these are of less generality than the axioms and are subject to the conditions of methodical proof. The law of Gravitation in Astronomy, the law of definite proportions in Chemistry are examples of such laws. Then there are Secondary laws which are of still less generality. Such laws are either Derivative or Empirical. The Derivative laws are those Secondary laws that have been analysed into and deduced from Primary laws. Empirical laws are those Secondary laws, that, given sufficient time and ingenuity, may be deduced from primary laws but have not yet been so deduced and meanwhile rest upon some unsatisfactory sort of induction by agreement or simple enumeration. A Secondary law that is proved only by the cannon of difference can be called

Derivative if some special account of it in relation to Primary laws is given. Many Secondary laws are partially Derivative: we can give general reasons for them without being able theoritically to determine the precise relations of the phenomena they describe. The Derivative laws make up the body of the exact sciences, having been assimilated and generalised; while Empirical laws are the undigested materials of science. The theorems of Euclid, Keplar's laws, the laws of Shadows etc. are the examles of derivative laws and Bode's, law of the planetary distances, the laws of expansion of different bodies by heat etc., are the examples of empirical laws. An Empirical law of one age may become promoted to a Derivative law at another age, and the derivation of empirical laws is the greater part of the explanation of Nature. Thus, the fact of the observed prticular is the material of science in the rawest state. Then comes the formulation of the empirical law; and lastly if possible, deduction or derivation, either from higher laws previously ascertained or from an hypothesis.

Secondary laws may also be classified according to their constancy into (1) the Invariable and (2) Approximate Generalisations in the form 'Most Xs are Y.' Examples of Secondary laws given above are also the examples of invariable laws. 'Most metals are solid at ordinary temperature,' 'most arctic animals are white in winter' etc., are examples of approximate generalisations. Now science demands that why most are and some are not, should be explained—and when this explanation is given i.e., when reasons are given for all the exceptions, the approximate generalisation may be converted into invariable laws. When statistics are obtainable it is proper to convert an approximate generalisation into a proportional statement of the fact, and in politics there is a special advantage in true Approximate generalisation amounting to 'Most cases.'

Secondary laws can only be trusted in 'Adjacent Cases'; that is, where the circumstances are similar to those in which the laws are known to be true. Thus, a Derivative law will be true wherever the forces concerned exist in the combinations upon which the law depends and there are no counteracting conditions. The extent of the prevalence of empirical laws is still less ascertainable. Where it has not been actually observed to be true, we cannot trust it, unless the circumstances on the whole, resemble so closely those amongst which it has been ob-

served that the unknown causes are likely to prevail there. Again, a considerable lapse of time may make an empirical law no longer trustworthy. The laws of concomitant variations may hold good only within certain limits. The bodies contract as the temperature falls is not true of water below 39°F.

Secondary Laws, again, are either of succession or of co-existence. Laws of succession are either (1) of direct causation, as that Evaporation reduces temperature, or (2) of the effect of a remote cause as 'Bad harvests tend to raise the price of bread, or (3) of the joint effects of the same cause as that 'Night follows day.' Laws of co-existence are of several classes : (i) One has the generality of primary law, though proved only by the method of agreement e.g., All gravitating bodies are inert (2) Next come laws of co-existence of those properties which are comprised in the definitions of Natural Kinds e.g., Gold is a metal of high specific gravity, high melting point etc. (3) There are again, certain coincidences of qualities not essential to any kind, and sometimes prevailing amongst many different kinds, e. g., Insects of nauseous taste have vivid colours, (4) Finally there may be constancy of relative position as of sides and angles in Geometry. All these cases of co-existence, except the geometrical, present the problem of deriving them from causation. Geometrical eo-existence, when it is not a matter of definition is deduced from definitions and axioms. And the coincidences of co-existence may sometimes be deduced and sometimes not.

EXPLANATION. (*See also supplementary note 2*)

Scientific Explanation consists in discovering, deducing and assimilating the laws of phenomena. (*See also supplementary note 3*) Generally what we are accustomed to, seems to need no explanation, unless our curiosity is particularly directed to it. What we are not accustomed to, immediately excites curiosity and demands explanation.

Explanation is a kind of classification. (*See also supplementary note 4*) It is the finding of resemblance between the phenomenon in question and other phenomena. In Mathematics the explanation of a theorem is the same as its proof. In Logic the major premise of every syllogism is an explanation of the conclusion. In concrete sciences to discover the cause of a phenomenon or to derive an empirical law from laws of causation is to explain it. The discovery of quantitative laws is a very satis-

factory form of explanation. The mere discovery of an Empirical law of co-existence, though not a very satisfactory form of explanation, is indeed something better than an isolated fact.

There are three modes of scientific explanation, viz., analysis, concatenation and subsumption. In the first there is an analysis of a phenomenon into the laws of its causes and the concurrence of those causes. Thus, the pumping of water implies (1) pressure of the air, (2) distribution of pressure in the liquid, (3) that motion takes the direction of the least resistance. In the second there is a discovery of steps of causation between a cause and its remote effects. The maxim, 'No cats no clover' is explained by assigning the intermediate steps in the following series; that the fructification of the red clover depends on the visits of humble-bees, who distribute the pollen in seeking honey; that if field-mice are numerous they destroy the humble-bees, nests; and that the destruction of field-mice depends upon the supply of cats, which, therefore, are a remote condition of the clover crop. In the third there is a subsumption of several laws under one more general assumption. The tendency of bodies to fall to the earth and the tendency of the earth itself to fall into the sun, are subsumed under the general law that 'All matter gravitates.' This process of subsumption bears the same relation to secondary laws that those do, to particular facts. The generalisation of many particular facts is a law, this generalisation of these laws is a higher law, and this process upwards or downwards, is characteristic of scientific progress.

These three modes of explanation (*see also supplementary note* (5) consist in generalising or assimilating the phenomena. The pressure of the air, of a liquid, and motion in the direction of least resistance, are all commoner facts than pumping. Cats kill mice, mice destroy humble-bees' nests etc. are more general laws than the law clover depends on cats. The law of gravitation is more general than the several laws subsumed under it.

Subsumption consists entirely in proving the existence of an essential similarity between things where it was not observed as that the gyrations of the moon, the fall of apples and the floatation of bubbles are examples of gravitation. But any sort of likeness does not suffice for scientific explanation. It demands likeness in respect of causation.

Explanation has its limits. Thus (i) the fundamental states of consciousness, (ii) the primary qualities of matter, (iii) the

ultimate laws, and (iv) an individual fact in its fulness cannot be explained.

Analogy

Analogy is a kind of probable proof between the *data* of comparison and the subject of our inference. Like deduction and induction, it assumes that things which are alike in some respects are also alike in others; but it differs from them in not appealing to a definite general law assigning the essential points of resemblance upon which the argument relies. In deductive proof, this is done by the major premise of every syllogism, in induction the law of causation and the canons serve the same purpose. But, in analogy the resemblance relied on cannot be stated categorically.

The cogency of any proof depends upon the *character* and *definiteness* of the likeness which one phenomenon bears to another; but analogy trusts to the general *quantity* of likeness between them. So the conclusion is merely probable. The degree of probability depends upon the proportionate relation between the number and importance of the points of resemblance and the number and importance of the points of difference taken in conjunction with the number of the unknown points.

SUPPLEMENTARY NOTES

1. Laws : The word law is rather ambiguous. It may in the first place mean a mandate or order that emanates from the will of some superior authority and is imposed upon a community subject to it. In this sense, we speak of the law of a state. Such is the nature of this law that it is made by a man or a group of men and can be violated and changed. In the second place, it may mean uniformity. In this sense we speak of the laws of nature. Such is the nature of these laws that they are not man-made and are "constant, inviolable and all-pervading." That is, these laws can neither be changed nor violated. In the third place it may mean a *norm* or standard to which we must conform if we are to attain some end. In this sense we speak of the Laws of Ethics, of Logic (i. e. of the normative science). These

laws are not man-made and cannot be changed but may by violated. Thus, the word law sometimes stands for some regularities that are created by men and can be both changed and violated by men. Again, it stands for some regularities or uniformities that are not man-made and can neither be changed nor violated. And finally, it stands for some standards or principles that are not man-made and can be violated though not changed.

2. Explanation : To explain is to relate and thereby to satisfy the curiosity that is excited. Now, a lay man and a scientist may relate and satisfy their curiosity in different ways. That is, a mere superficial similarity may satisfy a lay man but the scientist will not be satisfied by anything that falls short of essential similarity. So there are two kinds of explanation viz. popular and scientific. The explanation that satisfies a layman is a popular explanation while the explanation that satisfies a scientist is a scientific one. The distinction between these two kinds of explanation may be expressed as follows :

(1) Popular explanation is satisfied with the superficial points of similarity but scientific explanation concerns itself with deep-seated or fundamental similarity. That is, not any sort of likeness suffices for scientific explanation ; the only satisfactory explanation of concrete things or events, is to discover their likeness to others in respect of causation.

(2) As popular explanation is satisfied with the superficial points of similarity, so it often takes the help of what is called a 'Bad Analogy' for explaining an event. Thus, the explanation of the eclipse of the moon as the devouring by a great serpent or demon is a popular explanation, and it is done on the analogy of the swallowing of frogs by serpents. But for scientific explanation even a good analogy is not good enough.

(3) Again, as popular explanation often takes the form of bad analogy, so it also takes recourse to supernatural causes for the purpose of explaining natural events. The explanation of eclipse as due to the devouring by a demon aptly illustrates this characteristic of popular explanation. Scientific explanation is always in terms of natural causes or laws.

(4) Popular explanation intends to satisfy the curiosity of the laymen ; and only particular events excite such curiosities. Thus, a layman is interested in to-day's weather, his fever, her jaundice and so on. Ordinarily he is not interested in weather or fever or jaundice as such. And so, popular explanation is al-

ways of a particular fact, and not of a general law. But scientific explanation is not only of a particular fact but also of a general law.

(5) If it is a particular fact that excites the curiosity of the lay man, it is a particular fact that often satisfies this curiosity. Thus, if a lay man is curious of to-day's weather, he satisfies his curiosity by yesterday's storm. Thus, a popular explanation usually is of a particular facts in terms of another particular fact. But scientific explanation is in terms of general laws.

[There is a special and every day form of explanation that consists in assigning the agency in a particular occurrence; as when we ask—what stops the way? Who wrote Junius, who discovered gunpowder? These questions belong to our practical wants and urgencies, but the answer does not involve the process of scientific explanation. If, however, we proceed from the 'who' or 'what' to the 'why'—why does A's carriage stop the way? Why did the author of Junius write so bitterly?—there is an opening for the higher scientific process—(Bain.)]

(6) Popular explanation is rather an explanation in terms of the familiar, while "*in science those who speak of explaining any phenomenon mean (or should mean) pointing out not some more familiar but merely some more general phenomenon, of which it is a partial exemplification*"—*(Mill)*.

3. SCIENTIFIC EXPLANATION: *Scientific explanation consists in harmonizing fact with fact or fact with law or law with law, so that we may see them both to be cases of one uniform law of causation* —(Jevons).]

4. EXPLANATION IS A KIND OF CLASSIFICATION. ON THE RELATION OF CLASSIFICATION AND EXPLANATION HOBHOUSE WRITES: In classifying we deal with general attributes and their relations; in explaining with universal laws and their relations. Thus, in classifying we describe what is; in explaining, what must be and why it must be; in classifying we do not look for antecedents; in explaining we so analyse the antecedent as to show ground for the consequent: in classifying our ideal is to bring our contents under a *sumum genus* by some fixed principles of differentiation; in explaining it is to resolve complex laws into elements which are themselves connected. But though classifying and explaining can never be the same, the first is the true step to the second, in that in laying down resemblance it presumes identity of causation, and in pointing out differences

it sets the problem of discovering reasons why; while lastly, in identifying differences it indicates the solution which would cover in one explanation its total field of facts.

5. ON THE THREE MODES OF EXPLANATION MILL WRITES: There are three modes of explaining laws of causation, or which is the same thing resolving them into other laws. First, when the law of an effect of combined causes is resolved into the separate laws of the causes, together with the fact of their combination. Secondly when the law which connects any two links not proximate in a chain of causation is resolved into the laws which connect each with the intermediate links. Both of these are cases of resolving one law into two or more; in the third two or more are resolved into one....In all these three processes, laws are resolved into laws....more general than themselves, laws extending to all the cases which the former extended to and others besides. In the first two modes they are also resolved into laws more certain, in other words, more universally true than themselves; they are, in fact, proved not to be themselves laws of nature, the character of which is to be universally true, but *results* of laws of nature which may be true conditionally and for the most part. No difference of this sort exists in the third case, since here the partial laws are in fact, the very same law as the general one and any exception in them would be an exception to them too.

EXERCISES WITH HINTS

1. *What is meant by law and law of nature? Indicate the various senses in which the law is used.*

[See note 1 to summary.]

2. *How could you classify laws? Explain and illustrate the different kinds of laws.*

[See sec. 1—4.]

3. *The world is a system of laws—explain.*

[It is the assumption of the sciences that the world is governed by laws. Guided by this assumption, the sciences attempt to discover and prove them. As it is impossible for one science, to study the universe as a whole and as the universe presents

various aspects, so different sciences study different aspects of nature and find out the laws governing these different aspects. In its attempt to find out the laws every science proceeds with particular facts and brings them under a law ; this law again is brought under a still higher law which again is brought under still higher laws. This process continues so long as the primary laws or axioms are not reached. Now this bringing of less universal laws under more universal laws establishes the connection not only between them, but also among the laws that are under the more universal law and co-ordinate with the less universal one. Again, the laws established by one science are connected with the laws established by another science ; and in this way a systematic theory of nature is established. This shows that the world is a system of law and the different sciences study the different aspects of it. Such is the view of the monists. The pluralists object to it. They deny with the monists that the universe is a chaos. But they do not hold that the world is one system. It is quite in keeping with the progress of the sciences (they hold) to hold that the world is not one system of law but many systems of laws.]

4. *What is meant by scientific explanation ? How would you distinguish it from popular explanation ?*

[See sec. 5, and notes 2 and 3 to summary.]

5. *Explain and illustrate the different modes of scientific explanation.*

[See sec. 6 and note 5 to summary.]

6. *The object of science is to explain—Explain.*

[Science aims at reducing a mass of facts to an orderly and intelligible system—and this is explanation.]

7. *Discuss the following statements :—*

(i) To explain a phenomenon is to assign its cause.

(ii) To explain means to bring the particular or the less general under the universal or the more general.

8. *Indicate the limits of scientific explanation.*

[See sec.—7]

[The first or fundamental principles of science are themselves insusceptible of scientific explanation......the process of explaining must come somewhere to an end, with principles deducible from nothing prior to themselves—Joseph. The composite is explained by construction out of the elementary, the elementary by interconnection in a system......the whole itself

would not be explained as depending on anything outside itself but would be intelligible as a system of related elements—Hobhouse.

If we could deduce every possible law from the mere conception of being, still the question would recur, how being came about. Explanation must always assume something as given. No explanation can tell us anything of the ultimate origin of things.—Ibid.]

9. *What is an argument from analogy? How does it differ from deduction and induction?*

[See para 2 of sec. 8]

10. *How could you determine the strength or an analogical argument?*

[See para 5 of sec. 8 and add :

The strength of an analogical argument is not determined by taking into account the *number* of the points of agreement and difference. We should take into account the nature and importance of the points of agreement. Bosanquet has rightly observed that in analogy we must weigh the points of resemblance and not simply count them. In other words, while ascertaining the strength of an analogical agrument we should find out the nature of the relation that obtains between the implying properties or the observed points of similarities and the implied properties, *i.e.* the points of similarities we conclude to. If this relation is intimate or necessary the analogy is good, if not, it is bad. Thus, the analogy that Mars is inhabited like Earth as they resemble in being plancts, having similar atmosphere etc. is a good analogy because the relation between the implying property viz. moderate climate i.e., a climate favourable to the emergence and development of life and the implied property viz. the presence of life is a close one. But the analogy that Ram must have beard like Jadu as both of them are men is a bad analogy for the relation between being a man and having beard is not at all close.]

11. ANALOGICAL ARGUMENT IS FROM PARTICULAR TO PARTICULAR. DO YOU AGREE?

[No, for it depends on similarity or resemblance and, similarity is also partial identity. This partial identity or the universal is the ground of inference. In other words, in analogy we do not strictly speaking pass from a particular to another

particular. Here we pass from some observed points of similarity or other points of similarity and this passage of thought is made possible by the belief that there is a necessary relation or some identity between them. Thus when we pass from similarity of climate etc. of earth and Mars to the presence of life on Mars, we do it under the belief that there is a close relation between the two or that having a moderate climate and having life are causally connected. This universal which is not made explicit in analogy, is the basis of analogy and so, though analogy professes to pass from particular to particular, does not do this.]

12. *What is the value of analogy?*

[It helps science by suggesting hypothesis.]

13. *How does scientific Induction stand related to Induction by Simple Enumeration and inference from Analogy?*

[Scientific induction depends on the law of causation, while simple enumeration depends on counting and analogy on resemblance. So scientific induction gives certainty while the other two give probability. Induction by simple enumeration agrees with scientific Induction in passing from some to all. But analogy differs on this point—it passes from particular to particular. Analogy is more akin to seientific induction, as it involves analysis. All of them yield synthetic propositions.

Simple enumeration and Analogy are the most primitive forms of material inference. Men can count instances and observe similarity long before they can ascertain causal connection. Now, thinking that begins either by enumerating or by observing similarity, ought to develop into scientific induction. Thus if we begin with enumeration, we first of all arrive at analogy and then at scientific induction. For enumeration is not merely enumeration. We do not simply count while we count. Counting, says Bosanquet, is the construction of a total of units sharing a common nature. The strength of an induction by simple enumeration depends not merely upon the number of the instances observed. It also depends upon the nature of the instances i.e., upon the nature of the class of which the instances are members. Thus if one takes one hundred cakes prepared by a particular confectioner, and then concludes that all the cakes prepared by him are good the conclusion will be less probable than the conclusion that all crows are black even though one

concludes it by observing only one hundred crows. The reason for this is that while the instances observed in one case are members of an artificial class, in the other case, they are of a natural class. So in estimating the strength of induction by enumeration we take into accunt the nature of the class of which the instances are members i.e., the nature of the similarity that is present among the instances. So Induction by simple enumeration when it becomes conscious of its ground develops into analogy. And as the strength of the analogical argument depends on the nature of the relation that obtains between the implying property and the implied property, analogy when it becomes coscious of its ground develops into scientific Induction. Thus Induction by simple enumeration, Inference from Analogy and scientific Induction are not so much three different forms of inference as three different stages in scientific thinking.

14. *What is the argument from Analogy? How does it infer fram* (a) *Induction,* (b) *Metaphorical argument?*

15. *What are the various senses in which the word 'Analogy' has been used? Distinguish, giving instances, between good and bad analogies?*

CHAPTER XX

PROBABILITY

§ 1. MEANING OF CHANCE AND PROBABILITY: Chance was once believed to be a distinct power in the world, disturbing the regularity of Nature; though, according to Aristotle, it was only operative in occurrences below the sphere of the moon. As, however, it is now admitted that every event in the world is due some cause, if we can only trace the connection, whilst nevertheless the notion of Chance is still useful when rightly conceived, we have to find some other ground for it than that of a spontaneous capricious force inherent in things. For such a conception can have no place in any logical interpretation of Nature: it can never be inferred from a principle, seeing that every principle expresses an uniformity; nor, again, if the existence of a capricious power be granted, can any inference be drawn from it. Impossible alike as premise and as conclusion, for Reason it is nothing at all.

Chance as a distinct power:

Its unsoundness:

Every event is a result of causes: but the multitude of forces and the variety of collocations being immeasurably great, the overwhelming majority of events occurring about the same time are only related by Causation so remotely that the connection cannot be followed. Whilst my pen moves along the paper, a cab rattles down the street, bells in the neighbouring steeple chime the quarter, a girl in the next house is practising her scales, and throughout the world innumerable events are happening which may never happen together again; so that should one of them recur, we have no reason to expect any of the others. This is Chance or chance coincidence. The word Coincidence is vulgarly used only for the inexplicable concurrence of *interesting* events—"quite a coincidence!

The meaning of chance Coincidence.

On the other hand, many things are now happening together or coinciding, that will do so, for assignable reasons, again and again; thousands of men are leaving the City, who leave at the same hour five days a week. But this is not chance; it is causal coincidence due to the custom of business in this country, as determined by our latitude and longitude and other circumstances. No doubt the above chance coincidences—writing, cab-rattling, chimes, scales, *etc.*—are causally connected at some point of past time. They were predetermined by the condition of the world ten minutes ago; and that was due to earlier conditions, one behind the other, even to the formation of the planet. But whatever connection there may have been, we have no such knowledge of it as to be able to deduce the coincidence, or calculate its recurrence. Hence Chance is defined by Mill to be: Coincidence giving no ground to infer uniformity.

Mill's definition of chance.

Still, some chance coincidences do recur according to laws of their own: I say *some*, but it may be all. If the world is finite, the possible combinations of its elements are exhaustible; and, in time, whatever conditions of the world have concurred will concur again, and in the same relation to former conditions. This writing, that cab, those chimes, those scales will coincide again; the Argonautic expedition, and the Trojan war, and all our other troubles will be renewed. But let us consider some more manageable instance, such as the throwing of dice. Every one who has played much with dice knows that double sixes are sometimes thrown, and sometimes double aces. Such coincidences do not happen once and only once; they occur again and again, and a great number of trials will show that, though their recurrence has not the regularity of cause and effect, it yet has a law of its own, namely—a tendency to average regularity. In 10,000 throws there will be some num-

Can there be laws of chance?

Law of average as the basis of Probability.

ber of double sixes; and the greater the number of throws the more closely will the average recurrence of double sixes, or double aces, approximate to one in thirty-six. Such a law of average recurrence is the basis of Probability. Chance being the fact of coincidence without assignable cause, Probability is expectation based on the average frequency of its happening.

Probability as a fraction.

§ 2. PROBABILITY AS A FRACTION OR PROPORTION: Probability is an ambiguous term. Usually, when we say that an event is 'probable,' we mean that it is more likely than not to happen. But, scientifically, an event is probable if our expectation of its occurrence is less than certainty, as long as the event is not impossible. Probability, thus conceived, is represented by a fraction. *Taking 1 to stand for certainty, and 0 for impossibility, probability may be* $\frac{999}{1000}$, or $\frac{1}{1000}$, or (generally) $^1/_m$. The denominator represents the number of times that an event happens, and the numerator the number of times that it coincides with another event. In throwing a die, the probability of ace turning up is expressed by putting the number of throws for the denominator and the number of times that ace is thrown for the numerator; and we may assume that the more trials we make the nearer will the resulting fraction approximate to $\frac{1}{6}$.

The denominator and numerator of such a fraction.

Instead of speaking of the 'throwing of the die' and its 'turning up ace' as two events, the former is called 'the event' and the latter 'the way of its happening.' And these expressions may easily be extended to cover relations of distinct events; as when two men shoot at a mark and we desire to represent the probability of both hitting the bull's eye together, each shot may count as an event (denominator) and the coincidence of 'bull's-eyes' as the way of its happening (numerator).

Probability as

It is also common to speak of probability as a proportion. If the fraction expressing the probability of ace being cast is $\frac{1}{6}$, the proportion of cases in which

it happens is 1 to 5; or (as it is, perhaps, still more commonly put) 'the chances are 5 to 1 against it.' *a proportion.*

§ 3. Probability depends upon Experience and Statistics: As to the grounds of probability opinions differ. According to one view the ground is subjective: probability depends, it is said, upon the quantity of our Belief in the happening of a certain event, or in its happening in a particular way. According to the other view the ground is objective, and, in fact, is nothing else than experience, which is most trustworthy when carefully expressed in statistics.

Is the ground of Probability Subjective?

To the subjective view it may be objected, (a) that belief cannot by itself be satisfactorily measured. No one will maintain that belief, merely as a state of mind, always has a definite numerical value of which one is conscious, as $\frac{1}{100}$ or $\frac{1}{10}$. Let anybody mix a number of letters in a bag, knowing nothing of them except that one of them is X, and then draw them one by one, endeavouring each time to estimate the value of his belief that the next will be X; can he say that his belief in the drawing of X next time regularly increases as the number of letters left decreases?

Objections Considered (a) No satisfactory measurement of belief is possible.

If not, we see that (b) belief does not uniformly correspond with the state of the facts. If in such a trial as proposed above, we really wish to draw X, as when looking for something in a number of boxes, how common it is, after a few failures, to feel quite hopeless and to say: "Oh, of course it will be in the last." For belief is subject to hope and fear, temperament, passion, and prejudice, and not merely to rational considerations. And it is useless to appeal to 'the Wise Man,' the purely rational judge of probability; unless he is producible. Or, if it be said that belief is a short cut to the evaluation of experience, because it is the resultant of all past experience, we may reply that this is not true. For one striking ex-

(b) No uniform correspondence of belief with facts.

perience, or two or three recent ones, will immensely outweigh a great number of faint or remote experiences. Moreover, the experience of two men may be practically equal, whilst their beliefs upon any question greatly differ. Any two English-men have about the same experience, personal and ancestral, of the weather; yet their beliefs in the saw that 'if it rain on St. Swithin's Day it will rain for forty days after,' may differ as confident expectation and sheer scepticism. Upon which of these beliefs shall we ground the probability of forty days' rain?

(c) Observation as the ground of probability.

But (c) at any rate, if Probability is to be connected with Inductive Logic, it must rest upon the same ground, namely—observation. Induction, in any particular case, is not content with beliefs or opinions, but aims at testing, verifying or correcting them by appealing to the facts; and Probability has the same object and the same basis.

Illustration

In some cases, indeed, the conditions of an event are supposed to be mathematically predetermined, as in tossing a penny, throwing dice, dealing cards. In throwing a die, the ways of happening are six; in tossing a penny only two, head and tail: and we usually assume that the odds with a die are fairly 5 to 1 against ace, whilst with a penny 'the betting is even' on head or tail. Still, this assumption rests upon another, that the die is perfectly fair, or that the head and tail of a penny are exactly alike; and this is not true. With an ordinary die or penny, a very great number of trials would, no doubt, give an average approximating to $\frac{1}{6}$ or $\frac{1}{2}$; yet might always leave a certain excess one way or the other which would also become more definite as the trials went on; thus showing that the die or penny did not satisfy the mathematical hypothesis. Buffon is said to have tossed a coin 4040 times, obtaining 1992 heads and 2048 tails; a pupil of De Morgan tossed 4092 times, obtaining 2048 heads and 2044 tails.

There are other important cases in which probability is esti-

mated and numerically expressed, although statistical evidence directly bearing upon the point in question cannot be obtained; as in betting upon a race; or in the prices of stockes and shares, which are supposed to represent the probability of their paying, or continuing to pay, a certain rate of interest. But the judgment of experts in such matters is certainly based upon experience; and great pains are taken to make the evidence as definite as possible by comparing records of speed, or by financial estimates; though something must still be allowed for reports of the condition of horses, or of the prospects of war, harvests, *etc.*

Experience as the ground of probability.

However, where statistical evidence is obtainable, no one dreams of estimating probability by the quantity of his belief. Insurance offices, dealing with fire, shipwreck, death, accident, *etc.*, prepare elaborate statistics of these events, and regulate their rates accordingly. Apart from statistics, at what rate ought the lives of men aged 40 to be insured, in order to leave a profit of 5 per cent. upon £1000 payable at each man's death? Is 'quantity of belief' a sufficient basis for doing this sum?

Belief gives way to statistics.

§ 4. IT IS A KIND OF INDUCTION AND PRE-SUPPOSES CAUSATION: The ground of probability is experience, then, and, whenever possible, statistics; which are a kind of inductions. It has indeed been urged that induction is itself based upon probability; that the subtlety, complexity and secrecy of nature are such, that we are never quite sure that we fully know even what we have observed; and that, as for laws, the conditions of the universe at large may at any moment be completely changed; so that all imperfect inductions, including the law of causation itself, are only probable. But clearly, this doctrine turns upon another ambiguity in the word 'probable.' It may be used in the sense of 'less than absolutely certain';

The ground of Probability as inductively reached. Is Induction based upon probability?

Ambiguity in

the word 'Probable'.

and such doubtless is the conditions of all human knowledge, in comparison with the comprehensive intuition of arch-angles : or it may mean 'less than certain according to *our* standard of certainty,' that is, in comparison with the law of causation and its derivatives.

We may suppose some one to object that "by this relative standard even empirical laws cannot be called 'only probable' as long as we 'know no exception to them' ; for that is all that can be said for the boasted law of causation; and that, accordingly, we can frame no fraction to represent their probability. That 'all swans are white' was at one time, from this point of view, not probable but certain ; though we now know it to be false. It would have been an indecorum to call it only probable as long as no other-coloured swan had been discovered ; not merely because the number of events (seeing a swan) and the number of their happenings in a certain way (being white) were equal, and therefore the evidence amounted to 1 or certainty." But, in fact, such an empirical law is only probable ; and the estimate of its probability must be based on the number of times that similar laws have been found liable to exceptions. Albinism is of frequent occurrence ; and it is common to find closely allied varieties of animals differing in colour. Had the evidence been duly weighed, it could never have seemed more than probable that 'all swans are white.' But what law, approaching the comprehensiveness of the law of causation, presents any exceptions ?

Are empirical laws called to be probable ?

Supposing evidence to be ultimately nothing but accumulated experience, the amount of it in favour of causation is incomparably greater than the most that has ever been advanced to show that probability of any other kind of event ; and every relation of events which is shown to have the marks of causation obtains the support of that incomparably greater body of evidence. Hence the only way in which causation can

Is causation called to be probable ?

be called probable, for us, is by considering it as the upward limit (1) to which the series of probabilities tends; as impossibility is the downward limit (0). Induction, 'humanly speaking,' does not rest on probability; but the probability of concrete events (not of mere mathematical abstractions like the falling of absolutely true dice) rests on induction and, therefore, on causation. The inductive evidence underlying an estimate of probability may be of three kinds : *(a)* direct statistics of the events in question ; as when we find that, at the age of 20, the average expectation of life is 39-40 years. This is an empirical law, and, if we do not know the causes of any event, we must be content with an empirical law. But *(b)* if we do know the causes of an event, and the causes which may prevent its happening, and can estimate the comparative frequency of their occurring, we may deduce the probability that the effect (that is, the event in question) will occur. Or (c) we may combine these two methods, verifying each by means of the other. Now either the method (b) or (*a fortiori*) the method (c) (both depending on causation) is more trustworthy than the method (a) by itself.

Probability rests on Induction but not Vice versa.

Inductive evidence are of three kinds for an estimate of probability: (a) direct statistics

(b) Comparative frequency

(c) Combination of both.

But, further, a merely empirical statistical law will only be true as long as the causes influencing the event remain the same. A die may be found to turn ace once in six throws, on the average, in close accordance with mathematical theory ; but if we load it on that facet the results will be very different. So it is with the expectation of life, or fire, or shipwreck. The increased virulence of some epidemic such as influenza, an outbreak of anarchic incendiarism, a moral epidemic of over-loading ships, may deceive the hopes of insurance offices. *Hence we see, again, that probability depends upon causation, not causation upon probability.*

Truth of merely empirical Statistical law.

Conclusion.

15

That uncertainty of an event which arises not from ignorance of the law of its cause, but from our not knowing whether the cause itself does or does not occur at any particular time, is Contingency.

Contingency defined

§ 5. Of Averages and the Law of Error. The nature of an average supposes deviations from it. Deviations from an average, or "errors", are assumed to conform to the law (1) that the greater errors are less frequent than the smaller, so that most events approximate to the average; and (2) that errors have no "bias," but are equally frequent and equally great in both directions from the mean, so that they are scattered symmetrically. Hence their distribution may be expressed by some such figure as the following:

The concept of average

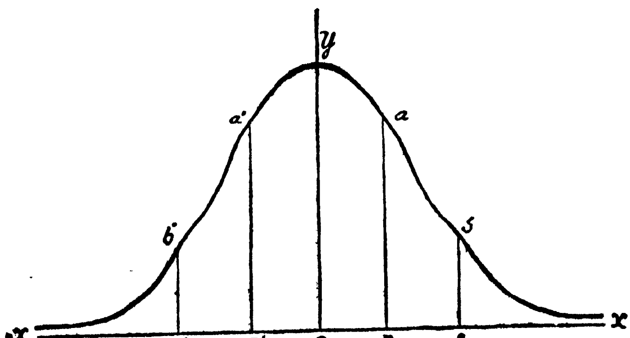


Fig. 11.

Here *o* is the average event, and *oy* represents the number of average events. Along *ox*, in either direction, deviations are measured. At *p* the amount of error or deviation is *op*; and the number of such deviations is represented by the line or ordinate *pa*. At *s* the deviation is *os*; and the number of such deviations is expressed by *sb*. As the deviations grow greater, the number of them grows less. On the other side of *o*, toward *-x*, at dis-

Interpretation of the diagram

tances, op', os' (equal to op, os) the lines $p'a'$, $s'b'$ represent the numbers of those errors (equal to pa, sb).

If o is the average height of the adult men of a nation (say) 5 ft. 6 in., s' and s may stand for 5 ft. and 6 ft.; men of 4 ft. 6 in. lie further toward $-x$, and men of 6 ft. 6 in. further toward x. There are limits to the stature of human beings (or to any kind of animal or plant) in both directions, because of the physical conditions of generation and birth. With such events the curve $b'yb$ meets the abscissa at some point in each direction; though where this occurs can only be known by continually measuring dwarfs and giants. But in throwing dice or tossing coins, whilst the average occurrence of ace is once in six throws, and the average occurrence of 'tail' is once in two tosses, there is no necessary limit to the sequences of ace or of 'tail' that may occur in an infinite number of trials. To provide for such cases the curve is drawn as if it never touched the abscissa.

Two ways of describing an average.

That some such figure as that given above describes a frequent characteristic of an average with the deviations from it, may be shown in two ways: (1) *By arranging the statistical results of any homogeneous class of measurements*; when it is often found that they do, in fact, approximately conform to the figure; that very many events are near the average; that errors are symmetrically distributed on either side, and that the greater errors are the rarer. (2) *By mathematical demonstration based upon the supposition that each of the events in question is influenced, more or less, by a number of unknown conditions common to them all, and that these conditions are independent of one another.* For then, in rare cases, all the conditions will operate favourably in one way, and the men will be short; in more numerous cases, many of the conditions will operate in one direction, and will be partially cancelled by a few opposing them; whilst in still more cases opposed conditions will approximately balance one another and produce the average event or something near it. The results will then conform to the above figure.

From the above assumption it follows that the symmetrical curve describes only a 'homogeneous class' of measurements; that is, a class no portion of which is much influenced by conditions peculiar to itself. If the class is not homogeneous, because some portion of it is subject to *peculiar* conditions, the curve will show a hump on one side or the other. Suppose we are tabulating the ages at which Englishmen die who have reached the age of 20, we may find that the greatest number die at 39 (19 years being the average expectation of life at 20) and that as far as that age the curve upwards is regular, and that beyond the age of 39 it begins to descent regularly, but that on approaching 45 it bulges out some way before resuming its regular descent—thus:

A homogeneous class defined.

Why is such a class necessary.

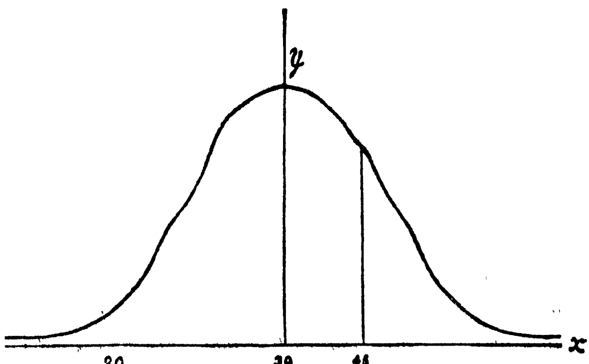


FIG. 12.

Such a hump in the curve might be due to the presence of a considerable body of teetotalers, whose longevity was increased by the peculiar condition of abstaining from alcohol, and whose average age was 45, 6 years more than the average for common men.

The nature of a group which is subject to selection.

Again, if the group we are measuring be subject to selection (such as British soldiers, for which

profession all volunteers below a certain height—say, 5 ft. 5 in. —are rejected), the curve will fall steeply on one side, thus :

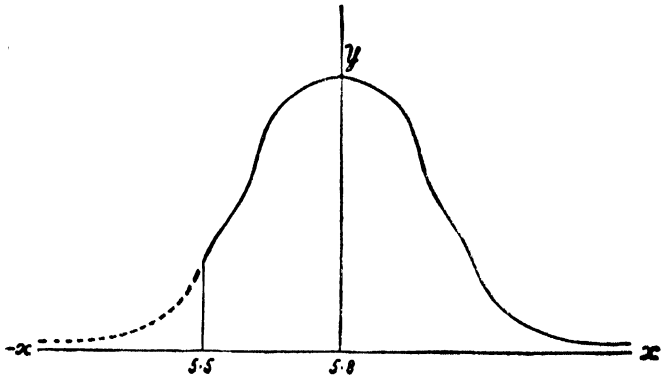


Fig. 13.

If, above a certain height, volunteers are also rejected, the curve will fall abruptly on both sides. The average is supposed to be 5 ft. 8 in.

The distribution of events is described by 'some such curve' as that given in Fig. 11 ; but different groups of events may present figures or surfaces in which the slopes of the curves are very different, namely, more or less steep ; and if the curve is very steep, the figure runs into a peak; whereas, if the curve is gradual, the figure is comparatively flat. In the latter case, where the figure is flat, fewer events will closely cluster about the average, and the deviations will be greater.

Significance of different Curves.

Suppose that we know nothing of a given event except that it belongs to a certain class or series, what can we venture to infer of it from our knowledge of the series? Let the event be the cephalic index of an Englishman. The cephalic index is the breadth of a skull $\times$ 100 and divided by the length of it ; *e.g.*, if a skull is 8 in. long and 6 in. broad, $\frac{6 \times 100}{8} = 75$. We know

Inference from bare knowledge of a class or series to which an event belongs.

that the average English skull has an index of 78. The skull of the given individual, therefore, is more likely to have that index than any other. Still, many skulls deviate from the average, and we should like to know what is the probable error in this case. The probable error is the measurement that divides the deviations from the average in either direction into halves, so that there are as many events having a greater deviation as there are events having a less deviation. If, in Fig. 11 above, we have arranged the measurements of the cephalic index of English adult males, and if at o (the average or mean) the index is 78, and if the line *pa* divides the right side of the fig. into halves, then *op* is the probable error. If the measurement at *p* is 80, the probable error is 2. Similarly, on the left hand, the probable error is *op'*, and the measurement at *p'* is 76. We may infer, then, that the skull of the man before us is more likely to have an index of 78 than any other; if any other, it is eqully likely to lie between 80 and 76, or to lie outside them; but as the numbers rise above 80 to the right, or fall below 76 to the left, it rapidly becomes less and less likely that they describe this skull.

The medium and the average.

In such cases as heights of men or skull measurements, where great numbers of specimens exist, the average will be actually presented by many of them; but if we take a small group, such as the measurements of a college class, it may happen that the average height (say, 5 ft. 8 in.) is not the actual height of any one man. Even then there will generally be a closer cluster of the actual heights about that number than about any other.

Advantages of taking the median.

Still, with very few cases before us, it may be better to take the median than the average. The median is that event on either side of which there are equal numbers of deviations. One advantage of this procedure is that it may save time and trouble. To find approximately the average height of a class, arrange the men in order of height, take the middle one and measure him. A further advantage of this method is that it ex-

cludes the influence of extra-ordinary deviations. Suppose we have seven cephalic indices, from skeletons found in the same barrow, $75\frac{1}{2}$, 76, 78, 78, 79, $80\frac{1}{2}$, 86. The average, is 79; but this number is swollen unduly by the last measurement; and the median, 78, is more fairly representative of the series; that is to say, with a greater number of skulls the average would probably have been nearer 78.

REDUCED OBSERVATION:

To make a single measurement of a phenomenon does not give one much confidence. Another measurement is made; and then, if there is no opportunity for more, one takes the mean or average of the two. But why? For the result may certainly be worse than the first measurement. Suppose that the events I am measuring are in fact fairly described by Fig. 11, although (at the outset) I know nothing about them; and that my first measurement gives p, and my second s; the average of them is worse than p. Still, being yet ignorant of the distribution of these events, I do rightly in taking the average. For, as it happens, $\frac{3}{4}$ of the events lie to the left of p; so that if the first trial gives p, then the average of p and any subsequent trial that fell nearer than (say) s' on the opposite side, would be better than p; and since deviations greater than s' are rare, the chances are nearly 3 to 1 that the taking of an average will improve the observation. Only if the first trial give o, or fall within a little more than $\frac{1}{2}p$ on either side of o, will the chances be against any improvement by trying again and taking an average. Since, therefore, we cannot know the position of our first trial in relation to o, it is always prudent to try again and take the average; and the more trials we can make and average, the better is the result. The average of a number of observations is called a "Reduced Observation."

Definition of 'Reduced observation'.

We may have reason to believe that some of our measurements are better than others, because they have been taken by

a better trained observer, or by the same observer in a more deliberate way, or with better instruments, and so forth. If so, such observations should be 'weighted,' or given more importance in our calculations ; and a simple way of doing this is to count them twice or oftener in taking the average.

How an observations 'weighted'

§ 6. INTERPRETATION OF PROBABILITIES : These considerations have an important bearing upon the interpretation of probabilities. The average probability for any *general class* or series of events cannot be confidently applied to any *one instance* or to any *special class* of instances, since this one, or this special class, may exhibit a striking error or deviation ; it may, in fact, be subject to special causes. Within the class whose average is first taken, and which is described by general characters as 'a man,' or 'a die,' or 'a rifle shot,' there may be classes marked by special characters and determined by special influences. Statistics giving the average for 'mankind' may not be true of 'civilised men,' or of any still smaller class such as 'Frenchmen.' Hence life-insurance offices rely not merely on statistics of life and death in general, but collect special evidence in respect of different ages and sexes, and make further allowance for teetotalism; inherited disease, *etc*. Similarly with individual cases : the average expectation for a class, whether general or special, is only applicable to any particular case if that case is adequately described by the class characters. In England, for example, the average expectation of life for males at 20 years of age is 39.40; but at 60 it is still 13.14, and at 73 it is 7.07 ; at 100 it is 1·61. Of men 20 years old those who live more or less than 39.40 years are deviations or errors ; but there are a great many of them. To insure the life of a single man at 20, in the expectation of his dying at 60, would be a mere bet, if we had no special knowledge of him ; the safety of an insurance office lies in having so many clients that opposite deviations cancel one another : the more clients the safer the business. It is quite possible that a hundred men aged 20 should be insured in

one week and all of them die before 25; this would be ruinous, if others did not live to be 80 or 90.

PERSONAL EQUATION:

Not only in such a practical affair as insurance, but in matters purely scientific, the minute and subtle peculiarities of individuals have important consequences. Each man has a certain case of mind, character, physique, giving a distinctive turn to all his actions even when he tries to be normal. In every employment this determines his *Personal Equation, or the normal average deviation from the normal.* The term Personal Equation is used chiefly in connection with scientific observation, as in Astronomy. Each observer is liable to be a little wrong, and this error has to be allowed for and his observations corrected accordingly.

Personal equation, or average deviation from

MEANING OF EXPECTATION:

The use of the term 'expectation,' and of examples drawn from insurance and gambling, may convey the notion that probability relates entirely to future events; but if based on laws and causes, it can have no reference to point of time. As long as conditions are the same, events will be the same, whether we consider uniformities or averages. We may therefore draw probable inferences concerning the past as well as the future, subject to the same hypothesis, that the causes affecting the events in question were the same and similarly combined. On the other hand, if we know the conditions bearing on the subject of investigation, have changed since statistics were collected, or were different at some time previous to the collection of evidence, every probable inference based on those statistics must be corrected by allowing for the altered conditions, whether we desire to reason forwards or backwards in time.

§ 7. THE RULES FOR THE COMBINATION OF PROBABILITIES ARE AS FOLLOWS:

(1) If two events or causes do not concur, the probability of one or

the other occurring is the sum of the separate probabilities. A die cannot turn up both ace and six; but the probability in favour of each is $\frac{1}{6}$: therefore, the probability in favour of one or the other is $\frac{1}{3}$. Death can hardly occur from both burning and drowning: if 1 in 1000 is burned and 2 in 1000 are drowned, the probability of being burned or drowned is $\frac{3}{1000}$.

(2) *If two events are independent, having neither connection nor repugnance, the probability of their concurring is found by multiplying together the separate peobabilities of each occurring.* If in walking down a certain street I meet A once in four times, and B once in three times, I ought (by mere chance) to meet both once in twelve times: for in twelve occasions I meet B four times; but once in four I meet A.

DETECTION OF A HIDDEN CAUSE.

This is a very important rule in scientific investigation, since it enables us to detect the presence of causation. For if the coincidence of two events is more or less frequent than it would be if they were entirely independent, their is either connection or repugnance between them. If, *e.g.*, in walking down the street I meet both A and B oftener than once in twelve times, they may be engaged in similar business, calling them from their offices at about the same hour. If I meet them both less often than once in twelve times, they may belong to the same office, where one acts as a substitute for the other. Similarly, if in a multitude of throws a die turns six oftener than once in six times, it is not a fair one: that is, there is a cause favouring the turning of six. If of 20,000 people 500 see apparitions and 100 have friends murdered, the chance of any man having both experiences is $\frac{1}{8000}$; but if each lives on the average 300,000 hours, the chance of both events occurring in the same hour is $\frac{1}{2400000000}$. If the two events occur in the same hour oftener than this, there is more than a chance coincidence.

Illustrations

The more minute a cause of connection or repugnance be-

tween events, the longer the series of trials or instances necessary to bring out its influence: the less a die is loaded, the more casts must be made before it can be shown that a certain side tends to recur oftener than once in six.

Length of the series of trials

(3) The rule for calculating the probability of a dependent event is the same as the above; for the concurrence of two independent events is itself dependent upon each of them occurring. My meeting with both A and B in the street is dependent on my walking there and on my meeting one of them. Similarly, if A is sometimes a cause of B (though liable to be frustrated), and B sometimes of C (C and B having no causes independent of B and A respectively), the occurrence of C is dependent on that of B, and that again on the occurrence of A. *Hence we may state the rule: If two events are dependent each on another, so that if one occur the second may (or may not), and if the second a third; whilst the third never occurs without the second, nor the second without the fiist; the probability that if the first occur the third will, is found by multiplying together the fractions expressing the probability that the first is a mark of the second and the second of the third.*

ORAL TRADITION:

Upon this principle the value of hearsay evidence or tradition deteriorates, and generally the cogency of any argument based upon the combination of approximate generalisations dependent on one another or "self-infirmative." If there are two witnesses, A and B, of whom A saw an event, whilst B only heard A relate it (and is therefore dependent on A), what credit is due to B's recital? Suppose the probability of each man's being correct as to what he says he saw, or heard, is $\frac{3}{4}$: then ($\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$) the probability that B's story is true is a little more than $\frac{1}{2}$. For if in 16 attestations A is wrong 4 times, B can only be right in $\frac{3}{4}$ of the remainder, or 9 times in 16. Again, if we have the Approximate Generalisations, 'Most attempts to reduce wages are

Illustrations.

met by strikes,' and 'Most strikes are successful,' and learn, on statistical inquiry, that in every hundred attempts to reduce wages there are 80 strikes, and that 70 p.c. of the strikes are successful, then 56 p.c. of attempts to reduce wages are unsuccessful.

Of course this method of calculation cannot be quantitatively applied if no statistics are obtainable, as in the testimony of witnesses; and even if an average numerical value could be attached to the evidence of a certain class of witnesses, it would be absurd to apply it to the evidence of any particular member of the class without taking account of his education, interest in the case, prejudice, or general capacity. Still, the numerical illustration of the rapid deterioration of hearsay evidence, when less than quite veracious, puts us on our guard agaist rumour. To retail rumour may be as bad as to invent an original lie.

The method cannot be applied in absence of statistics

(4) *If an event may coincide with two or more other independent events, the probability that they will together be a sign of it, is found by multiplying together the fractions representirg the improbability that each is a sign of it, and subtracting the product from unity.*

CIRCUMSTANTIAL AND ANALOGICAL EVIDENCE :

This is the rule for estimating the cogency of circumstantial evidence and analogical evidence; or, generally, for combining approximate generalisations "self-corroboratively." If, for example, each of two independent circumstances, A and B, indicates a probability of 6 to 1 in favour of a certain event; taking 1 to represent certainty, $1-\frac{6}{7}$ is the improbability of the event, notwithstanding each circumstance. Then $\frac{1}{7}\times\frac{1}{7}=\frac{1}{49}$, the improbability of both proving it. Therrfore the probability of the event is 48 to 1. Tne matter may be plainer if put thus: A's indication is right 6 times in 7, or 42 in 49; in the remaining 7 times in 49, B's indication will be right 6 times. Therefore, together they will be right 48 times in 49. If each of two witnesses is truthful

Illustrations

6 times in 7, one or the other will be truthful 48 times in 49. But they will not be believed unless they agree; and in the 42 cases of A being right, B will contradict him 6 times; so that they only concur in being right 36 times. In the remaining 7 times in which A is wrong, B will contradict him 6 times, and once they will both be wrong. It does not follow that when both are wrong they will concur; for they may tell very different stories and still contradict one another.

If in an analogical argument there were 8 points of comparison, 5 for and 3 against a certain inference, and the probability raised by each point could be quantified, the total value of the evidence might be estimated by doing similar sums for and against, and subtracting the unfavourable from the favourable total.

When approximate generalisations that have not been precisely quantified combine their evidence, the cogency of the argument increases in the same way, though it cannot be made so definite. If it be true that most poets are irritable, and also that most invalids are irritable, a still greater proportion will be irritable of those who are both invalids and poets.

Four principal causations.

On the whole, from the discussion of probabilities there emerge four principal cautions as to their use: (1) Not to make a pedantic parade of numerical probability, where the numbers have not been ascertained; (2) Not to trust to our feeling of what is likely, if statistics can be obtained; (3) Not to apply an average probability to special classes or individuals without inquiring whether they correspond to the average type; and (4) Not to trust to the empirical probability of events, if their causes can be discovered and made the basis of reasoning which the empirical probability may be used to verify.

The reader who wishes to pursue this subject further should read a work to which the foregoing chapter is greatly indebted, Dr. Venn's *Logic of Chance.*

SUPPLEMENTARY NOTES

THE FUNDAMENTAL PRINCIPLES OF PROBABILITY

The symbol p/h *denotes* "The probability of p given h." This is an undefined notion in the sense that it defined only by the axioms or postulates which are given below :

(1) If p and h are given, p/h can have only one value.

(2) The value of p/h may be any real number between 1 and 0, both included.

(3) If it so happens that h, implies p, then $p/h = I$. By 'I' we mean certainty.

(4) If h, implies the falsity of p or the truth of not-p, then $p/h = o$. By '0' we mean absolute impossibility.

(5) The disjunctive axiom :—The probability of p and I or q given h is equal to the probability of p given h plus the probability of q given h minus the probability of both p and q given h. To put it symbolically :—

$$(p \vee q)/h = p/h + q/h - p \;\&\; q/h$$

If p and q do not concur, $p \;\&\; q/h = o$. In this case the formula is simplified and corresponds to the principle (1) section 7 in the text.

(6) The conjunctive axiom :—If p, q and h are given, the probability of both p and q given h is equal to the probability of p given h multiplied by the probability of q given p and h. This is also equal to the probability of q given h multiplied by the probability of p given q and h. To put it symbolically :—

$$p \;\&\; q/h = p/h \times q/p \;\&\; h$$
$$= q/h \times p/q \;\&\; h$$

But if p and q are independent, $p/q \;\&\; h = p/h$ ann $q/p \;\&\; h = q/h$.

Thus the formula is simplified as follows :—

$$p \;\&\; q/h = p/h \times q/h$$

The formula corresponds to the principle (2) in section 7 in the text.

(The above account has been taken from chapter II part V of Russell's Human Knowledges : its scope and limits.)

SUMMARY

Chance was once believed to be a distinct power among other powers operating in the world. The unsoundness of the doctrine has been recognised. In nature there is a multitude of forces all of which are not known. By chance coincidence as vulgarly used we mean therefore an in-explicable concurrence of interesting events. A coincidence is according to Mill, no ground to infer uniformity. Some chance coincidences, however, are believed to recur according to laws of their own. This belief is responsible for the fact that the law of average has been taken as the basis of probability. In short, chance is said to operate so far as causes are not assignable; but when the average frequency is determined, probability or expectation based on it comes into being.

The word probability is ambiguous. In science, however, an event is probable if our expectation of its happening is less than certainty, as long as it is not impossible. '*I*' stands for certainty and *o* for impossibility. Thus probability can be expressed by one of the fractions lying between *I* and *o*. The denominator of such fraction represents the number of times that an event happens, and the numerator the number of times that it coincides with another event.

Probability may also be understood as a proportion. If an event happens in an X way in 20 per cent cases and in a Y way in 80 per cent cases, the chance of the event happening in an X way is 1 : 4. It may also be expressed as 'the chances are 4 : 1' against it. Some erroneously consider the ground of probability as subjective. But this is not a sound view. For (a) belief can not by itself be satisfactorily measured. (b) Belief is often not sound. It does not uniformly tally with facts. (c) If the verification, testing or correction of an induction is our goal, observation is indispensable. If probability is subjective this goal cannot be realised. Probability must therefore rest upon observation. It is therefore objective.

Without experience the judgment of expert is impossible and without statistical evidence no estimate of probability by the quantity of a person's belief is possible.

Now statistics is a kind of induction. Experience and statis-

tics constitute the ground of probability. But it is curious to note that some have urged that induction is itself based upon probability. The view requires thorough examination.

Carveth Read thinks that Induction does not rest on probability. He holds further that the probability of concrete events rests, on the contrary, on induction and, therefore, on causation. There are three ways by which we have an inductive estimate of probability :—(a) Direct statistics of the event in question when the causes are not definitely known. (b) The comparative frequency of occurrences. (c) A combination of both the methods verifying each by means of the other.

In any case we rely on empirical laws which rests on induciion. Such law too, presuppose or rest on causation. Thus probability depends on Induction and Causation but not *vice versa.*

When an event is uncertain because we do not know whether its cause does or does not occur at any particular time, uncertainly is known as contingency.

By formulating an average, errors can be understood as deviations from it. Errors are assumed to conform to the law (1) that the greater errors are less frequent than the smaller, so that most events approximate to the average and (2) that errors on both sides are equally frequent and equally great.

The frequent characteristic of an average with the deviations from it can be shown (1) by arranging the statistical results of any homogeneous class of measurements that is of a class, no portion of which is much influenced by conditions peculiar to itself as also by (2) mathematical demonstration based on the supposition that a number of unknown conditions common to them all influence the events in question and that this condition is independent of one another. If the class be not homogeneous, the relevant curve will show a hump on one side or the other. If the group we are to measure be subject to selection, the curve will fall steeply on one side. Different curves represent the different ways of distribution of events. By the median we mean that event on either side of which there are equal number of deviations. The median is, it will appear more fairly representing of a series than average,—specially when a small group is taken into consideration. The average of a number of observation is called a "Reduced Observation." Observations are weighted when they are better done. This is possible when there is a better trained observer or when the same observer ob-

serves either in a more deliberate way or with better instruments.

Probability is always with reference to a general class or series of events. The average probability for a class or series of events cannot therefore be confidently applied to an instance. A serious deviation may therefore be exhibited here. The general rule is therefore that the expectation for a class cannot be applied to a particular case unless that case is adequately described by the class characters.

In all cases, we should beware of personal equation. It means an average deviation from the normal. This deviation is due to the peculiarity of the observer's cast on mind, character and experience. Probability refers, not simply to future events but to past ones as well. If probability is based on laws and causes, it can have no reference to point of time.

The rules for the combination of probabilities.

(1) When two events do not meet together the probability of one is the same of the separate probability.

(2) If the events are independent, the probability of their occurring together is the product of the seperate probabilities of each occurring.

(3) Suppose two events are dependent each on another, so that if one occurs the second may or may not, and if the second occurs a third may or may not. It is further known that the second never occurs without the first, nor the third without the second. The probability that if the first occur a third will, is the product of the probability that the first is a mark of the second and the probability that the second is of the third.

This method cannot however be applied when there is no statistics.

(4) If an event may coincide with two or more independent events, the probability that they will together be a sign of it, is found by multiplying together the fractions representing the improbability that each is a sign of it, and subtracting the product from unity.

In ascertaining probabilities there are four principal cautions to be borne in mind :

(1) If we do not know the exact numerical value of a probability, it should not be used.

(2) If statistics is possible, we should not rely on our feelings.

(3) Without knowing whether a special class or individual corresponds to average type, an average probability must not be applied.

(4) The empirical probability of events are not to be relied on, if their causes can be discovered and made the basis of reasoning which the empirical probability may be used to verify.

EXERCISES WITH HINTS

1. *Explain and discuss the doctrine that Induction is based upon the theory of probability.*

[Read sec. 4]

2. *Write notes on the concept of chance.*

[Read sec. 1]

3. *Probability has been interpreted as a fraction as well as a proportion. Do you find any conflict between the two interpretations of probability.*

[Consult sec. 2 and try to show that both the interpretations point to the identical feature of probability. Thus the alleged conflict is baseless.]

4. *"The ground of probability is not subjective." How do you establish this proposition.*

[Consult sec. 3]

5. *Write notes on (a) Personal equation (b) Reduced Observation (c) Average and (d) Median.*

CHAPTER XXI

DIVISION AND CLASIFICATION

§ 1. CLASSIFICATION, SCIENTIFIC, SPECIAL AND POPULAR: Classification, in its widest sense, is a mental grouping of facts or phenomena according to their resemblances and differences, so as best to serve some purpose. A "mental grouping": for although in museums we often see the things themselves arranged in classes, yet such an arrangement only contains specimens representing a classification. The classification itself may extend to innumerable objects most of which have never been seen at all. Extinct animals, for example, are classified from what we know of their fossils; and some of the fossils may be seen arranged in a museum; but the animals themselves have disappeared for many ages.

Definition of classification

It is a mental grouping of facts.

Again, things are classed according to their resemblances and differences: that is to say, those that most closely resemble one another are classes together on that ground; and those that differ from one another in important ways, are distributed into other classes. The more the things differ, the wider apart are their classes both in thought and in the arrangements of a museum. If their differences are very great, as with animals, vegetables and minerals, the classing of them falls to different departments of thought or science, and is often represented in different museums, zoological, botanical, mineralogical.

According to heir resemblances and differences.

We must not, however, suppose that there is only one way of classifying things. The same objects may be classed in various ways according to the purpose in view. For gardening, we are usually content to classify plants into trees, shrubs, flowers,

The same object may be classed in vari-

ous ways according to the purpose in view.

grasses and weeds; the ordinary crops of English agriculture are distinguished, in settling their rotation, into white and green; the botanist divides the higher plants into gymnosperms and angiosperms, and the latter into monocotyledons and dicotyledons. The principle of resemblance and difference is recognised in all these cases; but what resemblances or differences are important depends upon the purpose to be served.

Purposes are either (a) special or practical or (b) general or scientific.

Purposes are either (*a*) special or practical, as in gardening or hunting, or (*b*) general or scientific, as in Botany or Zoology. The scientific purpose is merely knowledge; it may indeed subserve all particular or practical ends, but has no other end than knowledge directly in view. And whilst, even for knowledge, different classifications may be suitable for different lines of inquiry, in Botany and Zoology the Morphological Classification is that which gives the most general and comprehensive knowledge (see Huxley, *On the Classification of Animals*, ch. 1). Most of what a logician says about classification is applicable to the practical kind; but the scientific (often called 'Natural Classification'), as the most thorough and comprehensive, is what he keeps most constantly before him.

Scientific classification is most thorough; and comes late in human history.

Scientific classification comes late in human history, and at first works over earlier calassifications which have been made by the growth of intelligence, of language, and of the practical arts. Even in the distinctions recognised by animals, may be traced the grounds of classification: a cat does not confound a dog with one of its own species, nor water with milk, nor cabbage with fish. But it is in the development of language that the progress of instinctive classification may best be seen. The use of general names implies the recognition of classes of things corresponding to them, which form their denotation, and whose resembling qualities, so far as recognised, form their connota-

tion; and such names are of many degrees of generality. The use of abstract names shows that the objects classed have also been analysed, and that their resembling qualities have been recognised amidst diverse groups of qualities.

Of the classes marked by popular language it is worth while to distinguish two sorts (*cf.* chap. xix. § 4): Kinds, and those having but few points of agreement.

Two sorts of classes marked by popular language.

Scientific classification and popular contrasted.

But the popular classifications, made by language and the primitive arts, are very imperfect. They omit innumerable things which have not been found useful or noxious, or have been inconspicuous, or have not happened to occur in the region inhabited by those who speak a particular language; and even things recognised and named may have been very superficially examined, and therefore wrongly classed, as when a whale or porpoise is called a fish, or a showworm is confounded with snakes. A scientific classification, on the other hand, aims at the utmost comprehensiveness, ransacking the whole world from the depths of the earth to the remotest star for new objects, and scrutinising everything with the aid of crucible and dissecting knife, microscope and spectroscope, to find the qualities and constitution of everything, in order that it may be classed among those things with which it has most in common and distinguished from those other things from which it differs. A scientific classification continually grows more comprehensive, more discriminative, more definitely and systematically coherent. Hence the uses of classification may be easily perceived.

§ 2. Uses of Classification: The first use of classification is the better understanding of the facts of Nature (or of any sphere of practice): for understanding consists in perceiving and comprehending the likeness and difference of things, in assimilating and distinguishing them; and, in carrying out this process systematically, new correlations of properties are continually disclosed.

The first use of classification is the better un-

Understanding the facts of Nature.

Thus classification is closely analogous to explanation. Explanation has been shown (chap. xix § 5) to consist in the discovery of the laws or causes of changes in Nature; and laws and causes imply similarity, or like changes under like conditions: in the same way classification consists in the discovery of resemblances in the things that undergo change. We may say (subject to subsequent qualifications) that Explanation deals with Nature in its dynamic, Classification in its static aspect. In both cases we have a feeling of relief. When the cause of any event is pointed out, or an object is assigned its place in a system of classes, the gaping wonder, or confusion, or perplexity, occasioned by an unintelligible thing, or by a multitude of such things, is dissipated. Some people are more than others susceptible of this pleasure and fastidious about its purity.

Explanation and classification

Its second use is to aid memory

A second use of classification is to aid the memory. It strengthens memory, because one of the conditions of our recollecting things is, that they resemble what we last thought of; so that to be accustomed to study and think of things in classes must greatly facilitate recollection. But, besides this, a classification enables us easily to run over all the contrasted and related things that we want to think of. Explanation and classification both tend to rationalise the memory, and to organise the mind in correspondence with Nature.

Thought improves with the practice of explanation and

Every one knows how a poor mind is always repeating itself, going by rote through the same train of words, ideas, actions; and that such a mind is neither interesting nor practical. It is not practical, because the circumstances of life are rarely exactly repeated, so that for a present purpose it is rarely enough to remember only one former case; we need

cally) their resemblances and differences with the one before us, we may select a course of action, or a principle, or a parallel, suited to our immediate needs. Greater fertility and flexibility of thought seem naturally to result from the practice of explanation and classification. But it must be honestly added, that the result depends upon the spirit in which such study is carried on; for if we are too fond of finality, too eager to believe that we have already attained a greater precision and comprehension than are in fact attainable, nothing can be more petrific than 'science,' and our last state may be worse than the first. Of this, students of Logic have often furnished examples.

§ 3. CLASSIFICATION, DEDUCTIVE AND INDUCTIVE: Classification may be either Deductive or Inductive; that is to say, in the formation of classes, as in the proof of propositions, we may, on the whole, proceed from the more to the less, or from the less to the more general; not that these two processes are entirely independent.

Deductive and Inductive classification.

If we begin with some large class, such as "Animal,' and subdivide it deductively into Vertebrate and Invertebrate, yet the principle of division (namely, central structure) has first been reached by a comparison of examples and by generalisation; if, on the other hand, beginning with individuals, we group them inductively into classes, and these again into wider ones (as dogs, rats, horses, whales and monkeys into mammalia) we are guided both in special cases by hypotheses as to the best grounds of resemblance, and throughout by the general principle of classification—to associate things that are alike and to seperate things that are unlike. This principle holds implicitly a place in classification similar to that of causation in explanation; both are principles of intelligence. Here, then, as in proof, induction is implied in deduction, and deduction in induction. Still, the two modes of procedure may be usefully

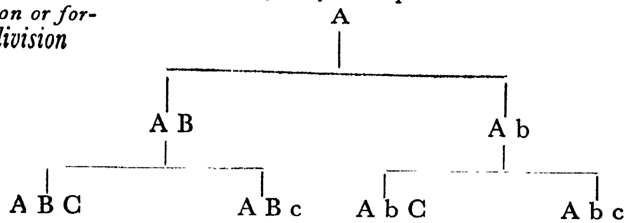
They imply each other

distinguished : in deduction, we proceed from the idea of a whole to its parts, from general to special ; in induction, from special (or particular) to general, from parts to the idea of a whole.

§ 4. Division, or Deductive Classification : it's rules :

The process of deductive classification or formal division

The process of Deductive Classification, or Formal Division, may be represented thus :



Given any class (A) to be divided :

To divide (i) Select fundamentum divisionis.

1. Select one important character, attribute, or quality (B), not common to all the individuals comprehended in the class, as the basis of division (*fundamentum divisionis*).

(ii) Proceed by Dichotomy

2. Proceed by Dichotomy; that is cut the given class into two, one having the selected attribute (say, B), the other not having it (b). This, like all formal processes, assumes the principles of Contradiction and Excluded Middle, that 'No A is both B and not-B,' and that 'Every A is either B or not-B' (chap. vi. § 3); and if these principles are not true, or not applicable, the method fails.

When a class is thus subdivided, it may be called, in relation to its subclasses, a Genus; and in relation to it, the subclasses may be called Species : thus—genus A, species AB and Ab, *etc.*

(iii) gradually until all the

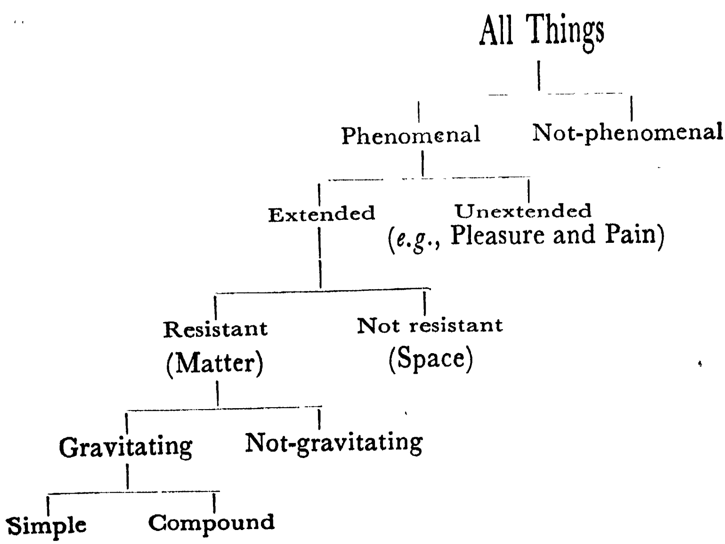
3. Proceed gradually in the order of the importance of characters; that is, having divided the given classes, sub-divide on the same principle the two classes thence arising ; and so again and again, step by

step, until all the characters are exhausted: *Divisio ne fiat per saltum.*

characters are exhausted

Illustration

Suppose we were to attempt an exhaustive classification of things by this method, we must begin with 'All Things,' and divide them (say) into phenomenal and not-phenomenal, and then subdivide phenomena, and so on, thus:



Having subdivided 'Simple' by all possible characters, we must then go back and similarly subdivide Not-phenomenal, Unextended, Not-resistant, Not-gravitating, and Compound. Now, if we know all possible characters, and the order of their importance, we might prepare *a priori* a classification of all possible things; at least, of all things that come under the principles of Contradiction and Excluded Middle. Many of our compartments might contain nothing actual; there may, for example, be nothing that is not phenomenal to some mind, or nothing that is extended and not-resistant (no vaccum), and so

forth. This would imply a breach of the rule, that the dividing quality be not common to the whole class ; but, in fact, doubts have been, and are, seriously entertained whether these compartments are filled or not. If they are not, we have concepts representing nothing, which have been generated by the mere force of grammatical negation, or by the habit of thinking according to the principle of Excluded Middle ; and, on the strength of these empty concepts, we have been misled into dividing by an attribute, which (being universal) cannot be a *fundamentum divisionis*. But though in such a classification places might be empty, there would be a place for everything ; for whatever did not come into some positive class (such as Gravitating) must fall under one of the negative classes (the 'Nots') that run down the right-hand side of the Table and of its subdivisions.

The ideal outside the mathematical sciences is hardly reached.

This is the ideal of classification. Unfortunately we have to learn what characters or attributes are possible, by experience and comparison ; we are far from knowing them all : and we do not know the order of their importance ; nor are we even clear what 'important' means in this context, whether 'widely prevalent,' or 'ancient,' or 'causally influential,' or 'indicative of others.' Hence, in classifying actual things, we must follow the inductive method of beginning with particulars, and sorting them according to their likeness and difference as discovered by investigation. The exceptional cases, in which deduction is really useful, occur where certain limits to the number and combination of qualities happen to be known, as they may be in human institutions, or where there are mathematical conditions. Thus, we might be able to classify orders of Architecture, or the classical metres and stanzas of English poetry ; though, in fact, these things are too free, subtle and complex for deductive treatment : for do not the Arts grow like trees ? The only sure cases are mathematical ; as we may show that there are possible only

three kinds of plane triangles, four conic sections, five regular solids.

§ 5. Rules for testing a Division: The rules for *testing* a Division are as follows:

Rules for testing a Division (1) Each sub-class should comprise less than the class.

1. Each Sub-class, or Species, should comprise less than the Class, or Genus to be devided. This provides that the division shall be a real one, and not based upon an attribute common to the whole class; that, therefore, the first rule for making a division shall have been adhered to. But, as in § 4, we are here met by a logical difficulty. Suppose that the class to be divided is A, and that we attempt to divide upon the attribute B, into AB and Ab; is this a true division, if we do not know any A that is not B? As far as our knowledge extends, we have not divided A at all. On the other hand, our knowledge of concrete things is never exhaustive; so that, although we know of no A that is not B, it may yet exist, and we have seen that it is a logical caution not to assume what we do not know. In a deductive classification, at least, it seems better to regard every attribute as a possible ground of division. Hence, in the above division of 'All Things'—'Not-phenomenal,' 'Extended-Not-resistant,' 'Resistant-Not-gravitating,' appear as negative classes (that is, classes based on the negation of an attribute), although their real existence may be doubtful. But, if this be justifiable, we must either rewrite the first test of a division thus: 'Each sub-class should *possibly* comprise less than the class to be divided'; or else we must confine the test to (*a*) thoroughly empirical divisions, as in dividing Colour into Red and Not-red, where we know that both sub-classes are real; and (*b*) divisions under demonstrable conditions—as in dividing the three kinds of triangles by the quality equilateral, we know that it is only applicable to acute-angled triangles, and do not attempt to divide the right-angled or obtuse-angled by it.

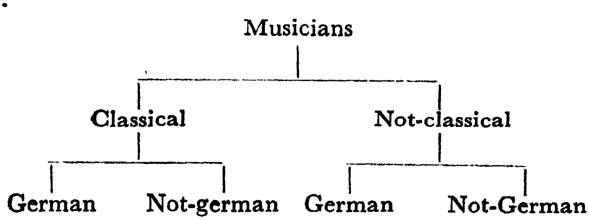
2. The sub-classes taken together should be equal to the

(2) The division should be exhaustive.

Class to be divided: the sum of the Species constitutes the Genus. This provides that the division shall be exhaustive; which dichotomy always secures, according to the principle of Excluded Middle; because whatever is not in the positive class, must be in the negative: Red and Not-red include all colours.

(3) The sub-classes must be mutually exclusive.

3. The Sub-classes must be opposed or mutually exclusive: Species must not overlap. This again is secured by dichotomy, according to the principle of Contradiction, provided the division be made upon one attribute at a time. But, if we attempt to divide simultaneously upon two attributes, as 'Musicians' upon 'nationality' and 'method,' we get what is called a Cross-division, thus: 'German Musicians.' 'Not-German,' 'Classical,' 'Not-Classical; for these classes may overlap, the same men sometimes appearing in two groups—Batch in 'German' and 'Classical,' Pergolesi in 'Not-German' and 'Classical.' If however, we divide Musicians upon these attributes successively, cross division will be avoided, thus:



Here no Musician will be found in two classes, *unless* he has written works in two styles, *or unless* there are works whose style is undecided. This "unless—or unless" may suggest caution in using dichotomy as a short cut to the classification of realities.

(4) The class must be predi-

4. No Sub-class must include anything that is not comprised in the class to be divided: the Genus comprises all the Species. We

must not divide Dogs into fox-terriers and dog-fish. *cable of the sub-classes.*

§ 6. INDUCTIVE CLASSIFICATION : The process of Inductive Classification may be represented thus :

Given any multitude of individuals to be classified :

(1) Place together in groups (or in thought) those things that have in common the most, the most widely diffused and the most important qualities.

(2) Connect those groups which have. as groups, the greater resemblance, and separate those that have the greater difference.

(3) Demarcate, as forming higher or more general classes, those groups of groups that have important characters in common ; and, if possible, on the same principle, form these higher classes into classes higher still : that is to say, graduate the classification upwards.

Whilst in Division the terms 'Genus' and 'species are entirely relative to one another and have no fixed positions in a gradation of classes, it has been usual, in Inductive Classification, to confine the term Species' to classes regarded as lowest in the scale, to give the term 'Genera' to classes on the step above, and at each higher step to find some new term such as 'Tribe,' 'Order,' 'Sub-kingdom,' 'Kingdom' ; as may be seen by turning to any book on Botany or Zoology. If, having fixed our Species, we find them subdivisible, it is usual to call the Sub-species 'Varieties.' *Species and Genera*

Suppose an attempt to classify by this method the objects in a sitting-room. We see at a glance carpets, mats, curtains, grates, fire-irons, coal-scuttles, chairs, sofas, tables, books, pictures, musical instruments, *etc.* These may be called Species'.

Carpets and mats go together; so do chairs and sofas; so do grates, fire-irons, and coal-scuttles; and so on. These greater groups, or higher classes, are 'Genera.' Putting together carpets, mats and curtains as 'warmth-fabrics'; chairs, sofas and tables as 'supports'; books, pictures and musical instruments as 'means of culture'; these groups we may call Orders. Sum up the whole as, from the housewife's point of view, 'furniture.' If we then subdivide some of the species, as books into poetry, novels, travels *etc.*, these Sub-species may be considered 'Varieties.'

Illustration

A classification thus made, may be tested by the same rules as those given for testing a Division; but if it does not stand the test, we must not infer that the classification is a bad one. If the best possible, it is good, though formally imperfect: whatever faults are found must then be charged upon the 'matter,' which is traditionally perverse and intractable. If, for example, there is a hammock in the room, it must be classed not with the curtains as a warmth-fabric, but with the sofas as a support; and books and pictures may be classed as, in a peculiar sense, means of culture, though all the objects in the room may have been modified and assorted with a view to gratifying and developing good taste.

A classification may be tested by the rules for testing a division

§ 7. Difficulty of Natural Classification: The difficulty of classifying natural objects is very great. It is not enough to consider their external appearance: exhaustive knowledge of their internal structure is necessary, and of the functions of every part of their structure. This is a matter of immense research, and has occupied many of the greatest minds for very many years. The following is a tabular outline of the classification of the Animal Kingdom.

Classification of animal kingdom

(See Page 255*)*.

Animal Kingdom

SUB-KINGDOM, or Phylum :	Vertebrates, Invertebrates (10 Sub-kingdom 5)
	Sauropsida, Ichthyopsida
CLASS :	Mammalia, Birds, Reptiles, Amphibia, Fishes
SUB-CLASS :	Placental, Implacental
DIVISION :	Monoeephia, Didelphia, Ornithodelphia
ORDER :	Quadrumana, Rodentia, Carnivora, Ungulata, Cætacea, *etc.*
SECTION :	Pinningrada (Seals, *etc.*), Plantigrada (Bears, *etc.*), Digitigrada
GENUS :	Mustelidæ (Weasels, *etc.*), Viverridæ (Civets, *etc.*), Hyænidæ, Canidæ, Felidæ
SPECIES :	Lion, Tiger, Leopard, Puma, Lynx, Cat, *etc.*
VARIETY :	African, Syrian, Cave-lion (extinct)

As there is not space enough to tabulate such a classification in full, I have developed at each step the most interesting groups : Vertebrates, Mammalia, Monodelphia, Carnivora, Digitigrada, Felidae, Lion. Most of the other groups in each

grade are also subdivisible, though some of them contain far fewer sub-classes than others.

To see the true character of this classification, we must consider that it is based chiefly upon knowledge of existing animals. Some extinct animals, known by their fossils, find places in it; for others new places have been made. But it represents, on the whole, a cross-section, or cross-sections of Nature as developing in time; and, in order to give a just view of the relations of animals, it must be seen in the light of other considerations. The older systems of classification, and the rules for making them, seem to have assumed that an actual system of classes, or of what Mill calls 'Kinds,' exists in nature, and that the relations of Kinds in this system are determined by quantity of resemblance in co-inherent qualities, as the ground of their affinity.

§ 8. Darwin's Influence on the Theory of Classification: Darwin's doctrine of the origin of species affects the conception of natural classification in several ways. (1) If all living things are blood-relations, modified in the course of ages according to their various conditions of life, 'affinity' must mean nearness of common descent'; and it seems irrational to propose a classification upon any other basis. We have to consider the Animal (or the Vegetable) Kingdom as a family tree, exhibiting a long line of ancestors, and (descended from them) all sorts of cousins, first, second, third, *etc.*, perhaps once, twice, or oftener 'removed.' Animals in the relation of first cousins must be classed as nearer than second cousins, and so on.

Darwins' doctrine and natural classification

(1) on this theory we should hold the Animal Kingdom as a family tree

But, if we accept this principle, and are able to trace relationship, it may not lead to the same results as would be reached simply relying upon the present 'quantity of resemblance,' unless we understand this in a very particular way. For the most obvious features of an animal may have been recently acquired;

and so in classifying should not rely on the

which often happens with those characters that adapt an animal to its habits of life, as the wings of a bat, or the fish-like shape of a dolphin; or as in cases of 'mimicry.' Some butterflies, snakes, *etc.*, have grown to resemble closely, in a superficial way, other butterflies and snakes, from which a stricter investigation widely separates them; and this superficial resemblance is probably a recent acquisition, for the sake of protection; the imitated butterflies being nauseous, and the imitated snakes poisonous. On the other hand, ancient and important traits of structure may, in some species, have dwindled into inconspicuous survivals or be still found only in the embryo; so that only great knowledge and sagacity can identify them; yet upon ancient traits, though hidden, classification depends. The seal seems nearer allied to the porpoise than to the tiger, the shrew nearer to the mouse than to the hedge-hog; and the Tasmanian wolf looks more like a true wolf, the Tasmanian devil more like a badger, than like a kangaroo: yet the seal is nearer akin to the tiger, the shrew to the hedge-hog, and the Tasmanian flesh-eaters are marsupial, like the kangaroo. To overcome this difficulty we must understand the resemblance upon which classification is based to include resemblance of Causation, that is, the fact itself of descent from common ancestors. For organic beings, all other rules of classification are subordinate to one: trace the genealogy of every form.

quantity of resemblance

By this rule we get a definite meaning for the phrase 'important or fundamental attribute' as determining organic classes; namely, most ancient, or 'best serving to indicate community of origin.' Grades of classification will be determined by such fundamental characters, and may correspond approximately to the more general types (now extinct) from which existing animals have descended.

(2) By the hypothesis of development the fixity of species is discredited. The lowest grade of a classification is made up not of well-defined types unchanging from age to age,

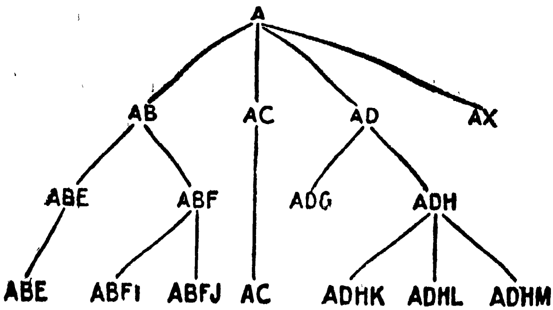
(2) The notion—fixity of species is discredited

but of temporary species, often connected by uncertain and indistinct varieties: some of which may, in turn, if the conditions of their existence alter, undergo such changes as to produce new species. Hence the notion that Kinds exist in organic nature must be greatly modified. During a given period of a few thousand years, Kinds may be recognised, because, under such conditions as now prevail in the world, that period of time is insufficient to bring about great changes. But, if it be true that lions, tigers, and leopards have had a common ancestor, from whose type they have gradually diverged, it is plain that their present distinctness results only from the death of intermediate specimens and the destruction of intermediate varieties. Were it possible to restore, by the evidence of fossils, all the ranks of the great processions that have descended from the common ancestor, there would nowhere occur a greater difference than between offspring and parents; and the appearance of Kinds existing in nature, which is so striking in a museum or zoological garden, would entirely vanish.

and so classification represents a cross-section of nature which is an everchanging, spreading, branching continuum

A classification, then, as formerly observed, represents a cross-section of nature as developing in time: could we begin at the beginning and follow this development down the course of time, we should find no classes, but an evermoving, changing, spreading, branching continuum. It may be represented thus: Suppose an animal (or plant) A, extending over a certain geographical areas, subject to different influences and conditions of climate, food, hill and plain, wood and prairie, enemies and rivals, and undergoing modifications here and there in adaptation to the varying conditions of life: then varieties appear. These varieties, diverging more and more, become distinct species (AB, AC, AD, AX). Some of these species, the more widely diffused, again produce varieties; which, in turn, become species (ABE, ABF, ADG,

ADH). From these, again, ABE, ABFI, ABFJ, AC, ADHK, ADHL, ADHM, the extant species, descend.



If in this age a classifier appears, he finds seven living species, which can be grouped into four genera (ABE, ABF, AC, ADH), and these again into three Families (AB, AC, AD), all forming one Order. But the animals which were their ancestors are all extinct. If the fossils of any of them—say AB, ADG and AX—can be found, he has three more species, one more genus (ADG), and one more family (AX). For AC, which has persisted unchanged, and AX, which has become extinct, are both of them Families, each represented by only one species. It seems necessary to treat such ancient types as species on a level with extant forms; but the naturalist draws our attention to their archaic characteristics, and tries to explain their places in the order of evolution and their relationships.

But now suppose that he could find a fossil specimen of every generation (hundred of thousands of generations), from ABFI, *etc.*, up to A; then, as each generation would only differ from the preceding as offspring from parents, he would be unable at any point to distinguish a species; at most, he would observe a slightly marked variety. ABFI and ABFJ would grow more and more alike, until they became indistinguishable in ABF; ABF and ABE would merge into AB; AB, AC, AD, and AX would merge into A. Hence, the appearance of species is due to our

taking cross-sections of time, or comparing forms that belong to periods remote from one another (like AX, ADG, and ADHK, or AD, ADH and ADHK), and this appearance of species depends upon the destruction of ancestral intermediate forms.

(3) Classification a deduction of co-inherent characters from the characters of earlier forms.

(3) The hypothesis of development modifies the logical character of classification; it no longer consists in a direct induction of co-inherent characters, but is largely a deduction of these from the characters of earlier forms, together with the conditions of variation; in other words, the definition of a species must, with the progress of science, cease to be a mere empirical law of co-inherence and become a derivative law of Causation. But this was already implied in the position that causation is the fundamental principle of the explanation of concrete things; and, accordingly, the derivative character of species or kinds extends beyond organic nature.

The classification of inorganic bodies also depends on causation.

§ 9. CLASSIFICATION OF INORGANIC BODIES ALSO DEPENDENT ON CAUSATION: The classification of inorganic bodies also depends on causation. There is the physical classification into Solids, Liquids, and Gases. But these states of matter are dependent on temperature; at different temperatures, the same body may exist in all three states. They cannot therefore be defined as solid, liquid, or gaseous absolutely, but only within certain degrees of temperature, and therefore as dependent upon causation. Similarly, the geological classification of rocks, according to relative antiquity (primary, secondary, tertiary, with their subdivisions), and mode of formation (igneous and aqueous), rests upon causation; and so does the chemical classification of compound bodies according to the elements that enter into them in definite proportions. Hence, only the classification of the elements themselves (amongst concrete things), at present, depends largely upon empirical Co-inherence. If the elements remain irresolvable into anything

simpler, the definitions of the co-inherent characters that distinguish them must be reckoned amongst the ultimate Uniformities of Nature. But if a definite theory of their origin both generally, and severally whether out of ether-vortices, or groups of electric corpuscles, or what-not, shall ever gain acceptance, similarity of genesis or causation will naturally be the leading consideration in classifying the chemical elements. To find common principles of causation, therefore, constitutes the verification of every Natural Classification. The ultimate explanation of nature is always causation; the Law of Causation is the backbone of the system of Experience.

SUMMARY

Classification, is a mental grouping of facts according to their resemblance, and difference is as best to serve some purpose. There is no one way of classifying things. The same object may be classed in various ways according to the purpose in view. Purposes are either (a) special or practical or (b) general or scientific. The scientific purpose is merely knowledge. It may subserve practical ends, but has no other end than knowledge directly in view.

Scientific classification comes late in human history. In the distinctions recognised by animals and children we can trace the grounds of classification. But it is in the development of language that the progress of instinctive classification may best be seen.

There are two sorts of classes marked by popular language viz., kinds and those having but few points of agreement. But the popular classification are very imperfect. They often omit important attributes and recognise unimportant ones. They are never comprehensive. A scientific classification aims at comprehensiveness, discrimination, definite and systematic coherence.

Classification is very useful. It helps us in the better under-

standing of the facts of Nature. It aids the memory. Greater fertility and flexibility of thought seems naturally to result from the practice of explanation and classification.

Classification may be either Deductive or Inductive. In the former we proceed from the more to the less, while in the latter we pass from the less to the more. The former is called division while it is usual to call the latter classification. They are entirely interdependent.

Given a class to be divided we (1) select an important character as the basis of division, (2) proceed by dichotomy and (3) gradually i.e., step by step. The ideal of division can, outside the mathematical sciences, be hardly realised.

The rules for testing a division are : (1) Each sub-class should comprise less than the class : (2) the sub-classes taken together should be equal to the class to be divided ; (3) the sub-classes must be mutually exclusive and (4) No sub-class should comprise anything that is it comprised in the class to be divided.

To classify the (1) place together in groups those things that have in common the most important qualities, (2) connect together those groups which have as groups greater resemblance and separate those that have the greater difference, and (3) demarcate as forming higher or more general classes, those group of groups, that have important characters in general— and if possible, on the same principle, from these higher classes into classes higher still. A classification thus made may be tested by the same rules as those for testing a Division.

Darwin's doctrine of the origin of species affects the conception of natural classification in several ways. (1) It makes us consider the Animal Kingdom as a family, there exhibiting a long line of ancestors and all sorts of cousins, first, second, third and so on. Animals in the relation of first cousins must be classed as nearer than the second cousins and so on. But, if we accept this principle, and are able to trace relationship it may not lead to the same results as would be reached by simply relying upon the quantity of resemblance. To overcome this difficulty we must understand the resemblance upon which classification is based to include resemblance of causation. (2) The hypothesis of development discredits the fixity of species. So classification represents a cross-section of nature that develops in time and is an evermoving, changing, spreading, branch-

ing continuum. (3) Classification logically ceases to be a direct induction of co-inherent characters, and becomes largely a deduction of these from the characters of earlier forms together with the conditions of variation. The classification of organic bodies depends on the Law of Causation. This is also true of the classification of inorganic bodies.

EXERCISES WITH HINTS

1. *Explain the nature of scientific classification. What are the rules of classification?*

[Read secs. 1 and 6]

2. *How does Darwin's doctrine of the origin of species affect the conception of natural classification?*

[Read sec. 8]

3. *Explain the relation of classification to division.*

[See sec. 3 and 6]

4. *What are the uses of classification?*

[See sec. 2]

5. *What is classification by series? Is it useful?*

[By classification by series we mean the arrangement of the classes of objects serially i.e. into a series in accordance with the varying degrees in which they possess some particular quality or qualities. For this we should, as Mill says, first of all "bring into one class all Kinds of things which exhibit that phenomenon, in whatever variety of forms or degrees; and secondly to arrange those Kinds in a series according to the degree in which they exhibit it, beginning with those which exhibit most of it and terminating with those which exhibit least." This classification applies, as Bain observes, "to all Kinds of objects where there is a progress or development and where the property developed has a commanding importance." According to Whewell, this classification is not a classification at all, but a mere linear progression. Indeed, he holds it to be a bad and narrow philosophy. But, Whewell is rather unsympathetic. For, though by an ordinary classification "our ideas of objects are brought into the order most conducive to the successful prosecution of inductive inquiries generally"; yet "when the purpose

is to facilitate some particular inductive inquiry more is required." "To be instrumental to that purpose the classification must bring those objects together the simultaneous contemplation of which is likely to throw most light upon the particular subject" (Mill). Indeed, the method of concomitant variations assumes this form of classification. Apart from a formation of such a series, this method cannot be applied. So, the usefulness of this form of classification cannot be denied.]

6. *What are the limits of classification ?*

[The highest class, the unique objects, and the marginal cases (i.e. those objects which like bats do not belong exclusively to either of the exclusive classes) cannot be classified.]

7. *What is division by dichotomy ? What are its advantages and disadvantages ?*

[Read secs. 4 and 5. Division by dichotomy is far from being purely formal. (1) In selecting a real *fundamentum divisionis*, it assumes something a posteriori or material. There is nothing in the process of division itself to indicate if the attribute chosen as the *fundamentum divisionis*, is a real attribute of the things to be divided and is not one which belong without *differentia* to the whole genus to be divided. (2) There is nothing in the process of division to show why the genus should be divided into just these species. (3) We end with a term about the reality of which the process leaves us entirely ignorant. (4) It represents co-ordinate species as if they were sub-ordinate to one another. (From Joseph, and Latta Macbeath.]

8. *What are the rules of logical division ?*

[Read secs. 4 and 5]

9. *What are the uses of division ?*

[It helps us to understand the extent or denotation of a general term and test classification. It also aids memory and inference.]

10. *What are the limits of logical division ?*

[An individual, the class whose instances do not exhibit difference, *infima* species and unique phenomena cannot be logically divided.]

CHAPTER XXII

NOMENCLATURE, DEFINITION, PREDICABLES

§ 1. **Precise thinking needs precise language**: Precision of thought needs precision of language for the recording of such thought and for communicating it to others.

Two conditions requisite for scientific language viz. Nomenclature and Terminology

We can often remember with great vividness persons, things, landscapes, changes and actions of persons or things, without the aid of language (though words are often mixed with such trains of imagery), and by this means may form judgments and inferences in particular cases; but for general notions, judgments and inferences, not merely about this or that man, or thing, but about all men or all kinds of things, we need something besides the few images we can form of them from observation. Even if we possess generic images, say, 'horse' or 'cat' (that is, images formed, like composite photographs, by a coalescence of the images of all the horses or cats we have seen, so that their common properties stand out and their differences frustrate and cancel one another), these are useless for precise thought; for the generic image will not correspond with the general appearance of horse or cat, unless we have had proportional experience of all varieties and have been impartially interested in all; and, besides, what we want for general thought is not a generic image of the appearance of things, though it were much more definite had fairly representative than such images ever are, but a general representation of their important characters; which may be connected with internal organs, such as none but an anatomist ever sees. We require a symbol connected with the general character of a thing, or quality, or process, as scientifically determined, whose representative truth may be trusted in ordinary cases, or may be verified whenever doubt arises. Such symbols are for most purposes

provided by language; Mathematics and Chemistry have their own symbols.

§ 2. NOMENCLATURE AND TERMINOLOGY: First there should be "a name for every important meaning": (*a*) A Nomenclature, or system of the names of all classes of objects, adapted to the use of each science. Thus, in Geology there are names for classes of rocks and strata, in Chemistry for the elements and their compounds, in Zoology and Botany for the varieties and species of animals and plants, their genera, families and orders.

Nomenclature

To have such names, however, is not the whole aim in forming a scientific language; it is desirable that they should be systematically significant, and even elegant. Names, like other instruments, ought to be efficient, and the efficiency of names consists in conveying the most meaning with the least effort. In Botany and Zoology this result is obtained by giving to each species a composite name which includes that of the genus to which it belongs. The species of Felidæ given in chap. xvii. § 7, are called *Felis leo* (lion), *Felis tigris* (tiger), *Felis leopardus* (leopard), *Felis concolor* (puma), *Felis lyncus* (European lynx), *Felis catus* (wild cat). In Chemistry, the nomenclature is extremely efficient. Names of the simpler compounds are formed by combining the names of the elements that enter into them; as Hydrozen Chloride, Hydrozen Sulphide, Carbon Dioxide; and these can be given still more briefly and efficiently in symbols, as HCl, H_2S, CO_2. The symbolic letters are usually initials of the names of the elements: as C—Carbon, S—Sulphur; sometimes of the Latin name, when the common name is English, as Fe—Iron. Each letter represents a fixed quantity of the element for which it stands, *viz.*, the atomic weight. The number written below a symbol on the right-hand side shows how many atoms of the element denoted enter into a molecule of the compound.

Illustration from sciences

(*b*) A Terminology is next required, in order to describe

and define the things that constitute the classes designated by the nomenclature, and to describe and explain their actions. *Terminology*

(i) A name for every integral part of an object, as head, limb, vertebra, heart, nerve, tendon; stalk, leaf, corolla, stamen, pistil; plinth, frieze, *etc.*

(ii) A name for every metaphysical part or abstract quality of an object, and for its degrees and modes; as extension, figure, solidity, weight; rough, smooth, elastic, friable; the various colours, red, blue, yellow, in all their shades and combinations; and so with sounds, smells, tastes, temperatures. The terms of Geometry are employed to describe the modes of figure, as angular, curved, square, elliptical; and the terms of Arithmetic to express the degrees of weight, elasticity, temperature, pitch of sound. When other means fail, qualities are suggested by the names of things which exhibit them in a salient way; figures by such terms as amphitheatre, bowl-like, pear-shaped, egg-shaped; colours by lias-blue, sky-blue, gentian-blue, peacock-blue; and similarly with sounds, smells and tastes. It is also important to express by short terms complex qualities, as harmony, fragrance, organisation, sex, symmetry, stratification.

(iii) In the explanation of Nature we further require suitable names for processes and activities: as deduction, conversion, verification, addition, integration, causation, tendency, momentum, gravitation, aberration, refraction, conduction, affinity, combination, germination, respiration, attention, association, development.

There may sometimes be a difficulty in distinguishing the terms which stand for qualities from those that express activities, since all qualities imply activities; weight, for example, implies gravitation; and the quality heat is also a kind of motion. The distinction aimed at lies between a quality as perceived by means of an effect upon our senses (as weight is resistance to our effort in lifting; heat, a sensation when we approach fire), and that property of a body which is conceived

to account for its energy (as gravitation that brings a body to the ground, or physical heat that expands an iron bar or works an engine). The former class of words, expressing qualities, are chiefly used in description: the latter class, expressing activities, are chiefly needed in explanation. They correspond respectively, like classification and explanation, with the static and dynamic aspects of Nature.

Terminology and Nomenclature of ordinary language

The terms of ordinary language fall into the same classes as those of science: they stand for things, classes of things, parts, or qualities, or activities of things; but they are far less precise in their signification. As long as popular thought is vague its language must be vague; nor it is desirable too strictly to correct the language whilst the thought is incorrigible. Much of the effect of poetry and eloquence depends upon the elasticity and indirect suggestiveness of common terms. Even in reasoning upon some subjects, it is a mistake to aim at an unattainable precision. It is better to be vaguely right than exactly wrong. In the criticism of manners, of fine art, or of literature, in politics, religion and moral philosophy, what we are anxious to say is often far from clear to ourselves; and it is better to indicate our meaning approximately, or as we feel about it, than to convey a false meaning, or to lose the warmth and colour that are the life of such reflections. It is hard to decide whether more harm has been done by sophists who take a base advantage of the vagueness of common terms, or by honest paralogists (if I may use the word) who begin by deceiving themselves with a plausible definiteness of expression, and go on to propagate their delusions amongst followers eager for systematic insight but ignorant of the limits of its possibility.

Definition is the explicit

§ 3. Definition: A Definition is necessary (if possible) for every scientific name. To define a name is to give a precise statement of its meaning or connotation. The name to be defined is the subject of a proposition, whose predicate is a list of the fundamental

qualities common to the things or processes which the subject denotes, and on account of possessing which qualities this name is given to them.

statement of the entire connotation of a name.

Thus, a curve is a line of which no part is straight. The momentum of a moving body is the product of its mass and its velocity (these being expressed in numbers of certain units). Nitrogen is a transparent colourless gas, atomic weight 14, specific gravity .9713, not readily combining, *etc.* A lion is a monodelphian mammal, predatory, walking on its toes, of nocturnal habits, with a short rounded head and muzzle; dental formula: Incisors, $\frac{3-3}{3-3}$, canines $\frac{1-1}{1-1}$, præmolars $\frac{3-3}{2-2}$, molars $\frac{1-1}{1-1}=30$; four toes on the hind and five on the foot, retractile claws, prickly tongue, light and muscular in build, about 9½ feet from muzzle to tip of tail, tawny in colour, the males maned, with a tufted tail. If anything answers to this description, it is called a lion; if not, not: for this is the meaning of the name.

Example:

For ordinary purposes, it may suffice to give an Incomplete Definition; that is, a list of qualities not exhaustive, but containing enough to identify the things denoted by the given name; as if we say that a lion is 'a large twany beast of prey with a tufted tail.' Such purposes may also be served by a Description; which is technically, a proposition mentioning properties sufficient to distinguish the things denoted, but not properties that enter into the definition; as if nitrogen be indicated as the gas that constitutes ⅘ of the atmosphere.

Definition and Description:

§ 4. Rules for testing a Definition: The rules for testing a Definition are: 1.—As to its Contents—

As to content

(1) It must state the whole connotation of the name to be defined.

(2) It must not include any quality derivative from the

connotation. Such a quality is called a Proprium. A breach of this rule can do no positive harm, but it is a departure from scientific economy. There is no need to state in the definition what can be derived from it; and whatever can be derived by causation, or by mathematical demonstration, should be exhibited in that manner.

(3) It must not mention any circumstance that is not a part of the connotation, even though it be universally found in the things denoted. Snch a circumstance, if not derivable from the connotation, is called an Accident. That, for example, the lion at present only inhabits the Old World, is an accident: if a species otherwise like a lion were found in Brazil, it would not be refused the name of lion on the score of locality. Whilst, however, the rules of Logic have forbidden the inclusion of proprium or accident in a definition, in fact the definitions of Natural History often mention such attributes when characteristic. Indeed, definitions of superordinate classes—Families and Orders—not infrequently give qualities as generally found in the subordinate classes, and at the same time mention exceptional cases in which they do not occur.

II.—As to its Expression—

As to expression

(4) A Definition must not include the very term to be defined, nor any cognate. In defining 'lion' we must not repeat 'lion,' nor use 'leo nine'; it would elucidate nothing.

(5) It must not be put in vague language.

(6) It must not be in a negative form if a positive form be obtainable. We must not be content to say that a lion is 'no vegetarian,' or 'no lover of daylight.' To define a curve as a line 'always changing its direction' may be better than as 'in no part straight.'

§ 5. Every Definition is relative to a Classification: The process of determining a Definition is inseparable from classification. We saw that classification consists in distributing things into groups according to their likenesses and differences,

regarding, as a class those individuals which have most qualities in common. In doing so we must, of course, recognise the common qualities or points of likeness; and to enumerate these is to define the name of the class. If we discover the qualities upon which a class is based by direct observation and induction, by the same method we discover the definition of its name.

Enumeration of qualities serving as basis of classification is the definition of the name of the class.

We saw also that classification is not merely the determination of isolated groups of things, but a systematic arrangement of such groups in relation to one another. Hence, again, Definitions are not independent, but relative to one another; and, of course, in the same way as classes are relative. That is to say, as a class is placed in subordination to higher or more comprehensive groups, so the definition of its name is subordinate to that of their names; and as a class stands in contrast with co-ordinate classes (those that are in the same degree of subordination to the same higher groups), so the definition of its name is in contrast or co-ordination with the definitions of their names. Lion is subordinate to *Felis*, to Digitigrade, to Carnivore and so on up to Animal; and, beyond the Animal Kingdom, to Phenomenon; it is co-ordinate with tiger, puma, *etc.*; and more remotely it is co-ordinate with dog, jackal, wolf, which come under *Canis*—a genus co-ordinate with *Felis*. The definition of lion, therefore, is subordinate to that of *Felis*, and to all above it up to Phenomenon: and is co-ordinate with that of tiger, and with all species in the same grade. This is the ground of the old method of definition *per genus et differentiam*.

Definitions like the classes not independent but relative.

Definition per genus et differentiam

The genus being the next class above any species, the *Differentia* or Difference consists of the qualities which mark that species in addition to those that

Differentia, the

connotation of the species in addition to that of the proximate genus

mark the genus, and which therefore distinguish it from all other species of the same genus. In the above definition of lion, for example, all the properties down the "light and muscular in build" are generic, that is, are possessed by the whole genus, *Felis*; and the remaining four (size, colour, tufted tail, and mane in the male) are the Difference or specific properties, because in those points the lion contrasts with the other species of that genus. Differences may be exhibited thus:

Lion.	*Tiger*
SIZE: about 9½ feet from nose to tip of tail.	About 10 feet.
COLOUR: tawny.	Warm tawny, striped with black.
TAIL: tufted.	Tapering.
MANE: present in the male.	Both sexes maneless.

There are other Differences in the shape of the skull. In defining lion, then, it would have been enough to mention the genus and the properties making up the Difference; because the properties of the genus may be found by turning to the definition of the genus; and, on the principle of economy, whatever it is enough to do it is right to do. To define 'by genus and difference' is a point of elegance, when the genus is known; but the only way of knowing it is to compare the individuals comprised in it and in co-ordinate genera, according to the methods of scientific classification. It may be added that, as the genus represents ancestral derivation, the predication of genus in a definition indicates the remote causes of the phenomena denoted by the name defined. And this way of defining corresponds with the method of double naming by genus and *species*; *Felis leo, Felis tigris, etc.*; *Vanessa Atalanta, Vanessa Io, etc.*

The so-called Genetic Definition, chiefly used in Mathematics, is a rule for constructing that which a name denotes, in such a way as to ensure its possessing the attributes conno-

ted by the name. Thus, for a circle: Take any point and, at any constant distance from it, trace a line returning into itself. In Chemistry a genetic definition of any compound might be given in the form of directions for the requisite synthesis of elements.

Genetic definition.

§ 6. Difficulties of Definition: The chief difficulty in the definition of scientific names consists in determining exactly the nature of the things denoted by them, as in classifying plants and animals. If organic species are free growths, continually changing, however gradually, according as circumstances give some advantage to one form over others, we may expect to find such species branching into varieties, which differ considerably from one another in some respects, though not enough to constitute distinct species. This is the case; and, consequently, there arises some uncertainty in collecting from all the varieties those attributes which are common to the species as a whole; and, therefore, of course, uncertainty in defining the species. The same difficulty may occur in defining a genus, on account of the extent to which some of its species differ from others, whilst having enough of the common character to deter the classfier from forming a distinct genus on their account. On the other hand the occurrence of numerous intermediate varieties may make it difficult to distinguish genera or species at all. Even the Kingdoms of plants and animals are hard to discriminate at the lowest levels of organisation. Now, where there is a difficulty of classification there must be a corresponding difficulty of definition.

The chief difficulty

It has been proposed in such cases to substitute a Type for a Definition; to select some variety of a species, or species of a genus, as exhibiting its character in an eminent degree, and to regard other groups as belonging to the same species or genus, acccording as they agree more with this type than with other

Is classification by type or by definition?

tion of one group as typical implies a recognition of its attributes as prevailing generally (though not universally) throughout the species or genus; and to recognise these attributes and yet refuse to enumerate them in a definition, seems to be no great gain. To enumerate the attributes of the type as an Approximate Definition of the species or genus, true of *most* of the groups constituting the species or genus, answers the same purpose, is more explicit, and can mislead no one who really attends to the exposition. An approximate definition is, indeed, less misleading than the indication of a type; for the latter method seems to imply that the group which is now typical has a greater permanence or reality than its co-ordinate groups; whereas, for aught we know, one of the outside varieties or species may even now be superseding and extinguishing it. But the statement of a definition as approximate, is an honest confession that both the definition and the classification are (like a provisional hypothesis) merely the best account we can give of the matter according to our present knowledge.

§ 7. THE LIMITS OF DEFINITION: The limits of Definition are twofold: (*a*) A name whose meaning cannot be analysed, cannot be defined. This limitation meets us only in dealing with the names of the metaphysical parts or simple qualities of objects under the second requisite of a Terminology. Resistance and weight, colour and its modes, many names of sounds, tastes, smells, heat and cold—in fact, whatever stands for an unanalysable perception, cannot be made intelligible to any one who has not had experience of the facts denoted; they cannot be defined, but only exemplified. A sort of genetic definition may perhaps be attempted, as if we say that colour is the special sensation of the cones of the retina, or that blue is the sensation produced by a ray of light vibrating about 650, 000, 000, 000, 000 times a second; but such expressions can give no notion of our meaning to a blind man, or

Limits of definition (a) A name whose meaning cannot be analysed cannot be defined.

to any one who has never seen a blue object. Nor can we explain what heat is like, or the smell of tobacco, to those who have never experienced them ; nor the sound of C 128 to one who knows nothing of the musical scale.

If we distinguish the property of an object from the sensation it excites in us, we may define any simple property as 'the power of producing the sensation' ; the colour of a flower as the power of exciting the sensation of colour in us. Still, this gives no information to the blind nor to the colour-blind. Abstract names may be defined by defining the corresponding concrete : the definition of 'human nature' is the same as of 'man.' But if the corresponding concrete be a simple sensation (as blue), this being indefinable, the abstract (blueness) is also indefinable.

(b) Where to define is to exhaust infinity, definition is impossible.

(*b*) The second limit of Definition is the impossibility of exhausting infinity, which would be necessary in order to convey the meaning of the name of any individual thing or person. For, as we saw in chap. iv., if in attempting to define a proper name we stop short of infinity, our list of qualities or properties may poossibly be found in two individuals, and then it becomes the definition of a class-name or general name, however small the actual class. Hence we can only give a Description of that which a proper name denotes, enumerating enough of its properties to distinguish it from everything else as far as our knowledge goes.

Significance of relations of predicables to a

§ 8. THE FIVE PREDICABLES: The five Predicables (Species, Genus, Difference, Proprium, Accident) may best be discussed in connection with Classification and Definition ; and in giving an account of Classsification, most of what has to be said about them has been anticipated. Their name, indeed, connects them with the doctrine of Propositions; for Predicables are terms that may be predicated, classified according to their connotative relation to the subject of a proposition (that is, according to the relation in

subject derivative from the general doctrine of classification.

which their connotation stands to the connotation of the subject): nevertheless, the significance of the relations of such predicates to a subject is derivative from the general doctrine of classification.

In X is Y, what Y is, depends on what X is.

For example, in the proposition 'X is Y.' Y must be one of the five sorts of predicables in relations to X; but of what sort, depends upon what X (the subject) is, or means. The subjects of the proposition must be either a definition, or a general connotative name, or a singular name.

X definition Y species.

If X be a definition, Y must be a species; for nothing but a general name can be predicated of a definition: and, strictly speaking, it is only in relation to a definition (as subject) that species can be a predicable; when it is called *Species predicabilis* (1).

X a connotative name, Y related to it in three different ways

If X be a connotative name, it is itself a species (*Species subjicibilis*); and the place of the subject of a proposition is the usual one for species. The predicate, Y, may then be related to the species in three different ways. First, it may be, a definition, exactly equivalent to the species;—in fact, nothing else than the species in an explicit form, the analysis of its connotation. Secondly, the predicate may be, or connote, some *part only* of the definition or connotation of the species; and then it is either genus (2), or difference (3). Thirdly, the predicate may connote *no part* of the definition, and then it is either derivable from it, being a proprium (4), or not derivable from it, being an accident (5). These points of doctrine will be expanded and illustrated in subsequent pages.

X a singular name, Y a genus

If X be a singular name, deriving connotation from its constituent terms (chap. iv. § 2), as 'The present Emperor of China,' it may be treated as a *Species subjicibilis*. Then that he is 'an absolute monarch,' predicates a genus; because that is a genus of 'Emperor,' a part of the

singular name that gives it connotation. That he wears a yellow robe is a proprium, derivable from the ceremonial of his court. That he is thirty years of age is an accident.

or a proprium or an accident

But if X be a proper name, having no connotation, Y must always be an accident; since there can then be no definition of X, and therefore neither species, genus, difference, nor proprium. Hence, that 'John Doe is a man' is an accidental proposition: 'man' is not here a *Species predicabilis*; for the name might have been given to a dog or a mountain. That is what enables the proposition to convey information: it would be useless if the proper name implied 'humanity.'

X a proper name, Y an accident

'Species' is most frequently used (as in Zoology) for the *class denoted* by a general name; but in Logic it is better to treat it as a general name used connotatively for the attributes possessed in common by the things denoted, and on account of which they are regarded as a class: it is sometimes called the Essence (§ 9). In this connotative sense, a species is implicitly what the definition is explicitly; and therefore the two are always simply convertible Thus, 'A plane triangle' (species) is 'a figure enclosed by three straight lines' (definition): clearly we may equally say.' A figure enclosed by three straight lines is a plane triangle.' It is a simple identity.

Species

A genus is also commonly viewed denotatively, as a class containing smaller classes, its species; but in Logic it is, again, better to treat it connotatively, as a name whose definition is part of the definition of a given species.

Genus

A difference is the remainder of the definition of any species after subtracting a given genus. Hence, the genus and difference together make up the up species; whence the method of definition *per genus et differentiam* (*ante*, § 5).

Difference

est step of the classification (varieties not being reckoned as classes), and the genus is also fixed on the step next above it, in Logic these predicables are treated as movable up and down the ladder: any lower class being species in relation to any higher; which higher class, wherever taken, thus becomes a genus. Lion may lagically be regarded as a species of digitigrade, or mammal, or animal; and then each of these is a genus as to lion: or, again, digitigrade may be regarded as a species of mammal, or mammal as a species of animal. The highest class, however, is never a species; wherefore it is called a *Summum Genus*: and the lowest class is never a genus; wherefore it is called an *Infima Species*. Between these two any step may be either species or genus, according to the relation in which it is viewed to other classes, and is then called Subaltern.

Summum genus

Infima species

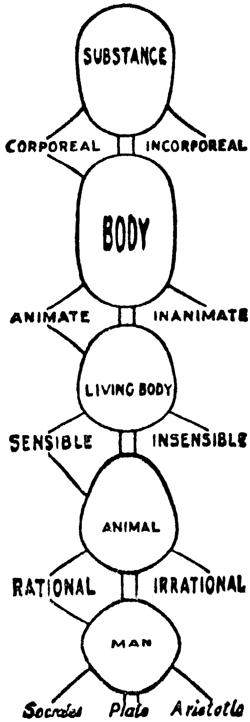
The *summum genus*, again, may be viewed in relation to a *given* universe or *suppositio* (that is, any limited area of existence now the object of attention), or to the *whole* universe. If we take the animal kingdom as our *suppositio*, Animal is the *summum genus*; but if we take the whole universe, 'All things' is the *summum genus*.

"Porphyry's tree" is used to illustrate this doctrine. It begins with a *summnm genus*, 'Substance,' and descends by adding differences, step by step, to the *infima species*, 'Man.' It also illustrates Division by Dichotomy. (For Prophyry's tree—*See Page* 279).

Porphyry's tree

Beginning with 'Substance,' as *summum genus*, and adding the difference 'Corporeal,' we frame the species 'Body.' Taking 'Body' as the genus and adding the difference 'Animate,' we frame the species 'Living Body; and so on till 'Man' is reached; which, being *infima species*, is only subdivisible into individuals. But the division of Man into individuals involves a change of principle; it is a division of the denotation, not an increase of the connotation as in the earlier steps. Only

the other side had been taken, Incorporeal Substance would be 'Spirit': which might be similarly subdivided.



Genus and species, then, have a double relation. In denotation the genus includes the species: in connotation the species includes the genus. Hence the doctrine that by increasing the connotation of a name we decrease its denotation: if, for example, to the definition of 'lion' we add 'inhabiting Africa,' Asiatic lions are no longer denoted by it. On the other hand, if we use a name to denote objects that it did not formerly apply to, some of the connotation must be dropped: if, for example, the name 'lion' be used to include 'pumas,' the tufted tail and mane can no longer be part of the meaning of the word; since pumas have not these properties.

Denotation and connotation vary universely.

This doctrine is logically or formally true, but it may not always be true in fact. It is logically true: because wherever we add to the connotation of a name, it is possible that some things to which it formerly applied are now excluded from its denotation, though we may not know of any such things. Still, as a matter of fact, an object may be discovered to have a property previously unknown, and this property may be fundamental and co-extensive with the denotation of its name, or even more widely prevalent. The discovery that the whale is a mammal did not limit the class 'whale': nor did the discovery that lions, dogs, wolves, *etc.*, walk upon their toes, affect the application of any of these names.

Criticism of the above

Similarly, the extension of a name to things not previously denoted by it, may not in fact alter its definition; for the extension may be made on the very ground that the things now first denoted by it have been found to have the properties enumerated in its definition, as when the name 'mammal' was applied to whales, dolphins, *etc.* If, however, 'mammal' had formerly been understood to apply only to land animals, so that its definition included (at least, popularly) the quality of 'living on the land,' this part of the connotation was of course lost when the denotation came to include certain aquatic animals.

Proprium

A proprium is an attribute derived from the definition: being either (*a*) implied in it, or deducible from it, as 'having its three angles equal to two right angles' may be proved from the definition of a triangle; or (*b*) causally dependent on it, as being 'dangerous to flocks' results from the nature of a wolf, and as 'moving in an ellipse' results from the nature of a planet in its relation to the sun.

Accident

An accident is a property accompanying the defining attributes without being deducible from them. The word suggests that such a property is merely 'accidental,' or there by chance'; but it only means that we do not understand the connotation.

Proprium and Accident bear the same relation to one an-

other as Derivative and Empirical Laws: the predication of a proprium is a derivative law, and the predication of an accident is an empirical law. Both accidents and empirical laws present problems, the solution of which consists in reducing them, respectively, to propria and derivative laws. Thus the colour of animals was once regarded as an accident for which no reason could be given; but now the colour of animals is regarded as an effect of their nature and habits, the chief determinants of it being the advantage of concealment whilst in other cases as among brightly coloured insects and snakes, the determinant may be the advantage of advertising their own noxiousness. If such reasoning is sound, colour is a proprium (and if so, it cannot *logically* be included in a definition; but it is better to be judicious than formal).

Relation between proprium and accident

If the colour of animals is a proprium, we must recognise a distinction between Inseparable and Separable Propria, according as they do, or do not, always accompany the essence: for mankind is regarded as one species: but each colour, white, black or yellow, is separable from it under different climatic conditions: whilst tigers are everywhere coloured and stripped in much the same way; so that we may consider their colouring as inseparable, in spite of exceptional specimens black or white or clouded.

The same distinction may be drawn between accidents. 'Inhabiting Asia' is an Inseparable Accident of tiger, but a Separable Accident of lion. Even the occasional characteristics and occupations of individuals are sometimes called separable accidents of the species; as, of man, being colour-blind, carpentering, or running.

A proprium in the original signification of the term (ἴδιον) was peculiar to a species, never found with any other, and was therefore convertible with the subject; but this restriction is no longer insisted on.

§ 9. REALISM AND NOMINALISM: Any predication of a genus,

difference or definition, is a verbal, analytic, or essential proposition : and any predication of a proprium or accident, is a real, synthetic, or accidental proposition (chap. v § 6) A proposition is called verbal or analytic when the predicate is a part, or the whole, of the meaning of the subject ; and the subject being species, a genus or difference is part, and a definition is the whole, of its meaning or connotation. Hence such a proposition has also been called explicative. Again, a proposition is called real or synthetic when the predicate is no part of the meaning of the subject ; and, the subject being species, a proprium or accident is no part of its meaning or connotation. Hence such a proposition has been called ampliative.

Verbal and real proposition

As to essential and Accidental, these terms are derived from the doctrine of Realism. Realists maintain that the essence of a thing, or that which makes a thing to be what (or of what kind) it is, also makes everything else of the same kind to be what it is. The essence, they say, is not proper to each thing or separately inherent in it, but is an 'Universal' common to all things of that kind. Some hold that the universal nature of things of any kind is an Idea existing (apart from the things) in the intelligible world, invisible to mortal eye and only accessible to thought; whence the Idea is called a noumenon: that only the Idea is truly real, and that the things (say, trees, bed-steads and cities) which appear to us in sense-perception, and which therefore are called phenomena, only exist by participating in, or imitating, the Idea of each kind of them. The standard of this school bears the legend *Universalia ante rem.*

Realism (Plato)

But others think that the Universal does not exist apart from particular things, but is their present essence; gives them actuality as individual substances; "informs" them, or is their formal cause, and thus makes them to be what they are of their kind according to the definition: the universal lion is in all lions, and

Realism (Aristotle)

is not merely similar, but identical in all; for thus the Universal Reason thinks and energises in Nature. This school inscribes upon its banners, *Universalia in re.*

To define anything, then, is to discover its essence, whether transcendent or immanent; and to predicate the definition or any part of it (genus or difference), is to enounce an essential proposition. But a proprium, being no part of a definition, though it always goes along with it, does not show what a thing is; nor of course does an accident; so that to predicate either of these is to enounce an accidental proposition.

Nominalism

Another school of Metaphysicians denies the existence of Universal Ideas or Forms; the real things, according to them, are individuals; which, so far as any of them, resemble one another, are regarded as forming classes; and the only Universal is the class-name, which is applied universally in the same sense. Hence, they are called Nominalists. The sense in which any name is applied, they say, is derived from a comparison of the individuals, and by abstraction of the properties they have in common; and thus the definition is formed. *Universalia post rem* is their motto.

Conceptualism

Some Nominalists, however, hold that, though Universals do not exist in nature, they do in our minds, as Abstract Ideas or Concepts; and that to define a term is to analyse the concept it stands for; whence, these philosophers are called Conceptualists.

Such questions belong to Metaphysics rather than to Logic; and the foregoing is a commonplace account of a subject upon every point of which there is much difference of opinion.

Aristotle doctrine of categories

§ 10. The Predicaments: The doctrine of the Predicaments, or Categories, is so interwoven with the history of speculation and especially of Logic that, though its vitality is exhausted, it can hardly be passed over unmentioned. The predicaments of Aristotle are the heads of a classification of terms as possible

predicates of a particular thing or individual. Hamilton (*Logic;* Lect, xi.) has given a classification of them; which, if it cannot be found in Aristotle, is an aid to the memory, and may be thrown into a table thus:

Substance		οὐσία	(1)
[Attribute]	—Quantity	ποσόν	(2)
	—Quality	ποιόν	(3)
	—Relation	πρός τι	(4)
[Modes of Relation]	[Where	ποῦ	(5)
	[When	πότε	(6)
	[Action	ποιεῖν	(7)
	[Passion	πάσχειν	(8)
	[Posture	κεῖσθαι	(9)
	[Habit	ἔχειν	(10)

Taking a particular thing or individual, as 'Socrates,' this is Substance in the proper sense of the word, and can never be a predicate, but is the subject of all predicates. We may assert of him (1) Substance in the secondary sense (species or genus) that he is a man or an animal; (2) Quantity, of such a height or weight; (3) Quality, fair or dark; (4) Relation, shorter or taller than Xanthippe; (5) Where, at Athens; (6) When, two thousand and odd years ago; (7) Action, that he questions or pleads (8) Passion, that he is answered or condemned; (9) Posture, that he sits or stands: (10) Habit, that he is clothed or armed.

Some comment

Thus illustrated (*Categoriæ:* c. 4), the predicaments seem to be a list of topics, generally useful for the analysis and description of an individual, but wanting in the scientific qualities of rational arrangement, derivation and limitation. Why are there just these heads, and just so many? It has been suggested that they were determined by grammatical forms; for Substance is expressed by a substantive; Quantity, Quality and Relation are adjectival; Where and When, adverbial; and the remaining four are verbal. It is true that the parts of speech were not systematically discriminated until some years after Aristotle's time; but, as

they existed, they may have unconsciously influenced his selection and arrangement of the predicaments. Where a principle is so obscure one feels glad of any clue to it *cf.* Grote's *Aristotle*, c. 3, and Zeller's *Aristotle*, c. 6). But whatever the origin and original meaning of the predicaments, they were for a long time regarded as a classification of things; and it is in this sense that Mill criticises them *(Logic*: Bk. I. c. 3).

Categories and predicables

If, however, the predicaments are heads of a classification of terms predicable, we may expect to find some connection with the predicables; and, in fact, secondary Substances are species and genus; whilst the remaining nine forms are generally accidents. But, again, we may expect some agreement between them and the fundamental forms of predication *(ante*, chap. i. § 5, and chap. ii. § 4): Substance, whether as the foundation of attributes, or as genus and species, implies the predication of coinherence, which is one mode of *Co-existence.* Quantity is predicated as equality (or inequality) a mode of *Likeness*; and the other mode of *Likeness* is involved in the predication of Quality. Relation, indeed, is the abstract of all predication, and ought not to appear in a list along with special forms of itself. 'Where' is position, or *Co-existence* in space; and 'When' is position in time, or *Succession.* Action and Passion are the most interesting aspect of *Causation.* Posture and Habit are complex modes of *Co-existence*, but too specialised to have any philosophic value. Now, I do not pretend that this is what Aristotle meant and was trying to say: but if Likeness, Co-existence, Succession and Causation are fundamental forms of predication, a good mind analysing the fact of predication is likely to happen upon them in one set of words or another.

Kant's use of

By Kant the word 'Category' has been appropriated to the highest forms of judgment, such as Unity, Reality, Substance, and Cause, under which the understanding reduces phenomena to order and thereby constitutes Nature. This change of meaning has not been made without a

the word category certain continuity of thought; for forms of judgment are modes of predication. But besides altering the lists of categories and greatly improving it, Kant has brought forward under an old title a doctrine so original and suggestive that it has extensively influenced the subsequent history of Philosophy. At the same time, and probably as a result of the vogue of the Kantian philosophy, the word 'category' has been vulgarised as a synonym for 'class,' just as 'predicament' long ago passed from Scholastic Logic into common use as a synonym for 'plight.' A minister is said to be 'in a predicament,' or to fall under the 'category of impostors.'

SUMMARY

Precision of thought needs precision of language. So, every science must have a nomenclature and a terminology. A nomenclature is a system of the names of all classes of objects, adapted to the use of each science while terminology is a system of names for describing and defining, the parts, qualities and activities of the things that constitute the classes designated by the nomenclature.

To define a name is to give a precise statement of its meaning or connotation. The name to be defined is the subject of a proposition, whose predicate is a test of fundamental qualities common to the things or processes which the subject denotes and on account of possessing which qualities the name is given to them. Thus, a curve is a line of which no part is straight is a definition. An incomplete or inadequate difinition is called a description which is popular and often useful but not scientific.

The rules for testing a definition are : I. As to its contents— (1) It must state the whole connotation of the name to be defined. (2) It must not include any quality derived from the connotation. (3) It must not mention any circumstance that is not a part of the connotation. II.—As to its expression—(4)

A definition must not include the very term to be defined nor any cognate. (5) It must not be put in vague language; (6) It must not be in a negative form, if a positive form be obtainable.

The process of determining a definition is inseparable from classification. In classification we recognise the common qualities or points of likeness ; and in definition we enumerate them. Again classification is not merely the determination of isolated groups of things, but a systematic arrangement of such groups in relation to another. Hence, definitions are not independent, but relative to one another, in the same way as classes are relative. This is the ground of the old method of definition *per genus et differentiam.*

Difference consists of the qualities which mark the species in addition to those that mark the genus, and which therefore distinguish it from all other species of the same genus.

The so-called Generic Definiton is a rule for constructing that which a name denotes, in such a way as to ensure its possessing the attributes connoted by the name.

The chief difficulty in the definition of scientific names consists in determining exactly the nature of the things denoted by them, as in classifying plants and animals. That is, where there is a difficulty of classification, there must be a corresponding difficulty of definition. In such cases it has been proposed to substitute a Type for Definition. But as a type presupposes a Definition this proposal is of no use. Hence, what we should do in such cases is to be satisfied with approximate definitions.

The limits of definition are twofold : (1) A name whose meaning cannot be analysed cannot be defined. This limitation meets us only in dealing with the metaphysical parts or simple qualities of objects. (2) The second limit of Definition is the impossibility of exhausting infinity, which would be necessary in order to convey the meaning of the name of any individual thing or person.

Predicables are terms that may be predicated, classified according to their connotative relation to the subject of a proposition. But the significance of the relations of such predicates to a subject is derivative from the general doctrine of classification. Thus, in 'X is Y' what Y is, depends on what X is. If X be a definition, Y must be a species (*Species predicabilis*). If X be a connotative name, it is itself a species (*species subjicibilis*)

and the predicate Y may be related to the species in three different ways. It may be a definition. Or it may be or connote some part only of the definition and be either genus or difference. Or it may connote no part of the definition and is either a proprium or an accident. If X be a significant singular name, Y is either a genus or a proprium or an accident. If X be a proper name, Y must always be accident.

Species is a general name used connotatively; for—the attributes possessed in common by the things denoted and on account of which they are regarded as a class. A genus is a name whose definition is part of the definition of a given species. A difference is the remainder of the definition of any species after subtracting a given genus. The highest genus is the *summum genus* and the lowest species is the *infima species*.

Genus and species have a double relation. In denotation the genus includes the species, in connotation, the species, includes the genus. Hence we have the doctrine that the denotation and the connotation of a name vary inversely. This doctrine is formally true, but it may not always be true in fact. For an object may be discovered to have a property previously unknown and this property may be fundamental and co-extensive with the denotation of its name. Similarly the extension of a name to things not previously denoted by it, may not in fact alter its definition.

A proprium is an attribute derived from the definition while an accident is a property accompanying the defining attributes without being deducible from them. Proprium and Accident bear the same relation to one another as Derivative and Empirical Laws : the predication of a proprium is a derivative law and the predication of an accident an empirical law.

Any predication of a genus, difference or definition is verbal, analytic or essential proposition and any predication of a proprium or accident is a real, synthetic or accidental proposition.

The distinction between Essential and Accidental is owing to the doctrine of Realism. According to the Realists, the essence of a thing, or that which makes a thing to be what (or of what kind) it is, also makes everything else of the same kind to be what it is. Some realists e.g. Plato hold that the universal nature of things of any kind is an idea existing (apart from the things) in the intelligible world, invisible to the mortal eye and only accessible to thought. This is the doctrine of *universalia*

ante rem. But some other realists e.g. Aristotle, think that the Universal does not exist apart from particular things, but is their present essence ; gives them actuality as individual substances ; 'informs' them, or is their formal cause. This is the doctrine of *universalia in re*.

To define anything, then, is to discover its essence, whether transcendent or immanent ; and to predicate the definition or any part of it, is to enounce an essential proposition. But as proprium and accident are no part of definition, the predication of them is an accidental proposition.

But there are metaphysicians who deny the existence of universal Ideas or Forms. The real things according to them are individuals which so far as any of them resemble one another are regarded as forming classes, and the only Universal is a class-name. These metaphysicians are called Nominalists. *Universalia post rem* is their motto. Definition is here is the state of the sense in which the name is employed. Some Nominalists, however, hold that though universals do not exist in nature, they do occur in our minds as Abstract Ideas or concepts and that to define a term is to analyse the concept it stands for. These philosophers are called conceptualists.

The predicaments or categories of Aristotle are the heads of a classification of terms as possible predicates of a particular thing or individual. Hamilton has given a classification of them. (detail—see the text). This doctrine has been variously criticised and estimated by various thinkers. Now, if the predicaments are heads of a classification of terms predicable, we may expect to find some connection with the predicable ; and in fact, secondary substances are species and genus, whilst the remaining nine forms are generally accidents. Again there is some agreement between them and the fundamental form of predication. (For this agreement see the text.)

By Kant the word 'category' has been used for the highest forms of Judgment, such as unity, reality etc. This change of meaning has not been made without a certain continuity of thought, for forms of judgment are modes of predication.

EXERCISES WITH HINTS

1. *Define Predicables. What is meant by the classification of attributes into differentia, property and accidents ?*

[Read sec. 8]

2. *Discuss, the relation of definition, division and classification.*

[For the relation of division and classification see secs. 3 & 6 of ch. XXI.

For definition and classification see secs. 5 & 6.

For definition and division :—Definition is the explicit statement of the connotation of a name whereas division is the analysis of the denotation of a name. As there is a very close relation between the denotation and the connotation of a name, so there is a very close relation between definition and division. Division, thus, presupposes definition. If the term to be divided is indefinite, division will not be logical. Indeed, the limits of definition are the limits of division also. Again in so far as, definition is *per genus et differentiam*, it assumed division. Division always makes the definition clear.]

3. *What are the principal errors incidental to definition ?*

[Accidental definition :—If the definition is a statement of an accident, it is accidental e.g. Man is a featherless biped.

Description :—If the definition does not state the connotation of a name it is called a description e.g. Man is a tool using animal.

Too wide definition :—If the definition state less than the connotation of a name and thus includes some that are not its denotation, it is called a too wide definition e.g. A triangle is a figure.

Incomplete definition :—A too wide definition is also called an incomplete or partial definition.

Too narrow definition :—If the definition states more than the connotation of a name and thus excludes some of its denotation, it is called a too narrow definition e.g. a triangle is a plane rectilineal figure having three equal sides.

Obscure and figurative definition :—If the definition is given in obscure and figurative language we have an example of such

a definition: e.g. a net is a reticulated fabric decussated at regular intervals. Bread is the stuff of life, Time is the moving image of eternity.

Redundant, Superfluous, and over-complete definition—when more than the connotation is stated we have such a definition. Here, the statement of something more than the connotation does not involve the exclusion of some of the denotation of a name. For what are stated are either inseparable accidents or propria. *e.g.* A triangle is a plane rectilinear figure bounded by three straight lines and having three angles.

Circular definition :—If a name is defined by its synonym, the definition is circular *e.g.* Life is the sum of vital functions.

Negative definition :—When a term, that is capable of being affirmatively defined is defined negatively, we have such a definition e.g. Man is not a dog.]

4. *What are the rules for testing a definition ?*

[See sec. 4]

N.B. Welton has summed the rules into one : A definition should be *(i)* adequate, precise, and *(ii)* clear, and should not be *(iii)* tautologous or *(iv)* negative.

5. *What is a description ? Is it useful ?*

[To define a name is to state entire connotation of a name. If this is not done, there is no definition. Now, if instead of the connotation of a name, the inseparable accidents of it with or without a few of the properties be stated, we have what is called a description. Description thus technically is a proposition mentioning qualities sufficient to distinguish the things denoted, but not the qualities that enter into the definition e.g. nitrogen is the gas that constitutes 4/5 of the atmosphere.

Description is not scientific, but it is useful. It helps us to identify an object. Again, when it is not possible to give a logical definition we are satisfied with a description. Definitions are usually tense and abstract and consequently very hard for the beginner. Descriptions are not so—and they often help the beginner.]

6. *What is a definition logically ? What are its uses ?*

[Read secs. 3-5]

N.B. It makes our ideas distinct and adequate and so is a preliminary requisite to correct thought. Again, "the business

of Definition is part of the business of discovery. When it has been clearly seen what ought to be our Definition it must be pretty well-known what truth we have to state." (Whewell).]

7. *What are the limits of logical definition* ?

[Read sec. 7.]

8. *How far is it true that the denotation and connotation of a name vary in inverse ratio* ?

[It cannot be said that Connotation and Denotation vary in inverse ratio to each other ; such a mathematical concept is quite inappropriate. We can speak intelligibly of halving or of doubling the denotation of a term, but it is meaningless to talk about doubling or halving its connotation ; and even could we do so, there would be no ratio maintained in the variation of the two aspects of the term—(Welton).

It is a relation that could only hold between classes arranged in a certain order, namely, in which a smaller, or sub-class, is grouped under a wider class, which is grouped under a wider class, and so on. Such an arrangement of classes according to a definite plan is called *classification*. The doctrine of inverse variation results from the attempts of the traditional Logicians to deal with the characteristics of classificatory series—(Stebbing).

The increasing of the connotation of a name and thereby limiting the application of the term is a process of *specialization*, the opposite process of decreasing the connotation so as to embrace a large number of objects is one of Generalization—Welton.)

See also ex. 9.]

9. *What is meant by the connotation of a name*? *Can it be really increased* ?

[The connotation of a name are those qualities and characteristics of the things denoted, by virtue of which it cannot be used literally as the name of any other things. The traditional logicians use the term *intension* instead of connotation. But, the term *intension* does not exactly correspond to the term connotation as used by Mill. The *intension* of a word is what we intend to mean by it ; and so it admits of, as Dr. Keynes has pointed out, three different meaning. Thus, it may stand for conventional or scientific intension *i.e.* the defining attributes of the name. Again, it may stand for subjective or psychological intension i.e. the attributes which the use of a word calls up in the mind of a person using it. It varies from individual to individual. It may

also stand for objective intension or comprehension *i.e.* all these attributes that are in fact possessed in common by all the things denoted. Now, connotation, as Mill understands it, is the conventional or scientific intension of a name. It cannot be changed. To change it is to change the definition of the name. So by increasing or decreasing the connotation of a name, we really get a new name—and the connotation of a name can neither be increased nor decreased.]

10. *What is definition per genus et differentiam? What is the ground of this method of definition? Is it satisfactory?*

[To define a name as to state explicitly the entire connotation of a name. Now, "in unfolding the complete connotation of a name it is often practically impossible to express it in terms which denote simple attributes only ; and, in nearly every case, to do so would make the definition needlessly long and involved. It is, in all cases, allowable to employ terms expressive of groups of attributes. Hence we have the time-honoured rule that definition should be *per genus et differentiam.* In mentioning the genus we use a term which implies all the attributes common to the species whose name is the term to be defined and to all other co-ordinate species of that genus ; and by adding the differentia, we complete the statement of the connotation by giving those attributes which differentiate that species from all such co-ordinate species. In other words, when we have to define a term, we first decide what class of things it belongs to, and then we mark the attribute or group of attributes, which distinguishes it from other members of that class. The name of the class is the genus, the distinguishing attribute or group of attributes, forms the Differentia. The genus selected must be a proximate genus." (Welton). The definition man is a rational animal may be said to be an example of this method of definition; as we may be said to arrive at this definition by stating the proximate genus 'animal' and the differentiam 'rationality.'

The ground of this method of definition is that definitions are not independent, but like classes are relative to one another. (see para 2 of sec. 5)

Some logicians hold that this account of definition is in many ways defective. (1) It is based on classification which does not necessarily express the essential nature of the things classified or defined. It merely enables us to identify an object or a species of objects as distinct from all others. (2) It presupposes the

existence of fixed species or real kinds, for only thus is it possible to give any definite meaning to the essence or essential qualities of the species defined. The essential qualities are just the qualities of the fixed species. There is a definite core of such qualities and all other qualities which the species or the individual members of it may possess are more or less accidental. But we no longer believe in the reality of fixed species. (3) It seems to involve the idea that the qualities of a thing are more or less independent, loose and separate or at least that they are externally related to one another. Scientific definition does not consist in an enumeration of qualities. Its aim is really to show the position of the thing defined in one or another system. System is the basis of definition and the failure of the traditional view is due to its failure to recognise this. (adopted from Latta and Macbeath).]

11. *Examine the following definitions.*

(i) Man is a featherless biped:
(ii) Logic is the science of thought.
(iii) Iron is the cheaper metal.
(iv) Music is expensive noise.
(v) Women are husband hunting animals.
(vi) Man is an animal that is trained by women.
(vii) Love is what the lover feels.
(viii) Mind is a *tabula rasa*.
(ix) Time is the moving image of eternity.
(x) Flower is an object that is loved by the girls and poets.
(xi) Metal is a solid substance.
(xii) Life is the negation of death.
(xiii) Metaphysics is the science of finding out bad reasons for what we believe on instinct.
(xiv) An idle man is the man whose brain is the devil's workshop.
(xv) Porosity is that which is possessed by objects that are porous.
(xvi) Oxygen is a gas that supports combustion.
(xvii) The camel is the ship of the desert.
(xviii) The horse is a noble animal.
(xix) Necessity is the mother of invention.
(xx) Eccentricity is peculiar idiosyncrasy.
(xxi) Fluency is an exuberance of verbosity.

12. *Is classification by type or by Definition?*

[See sec. 6.]

[Dr. Whewell defined the Type of any natural group as "an example which possesses in a marked degree all the leading characters of the class." And according to him "A natural group is determined not by a boundary without, but by a central point within—not by what it strictly excludes, but by what it eminently includes;—by a Type, not by a Definition." But Mill has pointed out that a Type presupposes Definition. Jevons also has said: "The type itself is an individual, not a class, and no other object can exactly be like the type. But as soon as we abstract the individual peculiarities of the type and thus specify a finite number of qualities in which other objects may resemble the type, we immediately constitute a class. The notion of classification by type is in fact erroneous in a logical point of view." Dr. Welton also has pointed out, that this view regards "the connotation as secondary to denotation which is an inversion of the true logical method. We can no more classify by types than we can define by types. We can use the mental image of a type as an illustration of a class, and in this way the conception of a typical example is useful. But it is typical because the idea of the class is already formed; not because it is the one determining factor in that formation."]

CHAPTER XXIII

DEFINITION OF COMMON TERMS

§ 1. THE RIGOUR OF SCIENTIFIC METHOD MUST BE QUALIFIED: Ordinary words may need definition, if in the course of exposition or argument their meaning is liable to be mistaken. But as definition cannot give one the sense of a popular word for all occasions of its use, it is an operation of great delicacy. Fixity of meaning in the use of single words is contrary to the genius of the common vocabulary; since each word, whilst having a certain predominant character, must be used with many shades of significance, in order to express the different thoughts and feelings of multitudes of men in endlessly diversified situations; and its force, whenever it is used, is qualified by the other words with which it is connected in a sentence, by its place in the construction of the sentence, by the emphasis, or by the pitch of its pronunciation compared with the other words.

Fixity of meaning is not always possible.

Clearly, the requisite of a scientific language, 'that every word shall have one meaning well defined,' is too exacting for popular language; because the other chief requisite of scientific language cannot be complied with, 'that there be no important meaning without a name.' 'Important meanings,' or what seem such, are too numerous to be thus provided for; and new ones are constantly arising, as each of us pursues his business or his pleasure, his meditations or the excursions of his fancy. It is impossible to have separate term for each meaning; and, therefore, the terms we have must admit of variable application.

Terms must admit of variable application.

An attempt to introduce new words is generally disgusting. Few men have mastered the uses of half the words already to be found in our classics. Much more would be lost than gained by doubling the dictionary. It is true that, at certain stages in the growth

Introduction of new words: its disadvantages

of a people, a need may be widely felt for the adoption of new words: such, in our own case, was the period of the Tudors and early Stuarts. Many fresh words, chiefly from the Latin, then appeared in books were often received with reprobation and derision, sometimes disappeared again, sometimes established their footing in the language: see *The art of English Poetry* (ascribed to Puttenham), Book III. chap. 4, and Ben Jonson's *Poetaster*, Act. v. sc. 1. Good judges did not know whether a word was really called for: even Shakespeare thought 'remuneration' and 'accommodate' ridiculous. But such national exigencies rarely arise; and in our own time great authors distinguish themselves by the plastic power with which they make common words convey uncommon meanings.

Improper attitude to new words

Fluid, however, as popular language is and ought to be, it may be necessary for the sake of clear exposition, or to steady the course of an argument, to avoid either sophistry or unintentional confusion, that words should be defined and discriminated; and we must discuss the means of doing so.

Confusion should be avoided.

§ 2. Language: Scientific method is applicable, with some qualifications, to the definition of ordinary words. *Classification is involved in any problem of definition: at least, if our object is to find a meaning that shall be generally acceptable and intelligible.* No doubt two disputants may, for their own satisfaction, adopt any arbitrary definition of a word important in their controversy: or any one may define a word as he pleases, at the risk of being misunderstood, provided he has no fraudulent intention. But in exposition or argument addressed to the public, where words are used in some of their ordinary senses, it should be recognised that the meaning of each one involves that many others. For language has grown with the human mind as representing its knowledge of the world: this knowledge consists of the resemblances and

Classification

Classification and definition

Growth of language and irre-

gular classification

differences of things and of the activities of things, that is, of classes and causes; and as there is such order in the world, so there must be in language: language, therefore, embodies an irregular classification of things with their attributes and relations according to our knowledge and beliefs. The best attempt (known to me) to carry out this view is contained in Roget's *Thesaurus*, which is a classification of English words according to their meanings; founded, as the author tells us, on the models of Zoology and Botany, it has some of the requisites of a Logical Dictionary.

Practical purpose of popular language and imperfect classification

Popular language, indeed, having grown up with a predominantly practical purpose, represents a very imperfect classification philosophically considered. Things, or aspects, or processes of things, that have excited little interest, have often gone unnamed: so that scientific discoverers are obliged, for scientific purposes, to invent thousands of new names. Strong interests, on the other hand, give such a colour to language, that, where they enter, it is difficult to find any different expressions. *Consistency* being much prized, though often the part of a blockhead, *inconsistency* implies not merely the absence of the supposed virtue, but a positive vice: *Beauty* being attractive and *ugliness* the reverse, if we invent a word for that which is neither, 'plainness,' it at once becomes tinged with the ugly. We seem to love beauty and morality so much as to be almost incapable of signifying their absence without expressing aversion.

Play of human interests

Influence of erroneous theories on the development of language

Again, the erroneous theories of mankind have often found their way into popular speech, and their terms have remained there long after the rejection of the beliefs they embodied: as—lunatic, augury, divination, spell, exorcism: though, to be sure, such words may often be turned to good account, besides the interest of preserving their original sense. *Language is a record as well as an index of ideas.*

Language, then being essentially classificatory, any attempt to ascertain the meaning of a word, far from neglecting its relations to others should be directed toward elucidating them.

Necessity of elucidation

Every word belongs to a group, and this group to some other larger group. A group is sometimes formed by derivation, at least so far as different meanings are marked merely by inflections as *short*, *shorter*, *shorten*, *shortly*; but, for the most part, is a conflux of words from many different sources. *Repose*, *depose*, *suppose*, *impose*, *propose*, are not nearly connected in meaning; but are severally allied in sense much more closely with words philologically remote. Thus *repose* is allied with *rest*, *sleep*, *tranquillity*; *disturbance*, *unrest*, *tumult*; whilst *depose* is, in one sense, allied with *overthrow*, *dismiss*, *dethrone*; *restore*, *confirm*, *establish*; and, in another sense, with *declare*, *attest*, *swear*, *prove*, *etc.* Groups of words, in fact, depend on their meanings, just as the connection of scientific names follows the resemblance in character of the things denoted.

Words as related to one another

Relation of co-ordination. Nature of Synonyms

Words, accordingly, stand related to one another, for the most part, though very irregularly, as genus, species, and co-ordinate species. Taking *repose* as a genus, we have as species of it, though not exactly co-ordinate with one another, *tranquillity* with a mental differentia (repose of mind), *rest*, whether of mind or body, *sleep*, with the differentia of unconsciousness (privative). Synonyms are species, or varieties, wherever any difference can be detected in them; and to discriminate them we must first find the generic meaning; for which there may, or may not, be a single word. Thus, *equality*, *sameness*, *likeness*, *similarity*, *resemblance*, *identity*, are synonyms; but, if we attend to the ways in which they are actually used, perhaps none of them can claim to be a genus in relation to the rest. If so, we must resort to a compound term for the genus, such as 'absence of some sort of difference.' Then *equality* is ab-

sence of difference in quantity ; *sameness* is often absence of difference in quality, though the usage is not strict : *likeness*, *similarity*, and *resemblance* in their actual use, perhaps, cannot be discriminated ; unless *likeness* be the more concrete, *similarity* the more abstract ; but they may all be used compatibly with the recognition of more or less difference in the things compared, and even imply this. *Identity* is the absence of difference of origin, a continuity of existence, with so much sameness from moment to moment as is compatible with changes in the course of nature ; so that egg, caterpillar, chrysalis, butterfly may be identical for the run of an individual life, in spite of differences quantitative and qualitative, as truly as a shilling that all the time lies in a drawer.

Co-ordinate species

Co-ordinate Species, when positive, have the least contrariety ; but there are also opposites, namely, negatives, contradictories and fuller contraries. These may be regarded as either co-ordinate genera or the species of co-ordinate genera. Thus, *repose* being a genus, *not-repose* is by dichotomy a co-ordinate genus and is a negative and contradictory ; then *activity* (implying an end in view), *motion* (limited to matter), *disturbance* (implying changes from a state of calm), *tumult*, *etc.*, and co-ordinate opposites, or contraries, of the species of *repose*.

Correlative words

As for correlative words, like *master and slave*, *hnsband and wife*, *etc.*, it may seem far-fetched to compare them with the sexes of the same species of plants or animals ; but there is this resemblance between the two cases, that sexual names are correlative, as 'lioness,' and that one sex of a species, like a correlative name, cannot be defined without implying the other ; for if a distinctive attribute of one sex be mentioned (as the lions' mane), it is implied that the other wants it, and apart from this implication the species is not defined : just as the definition of 'master' implies a 'slave' to obey.

Common words, less precise than the terms of a scientific

nomenclature, differ from them also in this that the same word may occur in different genera. Thus, *sleep* is a species of *repose* as above; but it is also a species of *unconsciousness*, with co-ordinate species *swoon*, *hypnotic state etc.* In fact, every word stands under as many distinct genera, at least, as there are simple or indefinable qualities to be enumerated in its definition.

Common words as distinguished from scientific terms

§ 3. IMPERFECT TERMINOLOGY: Partially similar to a scientific nomenclature ordinary language has likewise a terminology for describing things according to their qualities and structure. Such is the function of all the names of colours, sounds, tastes, contrasts of temperature, of hardness, of pleasantness; in short, of all descriptive adjectives, and all names for the parts and processes of things. Any word connoting a quality may be used to describe many very different things, as long as they agree in that quality.

One word may describe many things

But the quality connoted by a word and treated as always the same quality, is often only analogically the same. We speak of a *great* storm, a *great* man, a *great* book; but *great* is in each case not only relative, implying small, and leaving open the possibility that what we call great is still smaller than something else of its kind, but it is also predicated with reference to some quality or qualities, which may be very different in the several cases of its application. If the book is prized for wisdom, or for imagination its greatness lies in that quality; if the man is distinguished for influence, or for courage, his greatness is of that nature; if the storm is remarkable for violence, or for duration, its greatness depends on that fact. The word *great* therefore, is not used for these things in the same sense, but only analogically and elliptically. Similarly with good, pure, free, strong, rich, and so on. 'Rest' has not the same meaning in respect of a stone and of an animal, nor 'strong'

Analogical sameness

Border between

literal and figurative use. in respect of thought and muscle, nor 'sweet' in respect of sugar and music. But here we come to the border between literal and figurative use; everyone sees that figurative epithets are analogical; but by custom any figurative use may become literal.

A concrete thing viewed as universals meeting together Again, many general names of widely different meaning, are brought together in describing any concrete object, as an animal, or a landscape, or in defining any specific term. This is the sense of the doctrine, that any concrete thing is a conflux of generalities or universals: it may at least be considered in this way; though it seems more natural to say, that an object presents these different aspects to a spectator, who, fully to comprehend it, must classify it in every aspect.

§ 4. MAXIMS AND PRECAUTIONS OF DEFINITION: The process of seeking a definition may be guided by the following maxims:

Help of usage (1) *Find the usage of good modern authors*; that is (as they rarely define a word explicitly), consider what in various relations they use it to denote; from which uses its connotation may be collected.

Qualities in which the things denoted agree (2) But if this process yield no satisfactory result, make a list of the things denoted, and of those denoted by the co-ordinate and opposite words; and *observe the qualities in which the things denoted agree, and in which they differ from those denoted by the contraries and opposites.* If 'civilisation' is to be defined, make lists of civilised peoples, of semi-civilised, of barbarous, and of savage: now, what things are common to civilised peoples and wanting in the other respectively? This is an exercise worth attempting. If poetry is to be defined, survey some typical examples of what good critics recognise as poetry, and compare them with examples of bad 'poetry', literary prose, oratory, and science. Having determined the characteristics of

each kind arrange them opposite one another in parallel columns. Whoever tries to define by this method a few important, frequently occuring words, will find his thoughts the clearer for it, and will collect by the way much information which may be more valuable than the definition itself, should he ever find one.

(3) If the genus of a word to be defined is already known, the process may be shortened. Suppose the genus of poetry to be *belles lettres* (that is, 'appealing to good taste'), this suffices to mark it off from science; but since literary prose and oratory are also *balles lettres*, we must still seek the differentia of poetry by a comparison of it with these co-ordinate species. A compound word often exhibits genus and difference upon its face: as 'return', 'interpenetrate,' 'tuning-fork,' 'cricket-bat'; but the two last would hardly be understood without inspection or further description. And however a definition be discovered, it is well to state it *per genus et differentiam*.

Per genus et differentiam when the genus is known,

(4) In defining any term we should avoid encroaching upon the meaning of any of the co-ordinate terms; for else their usefulness is lessened: as by making 'law' include 'custom', or 'wealth' include 'labour' or 'culture'.

avoid encroachment upon the meaning of co-ordinate terms.

(5) *If two or more terms happen to be exactly synonymous, it may be possible (and, if so, it is a service to the language) to divert one of them to any neighbouring meaning that has no determinate expression.* Thus, Wordswoth and Coleridge took great pains to distinguish between Imagination and Fancy, which had become in common usage practically equivalent; and they sought to limit 'imagination' to an order of poetic effect, which (they said) had prevailed during the Elizabethan age, but had been almost lost during the Gallo-classic, and which it was their mission to restore. Co-ordinate terms often

A synonymous term may be diverted to an analogous meaning.

Distinguish betweon co-ordinate and synony-mous terms,

tend to coalesce and become synonymous, or one almost supersedes the other, to the consequent impoverishment of our speech. At present *proposition* (that something is the fact) has almost driven out *proposal* (that it is desirable to co-operate in some action). Even good writers and speakers, by their own practice, encourage this confusion : they submit to Parliament certain 'propositions' (proposals for legislation), or even make 'a proposition of marriage'. Definition should counteract such a tendency.

avoid undue extension of denotation or connotation

(6) *We must avoid the temptation to extend the denotation of a word so far as to diminish or destroy its connotation ; or to increase its connotation so much as to render it no longer applicable to things which it formerly denoted* : we should neither unduly generalise nor unduly specialise, a term. It is desirable to define *education* so as to include the lessons of experience ; or is it better to restrict it as implying a personal educator ? If any word implies blame or praise, we are apt to extend it to everything we hate or approve. But *coward* cannot be so defined as to include all bullies, nor *noble* so as to include every honest man, without some loss in distinctness of thought.

why do we specialise words

The same impulses make us specialise words ; for, if two words express approval we wish to apply both to whatever we admire and to refuse both to whatever displeases us. Thus, a man may resolve to call no one great who is not good : greatness, according to him, connotes goodness : whence it follows that (say) Napoleon I was not great. Another man is disgusted with greatness : according to him, good and great are mutually exclusive classes, sheep and goats, as in Gray's wretched clench : "Beneath the good how far, yet far above the great." In fact, however 'good' and 'great' are descriptive terms sometimes applicable to the same object, sometimes to different : but 'great' is the wider term and applicable to goodness itself and also to badness ; whereas by making 'great' connote goodness it becomes the narrower term. And as we have seen (§ 3), such epithets may be applicable

to objects on account of different qualities : *good* is not predicated on the same ground of a man and of a horse.

Importance origin in defining a word

(7) *In defining any word, it is desirable to bear in mind its derivation and to preserve the connection of meaning with its origin* ; unless there are preponderant reasons for diverting it, grounded on our need of the word to express a certain sense, and the greater difficulty of finding any other word for the same purpose. It is better to lean to the classical than to the vulgar sense of 'indifferent,' 'impertinent,' 'aggravating' 'phenomenal.'

Rigorous definition not to be attempted where it is not possible.

(8) *Rigorous definition should not be attempted where the subject does not admit of it.* Some kinds of things are so complex in their qualities, and each quality may manifest itself in so many degrees without ever admitting of exact measurement, that we have no means of marking them off precisely from other things nearly allied, similarly complex and similarly variable. If so we cannot precisely define their names. Imagination and fancy are of this nature, civilisation and barbarism, poetry and other kinds of literary expression. As to poetry, some think it only exists in metre, but hardly maintain that the metre must be strictly regular : if not, how much irregularity of rhythm is admissible ? Others regard a certain mood of impressioned imagination as the essence of poetry ; but they have never told us how great intensity of this mood is requisite. We also hear that poetry is of such a nature that the enjoyment of it is an end in itself ; but as it is not maintained that poetry must be wholly impersuasive or uninstructive, there seems to be no means of deciding what amount or prominence of persuasion or instruction would transfer the work to the region of oratory or science. Such cases make the method of defining by the aid of a type really useful : the difficulty can hardly be got over without pointing to typical examples of each meaning and admitting that there may be

Definition by type

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many divergences and unclassifiable instances on the border between allied meanings.

§ 5. Words of common language in scientific use: As science began from common knowledge, the terms of the common vocabulary have often been adopted into the sciences, and many are still found there: such as weight, mass, work, attraction, repulsion, diffusion, reflection, absorption, base, salt, and so forth. In the more exact sciences, the vague popular associations with such words are hardly an inconvenience: since those addicted to such studies do not expect to master them without undergoing special discipline; and, having precisely defined the terms, they acquire the habit of thinking with them according to their assigned signification in those investigations to which they are appropriate. It is in the Social Sciences, especially Economics and Ethics, that the use of popular terminology is at once unavoidable and prejudicial. For the subject-matters, industry and the conduct of life, are every man's business; and, accordingly, have always been discussed with a consciousness of their direct practical bearing upon public and private interests, and therefore in the common language, in order that everybody may as far as possible benefit by whatever light can be thrown upon them. The general practice of Economists and Moralists, however, shows that, in their judgment, the good derived from writing in the common vocabulary outweighs the evil: though it is sometimes manifest that they themselves have been misled by extra-scientific meanings. To reduce the evil as much as possible, the following precautions seem reasonable:

Precaution

(1) To try to find and adopt the central meaning of the word (say rent or money) in its current or traditionary applications: so as to lessen in the greater number of cases the jar of conflicting associations. But if the central popular meaning does not correspond with the scientific conception to be expressed, it may be better to invent a new term.

(1) To find and adopt central meaning

(2) To define the term with sufficient accuracy to secure its clear and consistent use for scientific purposes.

(3) When a popular term has to be used in a sense that departs from the ordinary one in such a way as to incur the danger of misunderstanding, to qualify it by some adjunct or "interpretation-clause."

(2) To define term with accuracy

The first of these rules is not always adhered to; and, in the progress of a science, as subtler and more abstract relations are discovered amongst the facts, the meaning of a term may have to be modified and shifted further and further from its popular use. The term 'rent,' for example, is used by economists, in such a sense that they have to begin the discussion of the facts it denotes, by explaining that it does not imply any actual payment by one man to another. Here, for most readers, the meaning they are accustomed to, seems already to have entirely disappeared. Difficulties may, however, be largely overcome by qualifying the term in its various relations, as produce-rents, ground-rents, customary rents, and so forth. (*Cf.* Dr. Keynes' *Scope and Method of Political Economy*, chap. 5)

(3) Use interpretation-clauses where there may be mis-understanding.

Importance of these rules.

§ 6. How Definitions affect the cogency of arguments: Definitions affect the cogency of arguments in many ways, whether we use popular or scientific language. If the definitions of our terms are vague, or are badly abstracted from the facts denoted, all arguments involving these terms are inconclusive. There can be no confidence in reasoning with such terms; since, if vague, there is nothing to protect us from ambiguity; or, if their meaning has been badly abstracted, we may be led into absurdity—as if 'impudence' should be defined in such a way as to confound it with honesty.

Again, it is by definitions that we can best distinguish between Verbal and Real Propositions. Whether a term predicated is implied in

Definitions help us to distinguish be-

tween Verbal and Real propositions

the definition of the subject or adds something to its meaning, deserves our constant attention. We often persuade ourselves that statements are profound and important, when, in fact, they are mere verbal propositions. "It is just to give every man his due"; "the greater good ought to be preferred to the less," such dicta sound well—indeed, too well! For 'a man's due' means nothing else than what it is just to give him; and 'the greater good' may mean the one that ought to be preferred: these, therefore, are Truisms. The investigation of a definition may be a very valuable service to thought; but, once found, there is no merit in repeating it. To put forward verbal or analytic propositions, or truisms, as information (except, of course, in explaining terms to the uninstructed), shows that we are not thinking what we say; for else we must become aware of our own emptiness. Every step forward in knowledge is expressed in a real or synthetic proposition; and it is only by means of such propositions that information can be given (except as to the meaning of words) or that an argument or train of reasoning can make any progress.

Illustrations

Opposed to a truism is a Contradiction in Terms; that is, the denying of a subject something which it connotes (or which belongs to its definition), or the affirming of it something whose absence it connotes (or which is excluded by its definition). A verbal proposition is necessarily true, because it is tautologous; it is inconsistent. Yet, as a rhetorical artifice, or a contradiction in terms is necessarily false, because figure, it may be effective: that 'the slave is not bound to obey his master' may be a way of saying that there ought to be no slaves; that 'property is theft,' is an uncompromising assertion of the communistic ideal. Similarly a truism may have rhetorical value: that 'a Negro is a man' has often been a timely reminder, or even that "a man's a man." It is only when we fall into such contradiction or tau-

Contradiction in terms and how it is distinguished from a verbal proposition

tology by lapse of thought, by not fully understanding our own words, that it becomes absurd.

Real Propositions comprise the predication of Propria and Accidentia. Accidentia, implying a sort of empirical law, can only be established by direct induction. But propria are deduced from (or rather by means of) the definition with the help of real propositions, and this is what is called 'arguing from a Definition.' Thus, if increasing capacity for co-operation be a specific character of civilisation, 'great wealth' may be considered as a proprium of civilised as compared with barbarous nations. For co-operation is made most effectual by the division of labour, and that this is the chief condition of producing wealth is a real proposition. Such arguments from definitions concerning concrete facts and causation require verification by comparing the conclusion with the facts. The verification of this example is easy, if we do not let ourselves be misled in estimating the wealth of barbarians by the ostentatious "pearl and gold" of kings and nobles, where 99 per cent. of the people live in penury and servitude. The wealth of civilisation is not only great but diffused, and in its diffusion its greatness must be estimated.

Nature of real proposition: Propria and Accidentia

To argue from a definition may be a process of several degress of complexity. The simplest case is the establishing of a proprium as the direct conseqnence of some connoted attribute, as in the above example. If the definition has been correctly abstracted from the particulars, the particulars have the atttributes summarised in the definition; and, therefore, they have whatever can be shown to follow from those attributes. But it frequently happens that the argument rests partly on the qualities connoted by the class name and partly on many other facts.

Argument from a definition

In Geometry, the proof of a theorem depends not only upon the definition of the figure or figures directly concerned,

Nature of Geometrical proof and the utility of construction.

but also upon one or more axioms, and upon propria or constructions already established. Thus in Euclid's fifth Proposition, the proof that the angles at the base of an isosceles triangle are equal, depends not only on the equality of the opposite sides, but upon this together with the construction that shows how from the greater of two lines a part may be cut off equal to the less the proof that triangles that can be conceived to coincide are equal, and the axiom that if equals be taken from equals the remainders are equal. Similarly, in Biology, if colouring favourable to concealment is a proprium of carnivorous animals, it is not deducible merely from their predatory character or any other attribute entering into the definition of any species of them, but from their predatory character together with the causes summarised in the phrase 'Natural Selection'; that is, competition for a livelihood, and the destruction of those that labour under any disadvantages, of which conspicuous colouring would be one. The particular coloration of any given species, again, can only be deduced by further considering its habitat (desert, jungle or snowfield); a circumstance lying wholly outside the definition of the species.

The dependence of validity on definition.

Senses in which things corresponding with terms 'exist'.

The validity of an argument based partly or wholly on a definition depends, in the first place, on the existence of things corresponding with the definition—that is, having the properties connoted by the name defined. If there are no such things as isosceles triangles, Euclid's fifth Proposition is only formally true, like a theorem concerning the fourth dimension of space : merely consistent with his other assumptions. But if there be any triangles only approximately isosceles, the proof applies to them, making allowance for their concrete imperfection : the nearer their sides approach straightness and equality the more nearly equal will the opposite angles be.

Again, as to the things corresponding with terms defined, according to Dr. Venn, their 'existence' may be understood in several senses: (1) merely for the reason like the pure genera and species of Porphyry's tree: the sole condition of whose being is logical consistency; or (2) for the imagination, like the giants and magicians of romance, the heroes of tragedy and the fairies of popular superstition; whose properties may be discussed, and verified by appeal to the right documents and authorities (poems and ballads): or (3) for perception, like plants, animals, stones and stars. Only the third class exist in the proper sense of the word. But under a convention or hypothesis of existence, we may argue from the definition of a fairy, or a demigod, or a dragon, and deduce various consequences without absurdity, if we are content with poetic consistency and the authority of myths and romances as the test of truth.

(1) Logical consistency

(2) Appeal to documents and authorities

(3) Perception

In the region of concrete objects, whose properties are causes, and neither merely fictions nor determinations of space (as in Geometry), we meet with another condition of validity of any argument depending on a definition: there must not only be objects corresponding to the definition, but there must be no other causes counteracting those qualities on whose agency our argument relies. Thus, though we may infer from the quality of co-operation connoted by civilisation, that a civilised country will be a wealthy one, this may not be found true of such a country recently devastated by war or other calamity. Nor can co-operation always triumph over disadvantageous circumstances. Scandinavia is so poor in the gifts of nature favourable to industry, that it is not wealthy in spite of civilisation: still, it is far wealthier than it would be in the hands of a barbarous people. In short, when arguing from a definition, we can only infer the *tendency* of any causal

Argument from definition is valid when counter-acting causes are absent

characteristics included in it; the unqualified realisation of such a tendency must depend upon the absence of counteracting causes. As soon as we leave the region of pure conceptions and make any attempt to bring our speculations home to the actual phenomena of nature or of human life, the verification of every inference becomes an unremitting obligation.

SUMMARY

The common vocabulary cannot always prescribe fixity of meaning. But scientific language should be such that every word shall have one meaning well defined. But this is too exacting in popular language. Here it is impossible to have a separate term for each meaning. Terms must therefore admit of variable application.

New words may be often necessary, but it has both advantages and disadvantages. Even good authors are found to have an improper attitude to new words. In short popular language is fluid; but to avoid confusion words should be as far as possible defined and discriminated.

Language grows along with the advancement of human knowledge. But old ideas and irregular classification are often preserved even when they are proved to be unscientific. Human interests have often been responsible for the lack of discrimination in the use of language.

Every word belongs to a group, and this group to some other larger group. Words stand thus related to one another. Such relations as Genus, Species and Co-ordinate Species help us understand the nature of words in a better a way. Since common words may occur in different Genera, they differ from scientific words.

Imperfect terminology is often unavoidable. For example, one word may describe many different things which are only analogically the same. Figurative epithets are analogical and it is by custom that a figurative use becomes liberal.

Many general names are often used and brought together in

describing a concrete thing. A concrete thing is thus regarded as the meeting together of universals.

In framing a definition the following maxims should be observed :—(1) Note the usage of good modern authors. (2) Observe the nature of different things and qualities denoted by a word and its co-ordinate opposite words. A comparison will help us determine the essential meaning of a word. (3) Define the word *per genus et differentiam* when the genus of the word is known. (4) Avoid encroaching upon the meaning of co-ordinate terms. (5) Divert wherever necessary a synonymous term to an analogous meaning. But the distinction between co-ordinate and synonymous terms should be maintained. (6) Avoid undue extension of the denotation or connotation of a term. (7) Enquire into the origin of a word for preserving the connection of meaning. (8) If a regorous definition is not possible, it should not be attempted. Definition by type is all that possible here.

Sometimes common words are used in science. In connection with their scientific use the following precautions will stand us in good stead :—(1) Find and adopt the Central meaning of the word in its current application. (2) with accuracy, (3) To avoid misunderstanding use interpretative clauses.

Definitions affect the cogency of arguments. They help us distinguish between verbal and real propositions. Real propositions comprise the predication of *propria* and *accidentia*. The propria follows from the definition while a verbal proposition is proved to be so by the investigation into the definition of the subject term. The Geometrical proof rests on both construction and definition.

The existence of things corresponding with terms defined, rests on any one of the following grounds :—

(1) Logical consistency of the ideas involved.

(2) Appeal to documents and authorities.

(3) Perception.

EXERCISES WITH HINTS

1. *State and explain some of the maxims of definition.*

[Read sec. 4.]

2. *Do you think that the rigour of scientific method must be qualified? Give reasons.*

[Read sec. 1.]

3. *Show how the cogency of arguments is affected by definitions.*

[Read sec. 6.]

4. *(a) Real propositions comprise the predication of Propria and Accidentia—Discuss.*

(b) It is by definitions that we can best distinguish between verbal and real propositions—Explain.

(c) "As to the things corresponding with terms defined, according to Dr. Venn, their 'existence' may be understood in several senses." What are the differences?

[Read sec. 6.]

5. *It is said that language comprises the nomenclature of an imperfect clssification, to which, every definition is relative. Explain.*

[Read sec. 2.]

6. *What are the marks of an imperfect terminology?*

[Read sec. 3]

CHAPTER XXIV

FALLACIES

§ 1. FALLACY DEFINED AND DIVIDED: A Fallacy is any failure to fulfil the conditions of proof. If we neglect or mistake the conditions of proof unintentionally, whether in our private meditations or in addressing others, it is a Paralogism: but if we endeavour to pass off upon other's evidence or argument which we know or suspect to be unsound, it is a Sophism.

Fallacy, Paralogism and Sophism.

Fallacies, whether paralogisms or sophisms, may be divided into two classes: *(a)* the Formal, or those that can be shown to conflict with one or more of the truths of Logic, whether Deductive or Inductive; as if we attempt to prove an universal affirmative in the Third Figure; or to argue that, as the average expectation of life for males at the age of 20 is $19\frac{1}{2}$ years, therefore Alcibiades, being 20 years of age, will die when he is $39\frac{1}{2}$; *(b)* the Material, or those that cannot be clearly exhibited as transgressions of any logical principle, but are due to superficial inquiry of confused reasoning, as in adopting premises on insufficient authority, or without examining the facts; or in mistaking the point to be proved.

Fallacies Formal and Material

§ 2. FORMAL FALLACIES OF DEDUCTION: Formal Fallacies of Deduction and Induction are, all of them, breaches of the rule 'not to go beyond the evidence. As a detailed account of them would be little else than a repetition of the foregoing chapters, it may suffice to recall some of the places at which it is easiest to go astray.

Formal fallacies of deduction

(1) It is not uncommon to mistake the Contrary for the Contradictory, as—A is not taller than B, ∴ he is shorter,

(1) mistaking the contrary for the contradictory

(2) To convert *A* or *O* simply, as—

All money is Wealth ∴ All Wealth is Money; or—Some Wealth is not Money ∴ Some Money is not Wealth.

(2) Simple Conversion of A or O

In both these cases, Wealth, though undistributed in the convertend, is distributed in the converse.

(3) To attempt to syllogise with two premises containing four terms, as—

(3) four terms

The Papuans are savages;
The Javanese are neighbours of the Papuans:
∴ The Javanese are savages.

Such an argument is excluded by the definition of a Syllogism, and presents no formal evidence whatever. We should naturally assume that any man who advanced it merely meant to raise some probability that 'neighourhood is a sign of community of ideas and customs.' But, if so, he should have been more explicit. There would, of course, be the same failure of conection, if a fourth term were introduced into the conclusion, instead of into the premises.

(4) To distribute in the conclusion a term that was undistributed in the premises (an error essentially the same as (2) above), *i.e.*, Illicit process of the major or minor term, as—

(4) Illicit major

Every rational agent is accountable;
Brutes are not rational agents:
∴ Brutes are not accountable.

In this example (from Whately), an illegitimate mood of Fig. I., the major term, 'accountable' has suffered the illicit process; since, in the premise it is predicate of an affirmative proposition and, therefore, undistributed; but, in the conclusion, it is predicate of a negative proposition and, therefore, distributed. The fact that nearly everybody would accept the conclusion as true, might lead one to overlook the formal inconclusiveness of the proof.

Again,

All men are two-handed;
All two-hundred animals are cooking animals:
∴ All cooking animals are men.

Here we have Bramantip concluding in A.; and there is, formally, an illicit process of the minor; though the conclusion is true; and the evidence, such as it is, is materially adequate. ('Two-handed,' being a peculiar differentia, is nugatory as a middle term, and may be cut out of both premises; whilst 'cooking' is a proprium peculiar to the species Man; so that these terms might be related in U., *All men are all cookers*; whence, by conversation, *All cookers are men.*)

and illicit minor

(5) To omit to distribute the middle term in one or the other premise, as—

(5) undistributed middle

All verbal propositions are self-evident;
All axioms are self-evident:
∴ All axioms are verbal propositions.

This is an illegitimate mood in Fig. II.; in which, to give any conclusion, one premise must be negative. It may serve as a formal illustration of Undistributed Middle; though, as both premises are verbal propositions, it is (materially) not syllogistic at all, but an error of classification; a confounding of co-ordinate species by assuming their identity because they have the generic attribute in common.

(6) To simply convert an hypothetical proposition, as—

(6) Simple conversion of an hypothetical proposition

If trade is free, it prospers;
∴ If trade prospers it is free.

This is similar to the simple conversion of the categorical A.; since it takes for granted that the antecedent is co-extensive with the consequent, or (in other words) that the freedom of trade is the sole condition of, or (at least) inseparable from, its prosperity.

The same assumption is made if, in an hypo-

affirming the

consequent and denying the antecedent thetical syllogism, we try to ground an inference on the affirmation of the consequent or denial of the antecedent, as—

If trade is free it prospers:
It does prosper;
∴ It is free
It is not free;
∴ It does not prosper.

Neither of these arguments is formally good; nor, of course, is either of them materially valid, if it be possible for trade to prosper in spite of protective tariffs.

An important example of this fallacy is the prevalent notion, that if the conclusion of an argument is true the premises must be trustworthy; or, that if the premises are false the conclusion must be erroneous. For, plainly, that—

If the premises are true, the conclusion is true, is a hypothetical proposition; and we argue justly—

The premises are true;
∴ The conclusion is true;
or, The conclusion is false;
∴ The premises are false (or one of them is).

This is valid for every argument that is formally correct; but that we cannot trust the premises on the strength of the conclusion, nor reject the conclusion because the premises are absurd, the following example will show:

All who square the circle are great mathematicians;
Newton squared the circle:
∴ Newton was a great mathematician.

The conclusion is true; but the premises are intolerable.

How the taking of Contraries for Contradictories may vitiate Disjunctive Syllogisms and Dilemmas has been sufficiently explained in the twelfth chapter.

§ 3. Formal Fallacies of Induction: *Formal fallacies of Induction* Formal Fallacies of Induction consist in supposing or inferring Causation without attempting to prove it, or in

pretending to prove it without satisfying the Canons of observation and experiment: as—

(1) To assign the Cause of anything that is not a concrete event: as, *e.g.*, why two circles can touch only in one point. We should give the 'reason'; for this expression includes, besides evidence of causation, the principles of formal deduction, logical and mathematical.

(1) assigning the cause of an event that is not concrete

(2) To argue, as if on inductive grounds, concerning the cause of the Universe as a whole. This may be called the fallacy of transcendent inference: since the Canons are only applicable to instances of events that can be compared; they cannot deal with that which is in its nature unique.

(2) arguing concerning the cause of the whole universe

(3) To mistake co-existent phenomena for cause and effect: as when a man, wearing an amulet and escaping shipwreck, regards the amulet as the cause of his escape. To prove his point, he must either get again into exactly the same circumstances without his amulet, and be drowned—according to the method of Difference; or, shirking the only satisfactory test and putting up with mere Agreement, he must show, (*a*) that all who are ship-wrecked and escape wear amulets, and (*b*) that their cases agree in nothing else; and (*c*), by the Joint Method, that all who are shipwrecked without amulets are drowned. And even if his evidence, according to Agreement, seemed satisfactory at all these points, it would still be fallacious to trust to it as proof of direct causation; since we have seen that unaided observation is never sufficient for this: it is only by experiment in prepared circumstances that we can confidently trace sequence and the transfer of energy.

(3) mistaking co-existence for causation

There is the reverse error of mistaking causal connection for independent co-existence: as if any one regards it as merely a curious coincidence that great rivers generally flow past great towns. In this case,

The riverse error

however, the evidence of connection does not depend merely upon direct Induction.

(4) Post hoc ergo proper hoc:

(4) *Post hoc, ergo propter hoc*: to accept the mere sequence of phenomena, even though often repeated, as proving that the phenomena are cause and effect, or connected by causation. This is a very natural error: for although, the antecedents of a phenomenon being numerous, most of them cannot be its cause, yet it is among them that the cause must be sought. Indeed, if there is neither time nor opportunity for analysis, it may seem better to accept any antecedent as a cause (or, at least, as a sign) of an important event than to go without any guide. And, accordingly, the vast and complicated learning of omens, augury, horoscopy and prophetic dreams, relies upon this maxim; for whatever the origin of such superstitions, a single coincidence in their favour triumphantly confirms them. It is the besetting delusion of everybody who has wishes or prejudices: that is, of all of us at some time or other; for then we are ready to believe without evidence. The fallacy consists in judging off-hand, without any attempt, either by logic or by common sense, to eliminate the irrelevant antecedents; which may include all the most striking and specious.

(5) Regarding co-effects as cause and effect

(5) To regard the Co-Effects (whether simultaneous or successive) of a common cause as standing in the direct relation of cause and effect. Probably no one supposes that the falling of the mercury in his thermometer causes the neighbouring lake to freeze. True, it is the antecedent, and (within a narrow range of experience) may be the invariable antecedent of the formation of ice; but, besides that the two events are so unequal, every one is aware that there is another antecedent, the fall of temperature, which causes both. To justify inductively our belief in causation, the instances compared must agree, or differ, in one circumstance only (besides the effect). The flowing tide is an antecedent of the ebbing tide; it is invariably

so, and is equal to it; but it is not the cause of it: other circumstances are present; and the moon is the chief condition of both flow and ebb. In several instances, States that have grown outrageously luxurious have declined in power: that luxury caused their downfall may seem obvious, and capable of furnishing a moral lesson to the young. Hence other important circumstances are overlooked, such as the institution of slavery, the corruption and rapacity of officials and tax-gatherers, an army too powerful for discipline, any or all of which may be present, and sufficient to explain both the luxury and the ruin.

(6) Mistaking a condition for the whole cause

(6) To mistake one condition of a phenomenon for the whole cause. To speak of an indispensable condition of any phenomenon as the cause of it, may be a mere conventional abbreviation; and in this way such a mode of expression is common not only in popular but also in scientific discussion. Thus we say that a temperature of 33° F. is a cause of the melting of ice, although that ice melts at 33° F., must further depend upon something in the nature of water, for every solid has its own melting-point. As long, then, as we remember that 'cause,' used in this sense, is only a convenient abbreviation, no harm is done, but, if we forget it fallacy may result: as when a man says that the cause of a financial crisis was the raising of the rate of discount, neglecting the other conditions of the market; whereas, in some circumstances, a rise of the Bank-rate may increase public confidence and prevent a crisis.

We have seen that the direct use of the Canons of Agreement and Difference may only enable us to say that a certain antecedent is a cause or an indispensable condition of the phenomenon under investigation. If, therefore, it is important to find the whole cause, we must either experiment directly upon the other conditions, or resort to the Method of Residues and deductive reasoning; nor must we be content, without showing (where such precision is possible) that the alleged cause and the given phenomenon are equal.

(7) Mistaking a single consequence for the whole effect

(7) To mistake a single consequence of a given cause for the whole effect, is a corresponding error, and none so common. Nearly all the mistakes of private conduct and of legislation are due to it: To cure temporary lassitude by a stimulant, and so derange the liver; to establish a new industry by protective duties, and thereby impoverish the rest of the country, to gag the press, and so drive the discontented into conspiracy; to build an alms-house, and thereby attract paupers into the parish, raise the rates, and discourage industry.

(8) Demanding greater exactness in the estimate of causes or effects than a given subjects admits of.

(8) To demand greater exactness in the estimate of causes or effects than a given subject admits of. In the more complex sciences, Biology, Psychology, Sociology, it is often impossible to be confident that all the conditions of a given phenomenon have been assigned, or that all its consequences have been traced. The causes of the origin of species and of the great French Revolution have been carefully investigated, and still we may doubt whether they have all been discovered, or whether their comparative importance has been rightly determined; but it would be very unreasonable to treat those things as miraculous and unintelligible. We read in the *Ethics*, that a properly cultivated mind knows what degree of precision is to be expected in each science. The greatest possible precision is always to be sought; but what is possible depends partly on the nature of the study and partly upon the state of scientific preparation.

(9) Treating an agent or condition remote in time as cause

(9) To treat an agent or condition remote in time as an unconditional cause: for every moment of time gives an opportunity for new combinations of forces and, therefore, for modifications of the effect. Thus, although we often say that Napoleon's Russian expedition was the cause of his downfall, yet the effect was subject to numerous further conditions.

Had the natives not burnt Moscow, had the winter been exceptionally mild, had the Prussians and Austrians not risen against him, the event might have been very different. It is rash to trace the liberties of modern Europe to the battle of Marathon. Indeed, our powers of perception are so unequal to the subtlety of nature, that even in experimental science there is time for molecular changes to occur between what we treat as a cause and the effect as we perceive it; and, in such cases, the strictly unconditional cause has not been discovered.

(10) Neglecting the negative conditions

(10) To neglect the negative conditions to which a cause is subject. When we say that water boils at 212° F., we mean "provided the pressure be the same as that of the atmosphere at about the sea-level"; for under a greater pressure water will not boil at that temperature, whilst under less pressure it boils at a lower temperature. In the usual statement of a law of causation, 'disturbing,' 'frustrating', counteracting' circumstances (that is, negative conditions) are supposed to be absent; so that the strict statement of such a law, whether for a remote cause, or for an immediate cause (when only positive conditions are included), is that the agent or assemblage of conditions, *tends* to produce such an effect, other conditions being favourable, or in the absence of contrary forces.

(11) It is needless to repeat what has already been said of other fallacies that beset inductive proof; such as the neglect of a possible plurality of causes where the effect has been vaguely conceived; the extension of empirical laws beyond adjacent cases; the chief errors to which the estimate of analogies and probabilities, or the application of the principles of classification are liable; and the reliance upon direct Induction where the aid of Deduction may be obtained, or upon observation where experiment may be employed. As to formal fallacies that may be avoided by adhering to the rules of logical method, this may suffice.

§ 4. MATERIAL FALLACIES CLASSIFIED: There remain many

ways in which arguments fall short of a tolerable standard of proof, though they cannot be exhibited as definite breaches of logical principles. Logicians, therefore, might be excused from discussing them; but out of the abundance of their pity for human infirmity they usually describe and label the chief classes of these 'extra-logical fallacies,' and exhibit a few examples.

Extra-logical fallacies

We may adopt Whately's remark, that a fallacy lies either (1) in the premises, or (2) in the conclusion, or (3) in the attempt to connect a conclusion with the premises.

(i) in the premise—bad observation and sham axiom

(1) Now the premises of a sound argument must either be valid deductions, or valid inductions, or particular observations, or axioms. In an unsound argument, then, whose premises are supported by either deduction, or induction, the evidence may be reduced to logical rules; and its failure is therefore a 'logical fallacy' such as we have already discussed. It follows that an extra-logical fallacy of the premises must lie in what cannot be reduced to rules of evidence, that is, in bad observations (§ 5), or sham axioms (§ 6).

(ii) in the conclusion

(2) As to the conclusion, this can only be fallacious if some other conclusion has been substituted for that which was to have been proved (§ 7).

(iii) in the connection between premise and conclusion

(3) Fallacies in the connection between premises and conclusion, if all the propositions are distinctly and explicitly stated, become manifest upon applying the rules of Logic. Fallacies, therefore, which are not thus manifest, and so are extra-logical, must depend upon some sort of slurring, confusion, or ambiguity of thought or speech (§ 8).

First kind, extralogical fallacy lying in

§ 5. Fallacies of Observation: Amongst Fallacies of Observation, Mill distinguishes (1) those of Non-observation, where either instances of the presence or absence of the phenomenon under inves-

tigation, or else some of the circumstances constituting it or attending upon it, though important to the induction, are overlooked. These errors are implied in the Formal Fallacies of Induction already treated of in § 3 (paragraphs (3) to (7)).

premises
(1) *Non-observation*

Mill's class (2) comprises fallacies of Malobservation. Malobservation may be due to obtuseness or slowness of perception; and it is one advantage of the physical sciences as means of education, that the training involved in studying them tends to cure these defects—at least, within their own range.

(2) *Mal-observation*

But the occasion of error upon which Mill most insists, is our proneness to substitute a hasty inference for a just representation of the fact before us ; as when a yachtsman, eager for marvels, sees a line of porpoises and takes them for the sea-serpent. Every one knows what it is to mistake a stranger for a friend, a leaf for a sparrow, one word for another. The wonder is that we are not oftener wrong ; considering how small a part present sensation has in perception, and how much of every object observed is supplied by a sort of automatic judgment. You see something brown, which your perceptive mechanism classes with the appearance of a cow at such a distance ; and instantly all the other properties of a cow are supplied from the resources of former experience : but on getting nearer, it turns out to be a log of wood. It is some protection against such errors to know that we are subject to them ; and the Logician fulfils his duty in warning us accordingly. But the matter belongs essentially to Psychology ; and whoever wishes to pursue it will find a thorough explanation in Prof. Sully's volume on *Illusions*.

Another error is the accumulation of useless, irrelevant observations, from which no proof of the point at issue can be derived. It has been said that an important part of an inductive inquirer's equipment consists in knowing what to observe. The

(3) *Useless and irrelevant observation*

study of any science educates this faculty by showing us what observations have been effective in similar cases; but something depends upon genius. Observation is generally guided by hypotheses: he makes the right observations who can frame the right hypotheses; whilst another overlooks things, or sees them all awry, because he is confused and perverted by wishes, prejudices or other false preconceptions; and still another gropes about blindly, nothing this and docketing that to no purpose, because he has no hypothesis, or one so vague and ill-conceived that it sheds no light upon his path.

Second kind of extralogical fallacy lying in premise

§ 6. Begging the Question: The second kind of extralogical Fallacy lying in the premises, consists in offering as evidence some assertion entirely baseless or nugatory, but expressed in such a way as to seem like a general truth capable of sub-suming the proposition in dispute: it is generally known as *petitio principii*, or begging the question. The question may be begged in three ways:

pititio princippii or begging the question. Its three forms:

(i) Taking senseless or false assertions is self evident.

(1) There are what Mill calls Fallacies *a priori*, mere assertions, pretending to be self-evident, and often sincerely accepted as such by the author and some infatuated disciples, but in which the cool spectator sees either no sense at all or palpable falsity. These sham axioms are numerous; and probably every one is familiar with the following examples: That circular motion is the most perfect; That every body strives toward its natural place; That like cures like; That every bane has its antidote; That what is true of our conceptions is true of Nature; That pleasure is nothing but relief from pain; That the good, the beautiful and the true are the same thing; That, in trade, whatever is somewhere gained is somewhere lost; That only in agriculture does nature assist man; That a man may do what he will with his own; That some men are naturally born to

rule and others to obey. Some of these doctrines are specious enough; whilst, as to others, how they could ever have been entertained arouses a wonder that can only be allayed by a lengthy historical and psychological disquisition.

(ii) offering verbal propositions is proof of some matter of fact

(2) Verbal propositions offered as proof of some matter of fact. These have, indeed, one attribute of axioms, they are self-evident to any one who knows the language; but as they only dissect the meaning of words, nothing but the meaning of words can be inferred from them. If anything further is arrived at, it must be by the help of real propositions. How common is such an argument as this: 'Lying is wrong, because it is vicious'—the implied major premise being that 'what is vicious is wrong'. All three propositions are verbal, and we merely learn from them that lying is *called* vicious and wrong; and to make that knowledge deterrent, it must be supplemented by a further premise, that 'whatever is called wrong ought to be avoided.' This is a real proposition; but it is much more difficult to prove it than 'that lying ought to be avoided.' Still, such arguments, though bad Logic, often have a rhetorical force: to call lying not only wrong but vicious, may be dissuasive by accumulating associations of shame and ignomity.

Definitions, being the most important of verbal propositions (since they imply the possibility of as many other verbal propositions as there are defining attributes and combinations of them), need to be watched with special care. If two disputants define the same word in different ways, with each of the different attributes included in their several definitions they may bring in a fresh set of real propositions as to the agency or normal connection of that attribute. Hence their conclusions about the things denoted by the word defined, diverge in all directions and to any extent. And it is generally felt that a man who is allowed to define his terms as he pleases, may prove anything to those who, through ignorance or inadvertence, grant that those

terms stand for have the attributes that figure in his definitions.

(3) *Circulus in demonstrando*, the pretence of giving a reason for an assertion, whilst in fact only repeating the assertion itself —generally in other words. In such cases the original proposition is, perhaps, really regarded as self-evident, but by force of habit a man says 'because'; and then, after vainly fumbling in his empty pocket for the coin of reason, the habit of symbolic thinking in words only, without reference to the facts, comes to his rescue, and he ends with a paraphrase of the assertion. Thus a man may try to prove the necessity of Causation: 'Every event must have a cause; because an event is a change of phenomena, and this implies a transformation of something pre-existing; which can only have been possible, if there were forces in operation capable of transforming it.' Or, again: 'We ought not to go to war, because it is wrong to shed blood.' But, plainly, if war did not imply bloodshed, the unlawfulness of this could be nothing against war. The more serious any matter is, the more important it becomes either to reason thoroughly about it, or to content ourselves with wholesome assertions. How many 'arguments' are superfluous!

(iii) Arguing 'in a circle

§ 7. SURREPTITIOUS CONCLUSION: The Fallacy of surreptitious conclusion (*ignoratio elenchi*), the mistaking or obscuring of the proposition really at issue, whilst proving something else instead. This may be done by substituting a particular proposition for an universal, or an universal for a particular. Thus, he who attacks the practice of giving in charity must not be content to show that it has, in this or that case, degraded the recipient; who may have been cxceptionally weak. Or, again, to dissuade another form giving alms in a praticular case, it is not enough to show that the general tendency of alms giving is injurious; for, by taking pains in a particular case, the general tendency may often be counteracted.

Ignoratio elenchi

Sometimes an argument establishing a wholly irrelevant conclusion is substituted for an *argumentum ad rem.* Macanlay complains of those apologists for Charles I, who try to defend him as a king, by urging that he was a good judge of paintings and indulgent to his wife.

Argumentum ad hominem

To this class of Fallacies belongs the *argumentum ad hominem*, which consists in showing not that a certain proposition is true, but that Critics ought to accept it in consistency with his other opinions. Thus: 'In every parish the cost of education ought to be paid out of the rates: you, at least, have said that there can be no sound economy, unless local expenses are defrayed from local funds.' But whether this is a fallacy depends, as Whately observes, upon whether it is urged as actually proving the point at issue, or merely as convicting the opponent of inconsistency. In the latter case, the argument is quite fair: whatever such a conclusion may be worth.

Argumentum ad populum

Similarly with the *argumentum ad populum*: 'this measure is favourable to such or such a class; let them vote for it.' An appeal to private greed, however base, is not fallacious, as long as the interest of the class is not *fraudulently* substituted for the good of the nation. And much the same may be said for the *argumentum ad verecundiam.* When a question of morals is debated as a question of honour among thieves, there is no fallacy, if the moral issue is frankly repudiated. The argument from authority is often brought under this head: 'such is the opinion of Aristotle.' Although this does not establish the truth of any proposition, it may be fairly argued as a reason for not hastily adopting a contrary conclusion: that is, if the subject under discussion be one as to which Aristotle (or whoever the authority may be) had materials for forming a judgment.

A negative use of this fallacy is very common. Some general doctrine, such as Positivism, Transcendentalism, Utilitarianism, or Darwinism, is held in common by a group of men; who,

however, all judge independently, and therefore are likely to differ in details. An opponent exhibits their differences of opinion, and thereupon pretends to have refuted the theory they agree in supporting. This is an *argumentum ad scholam*, and pushes too far the demand for consistency. In fact it recoils upon the sophist; for there is no sense in quoting men against one another, unless both (or all) are acknowledged to speak with the authority of learning and judgment, and therefore the general doctrine which the hold in common is the more confirmed.

Argumentum ad scholam

This is an example of the paralogism of 'proving too much'; when a disputant is so eager to refute an opponent as to lay down, or imply, principles from which an easy inference destroys his own position. To appeal to a principle of greater sweep than the occasion requires may easily open the way to this pitfall: as if a man should urge that 'all men are liars,' as the premise of an argument designed to show that another's assertion is less credible than his own.

The paralogism of proving too much

A common form of *ignoratio elenchi* is that which Whately called the 'fallacy of objections': namely, to lay stress upon all the considerations against any doctrine or proposal, without any attempt to weigh them against the cansiderations in its favour; amongst which should be reckoned all the considerations that tell against the alternative doctrines or proposals. Incontestable demonstration can rarely be expected even in science, outside of the Mathematics; and in practical affairs, as Butler says, 'probability is the very guide of life'; so that every conclusion depends upon the balance of evidence, and to allow weight to only a part of it is an evasion of the right issue.

Ignoratio elenchi

§ 8. Ambiguity: Fallacies in the connection of premises and conclusion, that cannot be detected by reducing the arguments to syllogistic form, must depend upon some juggling with language to dis-

Fallacies of ambiguity or accident

guise their incoherence. They may be generally described as Fallacies of Ambiguity, whether they turn upon the use of the same word in different senses, or upon ellipsis. Thus it may be argued that all works written in a classical language are classical, and that, therefore, the history of Philosophy by Diogenes Lærtius, being written in Greek, is a classic. Such ambiguities are sometimes serious enough; sometimes are little better than jokes. For jokes, as Whately observes, are often fallacies; and considered as a propœdeutic to the art of sophistry, punning deserves the ignominy that has overtaken it.

Fallacies of ellipsis usually go by learned names, as; (1) *a dicto secundum quid ad dictum simpliciter.* It has been argued that since, according to Ricardo, the value of goods depends solely upon the quantity of labour necessary to produce them, the labourers who are employed upon (say) cotton cloth ought to receive as wages the whole price derived from its sale, leaving nothing for interest upon capital. Ricardo, however, explained that by 'the quantity of labour necessary to produce goods' he meant not only what is immediately applied to them, but also the labour bestowed upon the implements and buildings with which the immediate labour is assisted. Now these buildings and implements are capital, the labour which produced them was paid for, and it was far enough from Ricardo's mind to suppose that the capital which assists present labour upon (say) cotton cloth has no claim to remuneration out of the price of it. In this argument, then, the word labour in the premise is used *secundum quid,* that is, with the suppressed qualification of including past as well as present lobour; but in the conclusion labour is used *simpliciter* to mean present labour only.

Three forms: (i) from a special cause to a geueral case

(2) *A dicto secundum quid ad dictum secundum alterum quid.* It may be urged that, since the tax on tea is uniform, therefore all consumers contribute equally to the revenue for their enjoyment

(ii) from one special case to another special case

of it. But written out fairly this argument runs thus: Since tea is taxed uniformly 4*d. per lb.*, all consumers pay equally for their enjoyment of it *whatever quality they use.* These qualifications introduced, no body can be deceived.

(iii) from a general rule to a special case

(3) *A dicto simpliciter ad dictum secundum quid,* also called *fallacia accidentis.* Thus: To take interest upon a loan is perfectly just, therefore, I do right to exact it from my own father in distress. The popular answer to this sort of blunder is that 'circumstances alter cases.' We commit this error in supposing that what is true of the average is likely to be true of each case; as if one should say: 'The offices are ready to insure my house [with thousands of others] against fire at a rate per annum which will leave them heavy losers unless it lasts a hundred years; so, as we are told not to take long views of life, I shall not insure.'

Fallacy of Division and Composition

The Fallacy of Division and composition consists in suggesting, or assuming, that what is true of things severally denoted by a term is true of them taken together. That every man is mortal is generally admitted, but we cannot infer that, therefore, the human race will become extinct. That the remote prospects of the race are tragic may be plausibly argued, but not from that premise.

Fallacy changing the premise

Changing the Premises is a fallacy usually placed in this division; although, instead oi disguising different meanings under similar words, it generally consists in using words or phrases ostensibly differing, as if they were equivalent: those addressed being expected to renounce their right to reduce the argument to strict forms of proof, as needless pedantry in dealing with an author so palpably straingthforward. If an orator says—'Nepoleon conquered Europe; in other words, he murdered five millions of his fellow creatures'—and is allowed to go on, he may infer from the latter of these propositions many things

which the former of them would hardly have covered. This is a short of hyperbole, and there is a corresponding meiosis, as: 'Mill *admits* that the Syllogism is useful'; when, in fact, that is Mill's *contention*. It may be supposed that, if a man be fool enough to be imposed upon by such transparent colours, it serves him right; but this harsh judgment will not be urged by any one who knows and considers the weaker brethren.

§ 9. Fallacies, a natural rank growth of the Human mind, not easy to classify, or exterminate: The above classification of Fallacies is a rearrangement of the plans adopted by Whately and Mill. But Fallacies resemble other spontaneous natural growths in not submitting to precise and definite classification. The same blunders, looked at from different points of view may seem to belong to different groups. Thus, the example given above to illustrate *fallacia accidentis*, 'that, since it is just to take interest, it is right to exact it from one's own father,' may also be regarded as *petitio principii*, if we consider the unconditional statement of the premise—'to take interest upon a loan is perfectly just'; for, surely, this is only conditionally true. Or, again, the first example given of a simple ambiguity—'that whatever is written in a classical language is classical, *etc.*,' may, if we attend merely to the major premise, be treated as a bad generalisation, an undue extension of an inference, founded upon a simple enumeration of the first few Greek and Latin works that one happened to remember.

It must also be acknowledged that genuine wild fallacies, roaming the jungle of controversy, are not so easily detected or evaded as specimens seem to be when exhibited in a Logician's collection; where one surveys them without fear, like a child at a menagerie. To assume the succinct mode of statement that is most convenient for refutation, is not the natural habit of these things. But to give reality to his account of fallacies an author needs a large space, that he may quote no inconsiderable part of literature ancient and modern.

As to the means of avoiding fallacies, a general increase of

sincerity and candour amongst mankind may be freely recommended. With more honesty there would be fewer bad arguments; but there is such a thing as well-meaning incapacity that gates unaffectedly fogged in converting A., and regards the refactoriness of O., as more than flesh and blood can endure. Mere indulgence in figurative language, again, is a besetting snare. "One of the fathers, in great severity called poesy *vinum dæmonum*," says Bacon: himself too fanciful for a philosopher. Surely, to use a simple for the discovery of truth is like studying beauty in the bowl of a spoon.

The study of the natural sciences trains and confirms the mind in a habit of good reasoning, which is the surest preservative against paralogism, as long as the terms in use are, like those of science, well defined; and where they are ill defined, so that it is necessary to guard against ambiguity, a thorough training in politics or metaphysics may be useful. Logic seems to me to serve, in some measure, both these purposes. The conduct of business, or experience, a sufficient time being granted, is indeed the best teacher, but also the most austere and expensive. In the seventeenth century some of the greatest philosophers wrote *de intellectus emendatione*; and if their successors have given over this very practical inquiry, the cause of its abandonment is not success and satiety but despair. Perhaps the right mind is not to be made by instruction, but can only be bred: a slow, haphazard process; and meanwhile the rogue of a sophist may count on a steady supply of dupes to amuse the tedium of many an age.

SUMMARY

A fallacy is a failure to fulfil the conditions of proof. If we neglect or mistake the conditions of proof unintentionally, it is a Paralogism ; but if we do this intentionally, it is a Sophism.

Fallacies may be devided into two classes : (a) the Formal and (b) the Material. Formal fallacies of Deduction and Induction are breaches of the rule not to go beyond the evidence.

"For the Important formal fallacies of Deduction and Induction, and other fallacies consult marginal notes in the text. See also fallacy at a glance."

NOTES ON : *Ignoratio Elenchi, Post hoc ergo propter hoc, circulus in demonstrando, non-observation, mal-observation and Non causa pro causa.*

We often try to refute an opponent by showing that his contention leads to absurdity. This way of refuting is known as *reductio ad absurdum* ; and the fallacy of *Non causa Pro causa* "is incident to the *reductio ad absurdum*. That argument disproves a thesis by showing that the assumption of its truth leads to absurd or impossible consequences, or proves one by showing the same for the assumption of its falsity. In *False cause*, the thesis alleged to be discredited is not really responsible for the absurd or impossible consequences, which would follow equally from other premises, whether that were affirmed or denied. It is ridiculous to suppose that the world can be flat ; for a flat world would be infinite and an infinite world could not be circumnavigated, as this has been" (Joseph).

"It is often identified with *post hoc, ergo propter hoc.* ...... But the locus *post hoc, ergo propter hoc* is not quite the same as that of *Non causa pro causa* : in other words, the type is a little different. In *False Cause* we are dealing with the logical sequence of premises and conclusion ; the fallacy lies in connecting the conclusion with a particular premise which might, so far as getting the conclusion is concerned, have been equally well included or omitted ; and because the conclusion is false, we erroneously infer this premise to be false also. In *post hoc, ergo propter hoc* we are dealing with the temporal relation of cause and effect ; the fallacy lies in connecting the effect with a particular event

which might equally well have happened or not happened, so far as the effect in question is concerned ; and we erroneously suppose that the effect, which did occur, occured because of that event." (*Ibid.*)

EXERCISES WITH HINTS

What is a fallacy ? How do you distinguish between Paralogism and Sophism ?

Test the following and discuss the methods employed, if any.

C. U. 1949.

(*a*) A bell struck in vacuo gives no sound. Therefore air is the medium of sound.

[It is observed under laboratory conditions that when there is air there is sound, and when there is no air, (i.e. in vacuo) there is no sound. 'And from this the conclusion is drawn that air is the medium of sound. There are two instances here—in one of which sound occurs and in the other of which it does not occur, and the instances have every other circumstances in common save air. It is a case of the correct employment of the Method of Difference. The conclusion is valid.]

(*b*) A body expands more and more as it is heated more and more. Therefore heat is the cause of expansion.

[It is observed under laboratory conditions that the more heat is applied, the more the body expands—and from this it is concluded that heat is the cause of the expansion of the bodies. This is a case of the correct employment of the method of concomitant variation. The conclusion is valid.]

(*c*) Both mosquitoes and cases of malaria have, in some parts of Bengal much rarer after the swamps of the area have been filled up. Therefore mosquitoes are the cause of malaria.

[It is observed in a large number of cases that where there are swamps there are mosquitoes and there is malaria, and that

where there are no swamps, there are no mosquitoes and there is no malaria. It has also been observed, that more swamps more mosquitoes and more malaria, and that less swamps, less mosquitoes and less malaria. On the basis of these observations it is inferred that mosquitoes cause malaria. Here the joint method and the method of Concomitant Variation have been employed. But as here reliance has been put on observation the conclusion is probable. The degree of probability is indeed very high.]

(*d*) Once an athelate weighed on his chest a load of 30 maunds of steel. A person who witnessed the performance concluded that the athelets was a great magician.

[The person witnessing performance concluded in this way only because he had the hypothesis that he who can weigh on his chest a load of 30 maunds is a magician. Now then hypothesis is the result of the simple conversion of A proposition. All magicians can weigh a load of 30 maunds on their chest. This is a hypothesis that is disproved by facts. And so the conclusion of the man is erroneous.]

C. U. 1948

Test the following and discuss the Methods employed, if there be any.

(*a*) In an instrument there is an iron ring and an iron ball, such that the ball can pass through the ring. Now the ball is heated to the highest degree. It is now found that the ball does not pass through the ring. Heat therefore must have expanded the ball.

[It has been observed under laboratory conditions that, in an instance the ball passes through the ring and in another instance it does not so pass, and that, the two instances have every other circumstance in common save one viz. heat of the highest degree. And from this it has been concluded that heat is the cause of the expansion of the ball. It is a case of the

correct application of the Method of Difference. The conclusion is valid.]

(*b*) Ram often suffers from cold in winter. He finds that he catches cold, if he gets out for a walk in the morning and that he does not catch cold if he keeps indoors.

[The conclusion here has been kept suppressed and it is his going out for a walk in the morning is a cause or condition of Ram's suffering from cold in winter. This conclusion is arrived at by the application of the Joint Method, i.e. after observing in a large number of cases that while going out is accompanied by an attack of cold, keeping indoors is not. As there, only pure observation is made use of, the conclusion is probable.]

(*c*) A number of persons died from snake-bite. Therefore snakes are poisonous.

[Observation that in a large number of cases of snake-bite is followed by death, it is concluded that all snakes are poisonous. It is a case of Induction by simple Enumeration. Here those cases of snake-bites that are not followed by death have not been observed. So it involves the fallacy of illicit generalisation and implies non-ovservation.]

(*d*) As the temperature of the body increases, the mercury in the thermometer rises. Heat is therefore the cause of the expansion of the mercury.

[It is observed that whenever the temperature of the body varies in any manner i.e. increases, mercury, another phenomenon also varies in some particular manner viz. rises. And from this it is concluded that heat is the cause of the expansion of the mercury. It is a case of the correct application of the Method of Concomitant Variation under experimental conditions. The conclusion is valid.]

(*e*) Rabi is a fickle-minded boy. One day, when he was walking by the river side near his house he perceived a snake at a distance in front of him. He however gathered courage and stepped forward to look more closely. He found to his surprise that there was no snake but only a piece of rope.

[Rabi had mal-observation, and this was corrected.]

C. U. 1947

Test the following and discuss the methods employed :—

(*a*) A B C and D live in their respective flats on the same premise, A occupying the top-most flat, and D the ground-floor flat. It is known to B C and D that A is in the habit of throwing all sorts of rubbish on the lower flat. One day C quarrels with D over a matter. After some time D sees from inside the room some rubbish fall down to his yard. Now D concludes that it is C who has thrown the rubbish to harass him.

[D, so concludes as he uses the Method of Difference outside the laboratory as follows :—no quarrel with D, no rubbish on the yard and quarrel with D and rubbish on the yard. ∴ D has thrown the rubbish. He concludes thus also because he forgets that D is in the habit of throwing such rubbish. His conclusion therefore involves the fallacy of *post hoc ergo propter hoc.*]

(*b*) A child was suffering from some pimples on the face. The father gave it a dose of 'Arnica' (homœopathic medicine) and the mother made it take a sufficient quantity of butter-milk for some days. And the pimples healed up. Now, both the father and the mother claimed full credit for cure.

[Both the father and the mother used the Method of Difference outside the laboratory, and committed the fallacy of *post hoc ergo propter hoc.*]

(*c*) Hari buys a plot of land which is said to be haunted. After sometime his son dies. Hari is now firmly convinced that the buying of that plot of land is the cause of his son's death. [*Post hoc ergo propter hoc.*]

(*d*) A and B are travelling in the same tram-car and are sitting side by side. After sometime A gets down. Now B finds that his moneybag is missing. B concludes that A must have stolen his bag and slipped away.

[*Post hoc ergo propter hoc.*]

(*e*) A certain football team is always found to win when it is captained by Mr. X although other players

frequently change. The team was defeated on several occasions when Mr. X was absent. It was therefore concluded that Mr. X's presence is the cause of the team's success.

[It is observed that many instances in which a phenomenon viz. victory of the team happens, have only one other antecedent circumstance viz. the captaincy of X, in common, while many instances in which it does happen i.e. the team is defeated have nothing else in common save the absence of that circumstance viz. the captaincy of X. And from this it is concluded that Mr. X's presence is the cause of the team's success. It is a case of the application of the Joint Method. The conclusion is probable.]

(*f*) Vaccination is believed to be a protection against a small-pox. But in times of epidemic vaccinated people are sometimes found to die of small-pox. Therefore vaccination is of doubtful value.

[Here the hypothesis, that vaccination is a protection against small-pox is put to test under experimental conditions. The deduction involved here is that if the hypothesis is true no vaccinated person should die of small-pox. But this is not true. For a person improperly vaccinated or already infected may die and this does not affect the truth of the hypothesis. The correct deduction is if the hypothesis is true, a person not improperly vaccinated or not already infected should not die.]

C. U. 1946

(*a*) The war-leaders of U.S.A., Italy, Germany, and Great Britain have been replaced by new leaders; hence Stalin will also be replaced.

[Bad analogy—observing that Stalin resembles the war-leaders of U.S.A., etc. in some respects it is concluded that he will resemble them in other respects also i.e. he will be replaced. The relation between the implying and the implied properties is not necessary, and so it is a case of bad analogy.]

(*b*) The excision of the thyroid gland dulls the intellect; hence the thyroid gland is the cause of our intelligence.

[Method of difference—a condition is taken from the whole cause.]

(*c*) A showman announced that children of both sexes were admitted free and then charged for admission to boys and girls on the plea that neither of them were children of both sexes.

[Fallacy of Division—The name both sexes is used in the first place collectively and then it is used distributively.]

(*d*) Paris did nothing wrong in carrying off Helen with her consent for her father left her free to choose her husband.

[Fallacy of accident—arguing from one special case to another. Helen was free to choose her husband before and not after marriage.]

(*e*) All punishment is degrading; for if it is justified, the offender is a brute; if it is not justified, the brutality is in the person who inflicts it.

["This reasoning appears to travel in a circle. Punishment they say is degrading, therefore it can work no moral improvement. But this begs the question. For if punishment could work a moral improvement it would not degrade but elivate. The humanitarian argument alternately proves that punishment can only intimidate because it is brutalising and that it is brutalising because it can only intimidate." (McTaggart)]

(*f*) As the fury of the storm increased, the pilot increased the speed of the plane; hence the storm is the cause of the aeroplane's speed.

[Method of Concomitant Variation—A remote condition is taken to be the cause.]

(*g*) The fittest survive; hence those who have survived the war are the best.

[Fallacy of accident—arguing from a general case to a special case.]

C. U. 1945

(*a*) All the crows I have seen are black, hence all crows are black.

[Induction by simple enumeration—Fallacy of illicit generalisation.]

(*b*) A comet was seen shortly before the outbreak of the war. It was therefore, if not the cause of the war, at least a heavenly messenger to proclaim its approach.

[Method of Difference—Fallacy of *post hoc ergo propter hoc.*]

(*c*) There has been an increase in the number of convictions for crime. There has therefore been an increase in criminality.

[This argument rests upon the assumption that the number of convictions for crime is a true index of the number of crime. But this assumption implies non-observation, for if the crimes are not detected conviction cannot take place. So the fact that there has been an increase in the number of convictions for crime does not prove that there has been an increase in criminality. For it may be due to the increased vigilance of the Police or more strict application of the Law.]

(*d*) The sun must move round the earth, for we have seen it rise and set.

[Malobservation.]

(*e*) If justice consists in keeping property safe, the just man must be a kind of thief, for the same kind of skill which enables a man to defend property also enable him to steal it.

[Bad analogy—From the fact that just man resembles the thief in some points viz., his knowledge of keeping of the property safe it is concluded that he resembles the thief in further points viz., in stealing it. But the relation between the implying property and the implied property are not the intimate one. For the character of a person is dependent not on his knowledge but on his actions.]

(*f*) Tariff walls hinder trade for a wall is always an obstacle to communication.

[Fallacy of ambiguity—To the metaphorical use of the word 'wall' the conclusion is due.]

(*g*) The dull colours of most female birds are due to protective colouration, for during incubation they are liable to attack. The female birds which have bright colours nest in holes in the ground or trees in a way that completely conceals the sitting bird.

[Joint method—It has been observed in a number of cases that those female birds who sit in the open, i.e. protect themselves have dull colour. It has also been observed that those female birds who nest in holes in the ground or trees in a way that completely conceals the sitting birds i.e. do not protect themselves have no dull colour i.e. have bright colours. So there is an agreement in presence and absence between self protection and dull colour. As the agrument relies on mere observation it is probable and the degree of probability very high.]

C. U. 1944

(*a*) The fall of mercury in the thermometer is followed by the freezing of the lake and is therefore its cause.

(*b*) Charles I was a good judge of paintings and indulgent to his wife. The English people therefore acted wrongly in revolting against him.

(*c*) Victory depends on superiority in shooting, for by superior archery the Parthians checked the Romans, Tiberious overcame Arminius, the Turks established their empire and the English defeated the French.

(*d*) The usefulness of their colouration to animals is shown by the fact that colour and marking are constant in each species of wild animals while there is great variation in colour among domestic animals.

(*e*) The sense of smell in flies and cockroaches is connected with the antennæ, for they can no longer find carrion if the antennæ are cut off.

[Method of Difference—valid.]

(*f*) Lack of education is the cause of crime, for the increase in education in the last fifty years has been accompanied by a decrease in crime.

[Method of Concomitant Variation. Remote condition taken to be the cause—Education—employment—decrease in crime.]

(*g*) Nepoleon's Russian expedition led to his downfall. Therefore Hitler's Russian expedition must lead to his downfall.

[Bad analogy.]

C. U. 1943

(*a*) Mosquitoes cause Malaria, because both mosquitoes and cases of malarial fever have become much rarer in parts of Italy and West Africa and elsewhere after these districts were well-drained.

(*b*) There is no such thing as colour inhering in external bodies because colours are more or less vivid in proportion to the light and if there be no light, then no colours are perceived.

[Method of Concomitant Variation and Difference. The conclusion rests upon the failure to distinguish between the existence of colours and the perception of colour. What the premises really conclude is that, perception of colours depends upon light. But it has been concluded here that existence of colours depends on light. So here we have the fallacy of Surreptitious Conclusion (*Ignoratio Elenchi*) the mistaking or obscuring the issue and proving the something else instead.]

(*c*) Everything that grows must also decay. Hence the British Empire must also fall by the hand of time.

[Bad analogy.]

(*d*) The earth cannot be round; for if it were, the water in the Suez Canal would flow out at both ends.

[Bad analogy—The earth is compared with ordinary round object. Then it is held that as water flows out at both ends when placed on a round object it should behave similarly on earth, i.e., the water in the Suez Canal flow out at both ends. But the earth is an ordinary round object. The water does not flow out on account of the law of gravitation. And this has not

been observed while this negative analogy has been propounded.]

(*e*) The low money-price of goods is not a proof of poverty, for China is a richer country than any part of Europe and yet gold has a higher purchasing power in China than in Europe.

[Accident—ambiguity of the term richer country—China is a agriculturally richer country while the European countries are industrially richer.]

(*f*) The object of war is durable peace; therefore soldiers are the best peace-makers.

[*Ignoratio Elenchi*—Even it is assumed that the objeet of war is durable peace it does not follow from this that war is the best way of establishing peace and that the soldiers are the best peace-makers.]

C. U. 1942

(*a*) It is ridiculous to suppose that the world can be flat; for a flat world would be infinite and infinite world could not be circumnavigated.

[Fallacy of *non causa pro causa*.]

["Here the supposition inconsistent with the fact of circumnavigation is not that the world is flat, but that it is infinite; it might be flat and still circumnavigable if it were found; the thesis of its flatness is therefore unfairly discredited."—(Joseph.)]

(*b*) Like the shark, the whale is a Vertebrate. Both are large marine animals, have wide mouths and feed only on living animal nutriment. But they are fishlike in form and have no hairy covering. Therefore like the shark the whale also breathes oxygen dissolved in the water and has no need to be supplied with atmospheric air.

[Bad analogy.]

(*c*) All scientific statements should be capable of proof. But it is impossible to prove everything, for that

would involve an infinite regress. Hence science is impossible.

[The conclusion that science is impossible rests on the assumption that like the scientific statement the postulates of science also are to be proved in the same way. But this assumption is illegitimate. Hence the agrument commits the Fallacy of undue assumption.]

(*d*) Consciousness is unaccountable on a purely physical or mechanical theory of the world. Hence, instead of attempting to explain consciousness in terms of physical law, we must find in physical law a manifestation of intelligence.

[The conclusion is due to the assumption that a philosophy must be monistic. That is, it holds that the philosophical explanation should be in terms of either consciousness or of matter. Either we should explain consciousness in terms of matter or we should explain matter in terms of consciousness. But this assumption may be challenged and so the argument involves the fallacy of undue assumption.]

(*e*) Epilepsy appears in animals born of parents which were rendered epileptic by an injury to the spinal cord. Hence acquired characters are transmitted to the offspring.

[Induction by simple enumeration—Fallacy of illicit generalisation.]

(*f*) The faster I run, the hotter I get. Therefore running makes a man hot.

C. U. 1941

(*a*) Sir D. Brewster proved that the colours seen upon the mother-of-pearls are not caused by the nature of the substance, but by the form of the surface. He took impressions of mother-of-pearls in wax, and found that though the substance was entirely different, the colours were exactly the same.

[Methods of agreement applied under laboratory condition. The conclusion is valid.]

(*b*) The place of a planet at a given time is calculated by the law of gravitation ; if it is half a second wrong, the fault is in the instrument, the observer, the clock or the law. Now, the more observations are made, the more of this fault is brought home to the instrument, the observer and the clock.

[The direct deductive method—The conclusion arrived at by deduction from the laws that were supposed to determine the position of the planet of a given time is put to test by induction and it is found that induction does not corroborate deduction. So an enquiry is made into the grounds of error and it is found that the fault of the instrument, the observer and the clock is responsible for this discrepancy.]

(*c*) A community consists of individuals, and as individual consists of cells. Both of them are complex structures. The individual grows, and so does the community. The community thrives when members co-operate with one another and the health of the individual is dependent on the organs working in harmony. There being such close resemblance between the individual and the community, it is obvious that the community, like the individual, is mortal.

[Bad analogy.]

(*d*) During the great war trade was very brisk and all businessmen made large profits. Since the end of the war, there has been depression of trade and businessmen have suffered heavy losses. It is clear that peace is unfavourable to commercial prosperity.

[Joint Method—while employing the method it has not been observed that during peace some industry thrives and during war some industries suffer heavy loss. So the conclusion is fallacious and implies non-ovserbation.]

(*e*) It has been observed that as education spreads, the number of crimes becomes less. People, therefore, advocate universal compulsory education on the ground that thus crime will disappear. But yet we see that

criminals belong to the educated classes, and that the majority of uneducated persons are not criminals.

See exercise—1944—Ex. (e)

(*f*) Tyndall found that of twenty-seven sterilized flasks containing infusion of organic matter, and opened in pure Alpine air, not one showed putrefaction ; while of twenty-three similar fiasks, opened in a hay loft, only two remained free from putrefaction after three days. He concluded that putrefaction is due to floating particles in the air.

[Method of Difference—valid.]

C. U. 1940

(*a*) The proportion of inmates in our lunatic asylums who can read and write is very high, from which we may infer education is one of the causes of insanity.

[The conclusion is due to our not observing that there are educated men who are not insane and that there are insane persons who are not educated.]

(*b*) The great famine in Ireland began in 1845 and reached its climax in 1848. During this time agrarian crime increased very rapidly until in 1848 it was more than three times as great as in 1845. After this time it decreased with the return of better crops until in 1851 it was only 50% more than what it was in 1845. It is evident from this that a close relation of cause and effect exists between famine and agrarian crime.

[Joint method and the method of concomitant variation. Highly probable.]

(*c*) The armament firms thrive on war, the glazers gain by broken windows, the operating surgeons depend on cancer for their childrren's bread. Therefore, fortune is everywhere made out of destruction, waste and disease.

[Method of agreement—Implies the non observation of those cases where fortune is not made out of destruction, waste and disease.]

(*d*) The nearer all bodies approach the earth, the greater is the velocity with which they approach it, but the further they are, the less is the force with which they tend to approach it. We may therefore infer that their greater (or less) nearness to the earth is the cause of the increased (or diminished) velocity.

[Methods of Concomitant Variation.]

(*e*) The metropolis of a country is similar in many respects to the heart of the animal body : therefore the increased size of the metropolis is a disease thereof.

[Bad analogy.]

(*f*) The principal cause of awakening in the country is the spread of Western ideas through the Western system of education. The awakening is more conspicuous in British India than it is in the Native States ; and in British India itself Provinces that have imbibed the spirit of Western education more and more progressive than others. Looking to the development of Political consciousness in the country as a whole, we find it has run parallel to the development of Western education.

[Method of Concomitant Variation and joint method.]

M. U. March. 1940

(*a*) Brute force never succeeded in the past and it will not succeed in the future.

[Induction by simple enumeration.]

(*b*) Cocoanut trees flourish in places which are not far removed from the sea.

[Induction by simple enumeration.]

(*c*) Famines and wars this year are evidently due to the wrath of Gods. If they are appeased, they will come to an end.

[The conclusion depends upon the assumption that famines and wars are due to the wrath of Gods and this assumption is taken to be self-evident which, it certainly is not. So it involves the fallacy of *petitio principii*.]

M. U. September, 1940

(*a*) At present no body cares for the normal and spiritual values of life.

[Here it is not observed that even at the present day some persons care for normal and spiritual value of life. The conclusion is arrived at by means of induction by simple enumeration i.e. by an observation of those persons only who do not care for such values of life.]

(*b*) Day is the cause of night.

[Co-effects—common cause are regarded as standing in the direct relation of cause and effect. A mere invariable antecedent is taken to be the cause.]

(*c*) I should not do this action because it is wrong.

[It is assumed to be self-evident that this action is wrong. But in fact this may not be the case. So it is a case of *petitio principii.*]

M. U. Septembe 1942

(*a*) Jupiter gives out more light than it receives from the Sun. What is the obvious conclusion and by what method is it reached ?

[Method of Residues—The conclusion being Jupiter has its own sources of light.]

(*b*) War is a blessing, not an evil. Show me a nation which has ever become great without waging wars.

[Induction by simple enumeration—Fallacy of illicit generalisation—implies the non-observation of those nations who have not become great by waging wars.]

(*c*) Over-driven cattle if killed before recovery from their fatigue become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death, cocks killed in fight, and

soldiers slain in the field of battle. Therefore it is concluded that severe exhaustion prior to death is the cause of rapid putrefaction.

[Method of agreement.]

(*d*) What inductive fallacy may David be said to have committed, when he said in his haste that all men are liars ?

[David Committed the fallay of hasty generalisation.]

(*e*) I am sure to pass my B.A. degree examination in the First class, because my brother passed his B.A. in the First class.

[Bad analogy.]

M. U. March 1943

(*a*) When a coin and a feather are dropped simultaneously in the receiver of an air-pump, the air being let in, the feather flutters to the bottom after the coin ; but when the air is pumped out of the receiver, the coin and the feather being dropped at the same instant reach the bottom of the receiver together. What is the apparent conclusIon and by what method is it established ?

[Method of Difference proving that air is the cause of the fluttering of the feather, so that the acceleration due to gravity is really uniform.]

(*b*) Intermittent fever is found only in places where there are marshes even though they differ in every other respect. Therefore, marshes are the cause intermittent fever.

[Method of agreement.]

(*c*) England, which is governed by the British is a wealthy and prosperous country : there is every reason to think that India which is also governed by the British should be prosperous too.

[The conclusion is due to the failure to recognise the distinction between governing in the interest of the governed and the

governing in the interest of those who govern. So there is a fallacy of accident. Arguing from one special case to another, it can be characterised as the case of bad analogy. For, the conclusion is drawn on the basis of the resemblance between India and England (being ruled by the British.)]

(*d*) Poverty must be the cause of increase of population; for we find that all poor countries are thickly populated, while those that are rich have a scanty population. Further what is true of countries is also true of individuals. It is usually the poor who have big families.

[The Joint Method—A remote condition is taken to be the cause—Poverty—absence of other forms of enjoying leisure—big population.]

(*e*) The naturalist gets his bone or his tooth and from it he can build up limb upon limb, muscle upon muscle, organ upon organ the unknown animal of which these things were parts.

[The naturalist can do this only because an organism displays a very intimate kind of unity. And he does this on the analogy of the bone or tooth of the unknown animal he gets with that of known animals.]

M. U. September 1943

(*a*) When Beggers die, there are no comets seen; the heavens themselves blaze forth the death of princes. There is therefore a necessary connection between the appearance of a comet and the death of a prince.

[*Post hoc ergo propter hoc.*]

(*b*) A certain school had 150 pupils on its roll. One morning the attendance suddenly fell to 50. The average attendance was 130, the 20 absentees being accounted for by slight ailments, engagements, and indifference. There was no festival nor epidemic in the neighbourhood to account for the sudden drop. There had been, however, a big political meeting, so the Headmaster concluded that this accounted for the unusual number of absentees.

[The Head master came to conclusion by eliminating other possible causes and by applying the method of difference not under laboratory condition.]

(*c*) A community consists of individuals and the individual consists of cells. Both are complex structures. The individual grows and so does the community. The community thrives when its members co-operate; so also the health of the individual depends upon the harmonious working of his organs. The individual is mortal; therefore the community is mortal.

[Bad analogy.]

(*d*) Having constantly observed the full moon in a clear sky, I assent that the weather is always fine when moon is full.

[Induction by simple enumeration.]

(*e*) We observe very frequently that very poor hand-writing characterises the manuscripts of able men, while the best hand-writing is as frequent with those whose penmanship is poor. We may, therefore, infer that poor penmanship is caused by the influence of severe mental labour.

[Joint Method—The conclusion implies the non-observation of those individuals who are able and yet have a good hand-writing.]

M. U. March 1944

(*a*) I have gone carefully through the list of members and I find that they are all Hindus.

[Perfect Induction.]

(*b*) 'Fortune favours fools.'

[The conclusion is owing to the non-observation of those cases where fortune does not favour fools and where fortune favours the not-fools.]

(*c*) D. Brewster proved that the colours seen upon mother-of-pearl are not caused by the substance but by

the form of the substance. He took impressions of mother-of-pearl in work and found that though the substance was entirely different, the colours were exactly the same.

(*d*) The planet Mars resembles the earth in possessing atmosphere, water and moderate temperature and we may therefore suppose it to be inhabited.

[Good analogy.]

(*e*) Lord Curzon argued, 'The hereditary principle is established in every branch and aspect of our National life. We have hereditary bankers, lawyers, and even hereditary cotton spinners. Why should it be a blot and an offence when applied to the House of Lords' ?

[Bad analogy.]

M. U. September. 1944

(*a*) The town must be unhealthy. I know three people who live there and not one of them is in good health.

[Induction by simple enumeration—hasty generalisation.]

(*b*) We should be guided by the decisions of our ancestors for old age is wiser than youth.

(*c*) A butter cup, a blade of grass, a fern, a moss, contain green colouring matter, I infer that all members in the vegetable kingdom contain green colouring matter.

[Induction by simple enumeration.]

(*d*) Both mosquitoes and malarial fever have in certain parts of West Africa, India, Malaya and elsewhere become much rare since these districts have been drained. Is malarial fever the effect of the presence of mosquitoes ?

M. U. March 1945

(*a*) We hear every day about the atrocities committed by the German soldiers. It is evident that all Germans are cruel.

[Induction by simple enumeration.]

(*b*) The proportion of inmates in our asylums who can read and write is very high, from which we may infer that education is among the cause of insanity.

(*c*) Whenever I take tea in the evening I don't sleep, I find I am able to sleep well if I avoid taking tea. Obviously tea is the cause of my sleeplessness.

[Joint Method.]

(*d*) Scarcity of food grains in the country is due to the lack of facilities of transport; for we find that scarcity of food increases when difficulties of transport increase.

[Concomitant variation—condition taken to be the cause—bad analogy.]

M. U. September, 1945

(*a*) Men live and die. Likewise every nation that lives must die.

[Bad analogy.]

(*b*) The whole of this street was inoculated. Yet some of its residents have died of cholera. Therefore, inoculation is no safeguard against the disease.

(*c*) The comet appeared and the king passed away, Heavenly bodies determine our fortunes.

[Post hoc ergo propter hoc.]

(*d*) Judge—You are the criminal because five peoples have deposed that they have seen you stealing.

Accused—I can produce ten witnesses, my Lord, who will swear they have not seen me stealing.

[In appropriate non-ovservation of event is not a proof of the non-existence of the event.]

(*e*) Recently there have been several cases ef typhoid in the city. On investigation it was found that all of them were being supplied milk from the same dairy.

[Perfect Induction.]

M. U. March 1946

(*a*) Dreams foretell future events for whenever I dream that I have crossed a river I pass the examination.

[Induction by simple enumeration.]

(*b*) There can never be equality among men. Look at the five fingers !

[Bad analogy.]

(*c*) Countries that are industrialised like England and America increase in power. Therefore increase in industrialisation lead to increase in power.

[Method of agreement—a condition taken to be the cause or co-effects of the same cause regarded as cause and effect. It is not observed that they cause each other.]

(*d*) Whenever a cat crosses my path, I fail in my undertaking. Obviously the crossing of the path by the cat is the cause of my failure.

[Post hoc ergo propter hoc.]

(*e*) I see the sun rise and set every day. I therefore conclude that the sun goes round the earth.

[Mal-observation.]

M. U. September 1946

(*a*) All religions lead to God, for do not all rivers fall into the sea ?

[Bad analogy.]

(*b*) Whenever I use a talisman my child recovers from its illness. Talismans cure diseases.

[Post hoc ergo propter hoc.]

(*c*) The short men I have known are unreliable. Beware of short men.

[Hasty generalisation.]

(*d*) The bigger the city the greater the number of crimes. The bigger the city the greater the number of cinema theatres. Cinemas are the cause of crimes.

[Co-existent phenomena are mistaken for cause and effect.]

(*e*) The boys who take milk are healthier than those who do not take it. Milk is therefore a nourishing diet.

[Joint Method.]

M. U. March 1947

(*a*) All ambitious men I have come across are selfish. Therefore I am entitled to infer that ambition leads to selfishness.

(*b*) The periodicity of trade-cycles tallies with that of the activity of Sun-spots. Sun-spot activity is therefore among the causes for fluctuations of trade.

[An invariable antecedent is taken to be a cause.]

(*c*) Since the capital town of a Province is its heart, the enormous growth of the Capital must be considered a symptom of social disease.

[Bad analogy.]

(*d*) 'He talks with Angels.'
'How do you know that' ?
'He himself admits it'.
'But, suppose he lies' ?
'What ! A man who talks with angels to be capable of lies ! Impossible.'

[Arguing in a circle. That he talks with Angels is proved by the fact that he says so, the truth of which again proved by the fact that he talks with Angels.]

(*e*) Whom the gods love die young. Ramu, who died very young, must have been beloved of the gods.

[The conclusion is deduced from a proposition which pretends to be self-evident.]

(*f*) The Sun and Moon at their rise are big, and when they reach mid-heavens they are small. The diminution in size must therefore be due to their contraction, as they travel in the sky.

[Bad analogy.]

M. U. September 1947

(*a*) Prohibition failed in America ; therefore, it is bound to fail in India.

[Bad analogy.]

(*b*) Some graduates from Indian Universities are found to be impractical and therefore unemployable. University education in India must therefore be deemed worthless.

[Hasty generalisation.]

(*c*) The proportion of inmates in our Mental Hospitals who can read and write is very high. From which we have to infer that education is among the causes of insanity.

(*d*) 'Every time I receive a telegram some near relation of mine is dead. Telegrams are therefore ominous things. I advise all my near relations not to send me telegrams.'

[Hasty generalisation.]

(*e*) The World War II is attended with a phenomenal progress in science and revolutions in our political ideas. There can be no progress without wars.

[Induction by simple enumeration—The conclusion implies non-observation of the fact that there can be progressive science and revolutions in our political ideas even when there was no world war.]

(*f*) 'There is a destiny shapes our ends,
Rough hew them how we will'.
That is why Germany lost the War.

[The proposition that destiny shapes our ends is taken to be self-evident, but this is not admitted.]

M. U. September 1948

(*a*) He must be an excellent man, for he talks so impressively.

[Undue assumption.]

(*b*) Women as a class have not hitherto been equal to men. Feminine inferiority therefore is an unalterable fact of life.

[Induction by simple enumeration—Hasty generalisation.]

(*c*) The sun and moon at their rise are big and when they reach mid-heavens they are small. This diminution must be due to their contraction in size as they travel up the sky.

(*d*) They are advocating a Law of Divorce for Hindu women. What would our ancestors say to this ? How does such a measure tally with our institutions ? Are we to put the wisdom of yesterday in competition with the wisdom of centuries ? (cries of Hear ! Hear ! !) Is beardless youth to show no respect for the decisions of the mature age ? (Lord cries of Hear ! Hear ! !) If this measure be right would it have been reserved for this modern and degenerate times ?

[*Argumentum ad populum.*]

(*e*) The inner world of mind attains the light of knowledge through seven organs of sense. Therefore, some mediaeval astronomers said, there must be seven planetary bodies to illumine the outer world of nature.

[Bad analogy.]

(*f*) Able men generally have very bad handwriting while good handwriting is frequently found in men doing comparatively little mental work. Hence, it is to be inferred, that mental strain is the cause of poor penmanship.

M. U. March 1949

(*a*) The Asian countries do not deserve independence ; see what has happend in Burma and Indonesia.

[Bad analogy.]

(*b*) He must be an excellent man : for, I have been favourably impressed with his manner of talking.

(*c*) He who deliberately kills another should suffer death. A soldier who deliberately kills the enemy should therefore suffer death.

[Fallacy of Accident.]

(*d*) India owes its independence to the study of English ; for all the Indian political leaders know the language.

[Method of agreement—a mere accidental accompaniment is taken to be the cause.]

(*e*) Really able scholars write a bad hand. Many who do little mental work possess good penmanship. We may infer that poor penmanship is caused by great scholarliness.

(*f*) Did you see the comet that appears in the Southern Sky ? It must be a portent of a third World War !

C. U. 1950

(*a*) Iron balls have been found to break glass panes when they strike against the latter ; therefore, 'being made of iron' is the cause of breakage of glass panes.

(*b*) This man, shot through the heart, drops down dead; therefore, shooting through the heart causes death in all cases.

(*c*) The weight of the load is the total weight less the weight of the cart.

(*d*) Cocoanut trees best flourish in places near the sea.

(*e*) The temperature rises in the highest degree when the sun is at midsky; therefore, the sun is the cause of the earth's heat.

(*f*) The malarial fever stopped with the adminis-

tration of quinine for two days ; therefore quinine cures malaria.

(*g*) Telegrams are ominous, for they bring the news of the death of some friend or relative.

C. U. 1951

(*a*) The number of students in Logic classes has considerably fallen. This must be due to some defect in the Science of Logic itself.

(*b*) Congestion in Calcutta can be appreciably removed by starting new colleges outside the city. For students reading in Calcutta will naturally drift there.

(c) The only factor disturbing the world peace has of late been the Korean War. So if the Korean question be settled there will be world peace again.

(*d*) The nearer a body approaches the earth the greater is its downward motion, and the farther it is from the earth the slower is that motion. Therefore the intensity of downward motion of a falling body depends on its distance from the earth.

(*e*) Machines work smoothly and successfully. In order, therefore, that our actions also be smooth and successful we should submit to complete mechanisation of our mind.

(*f*) The future must resemble the past, for does not Nature behave uniformly ?

C. U. 1952

(*a*) Extra-kind examination does not make a subject of study popular. The examination in Logic, it is said, was extra-kind for many years, and yet the number of Logic students had been steadily declining : and last year, it is said, the examination was not extra-kind, and yet there has been no marked fall in the number.

(*b*) Our country must have been prospering all these years. For are we not having big development plans like those in the U.S.S.R. and the U.S.A. ?

(*c*) The more we are eating bread the poorer in health we are becoming. Bread must, therefore, be an unhealthy food.

(*d*) So many people eat bread, and they are all in good health. Bread must, therefore, be a healthy food.

(*e*) Many merchants amassed wealth when the World War had been going on. This must be due to a good star.

(*f*) The old Oriental culture must be bad for modern man has discarded it.

APPENDIX—A

THE INDUCTIVE SYLLOGISM

In the history of Logic Induction has often been stated in the language of Deduction. So it has been often ignored that Induction and Deduction have really different forms. An extreme form of this endeavour has been to interpret Induction as not only expressible in the language of Deduction but identical with Deduction. Without considering this extreme position, we shall for the present be concerned with the study of the different possible ways of stating Induction in the language of Deduction.

The syllogism is ordinarily taken to be deductive in character. But it has sometimes been censiderd as a possible form of inductive argument also.

Aristotle

Of such attempts to treat induction syllogistically, the Inductive syllogism as formulated by Aristotle deserves our attention. The Inductive syllogism of Aristotle is a syllogism in the Third Figure. For example,

Man, the horse and the mule are long-lived ;
Man, the horse and the mule are all the gall-less animals ;
∴ All gall-less animals are long-lived.

Aristotle describes this form of argument as *proving the major term of the middle by means of the minor.*

The expressions, 'Major', 'Minor' and 'Middle' in Aristotle's terminology differ in meaning from such expressions as are used in the current terminology. In the current terminoloy, by the major term we mean the term which is used as the predicate of the conclusion. Similarly the minor term is the term which is used as the subject of the conclusion, and the middle term is what occurs in both the premises but does not occur in the conclusion.

Explanation

In Aristotle, however, the major term is the term which possesses the largest measure of denotation. The minor term would then be the term of the smallest measure of denotation and the middle will lie in between. In the given syllogism the denotation of the term 'long-lived' is he greatest and the term is therefore called the major term. The term 'man, the horse and the mule' has the smallest measure of denotation and is therefore the minor term of the argument. In the second premise we find no doubt a sort of identity between 'man, the horse and the mule' on the one hand and the term 'gall-less' on the other. But in spite of this identity the predicate of this premise viz., 'gall-less' has the possibility of being more extensive in denotation than the term 'man, the horse and the mule', the term 'gall-less' is therefore the middle term while the term 'man, the horse and the mule' is the minor term. In the conclusion we prove the major term (long-lived) of the middle term (gall-less) by means of the minor term (man, the horse and the mule). This is why Aristotle describes the given syllogism as proving the major term of the middle by means of the minor.

CHARACTERISTICS OF THE INDUCTIVE SYLLOGISM AS FORMULATED BY ARISTOTLE.

To Aristotle Induction must proceed by complete enumeration. But his Induction by complete enumeration is by no means identical with the modern form of Perfect Induction. In the modern form of perfect induction the particulars so enumerated are individuals while in Aristotle the species coming under a genus are enumerated. Secondly, the conclusion of a modern induction by complete enumeration is a collective proposition i.e., a summation of particular propositions. In Aristotle, however, we are supposed to get a genuine general proposition in the conclusion.

When I know that each one of the mangoes in a basket is sweet, I can sum up particular experiences by concluding that all the mangoes in the basket are sweet. This is

Explanation

the modern form of induction per complete enumeration. Here we enumerate bare instances while in Aristotle we enumerate the species like man, the horse and the mule. The conclusion in Aristotle viz., all gall-less animals are long-lived, is a genuine general proposition. But the so-called conclusion of a modern induction by complete enumeration gives only a collective judgment viz., all the mangoes in the basket are sweet. In a genuine general proposition the subject term represents an *open class* while that in a collective judgment represents a *closed class.*

Aristotle has often been accused of having distorted the nature of Induction. The Inductive syllogism has to rest on its premises which are general propositions. So if we recognise no other mode of establishing general propositions, his logic falls to pieces. But it is not true to say that he did not recognise any other mode of establishing general propositions. For 'he gives the name of Induction to the processes in which the particulars of our experience suggest to us the principles they exemplify.' (*P*—391 *An Introduction to Logic—2nd edition—Joseph.*) It is only when such principles are formulated that the Inductive Syllogism becomes possible. The formulation of principles in this way is further not a formal logical process from premises to conclusion.

What then is the nature of the Inductive Syllogism ?

Since each one of the premises of an Inductive Syllogism is a general proposition, the Inductive Syllogism is by no means an independent process. The Inductive Syllogism fails therefore to explain the genesis of general propositions. It is the cogency of the syllogistic form that is utilised in the Inductive Syllogism. But no way can we say that a syllogism is by itself responsible for establishing general propositions. Aristotle has clearly recognised this and we are of opinion that the inductive Syllogism is to be interpreted in the light of this truth.

FURTHER ATTEMPTS TO INTERPRET INDUCTION SYLLOGISTICALLY.

In the history of modern philosophy there have been *further attempts to interpret Induction as a form of syllogism.* This way of

interpreting Induction is carried through a form of Inductive Syllogism of which the major premise is the uniformity of nature or some other principle of Induction. In our opinion this way of interpreting Induction is liable to be criticised on a number of grounds :

(1) If some principle of Induction is taken up as the major premise, that major premise must have to be further proved. For, all premises in Induction requires proof. But since the Inductive principle will in that case require proof, the Inductive Syllogism concerned is by no means a self sufficient process.

(2) What is more important for us to consider is further that there is a fundamental distinction between a principle and a premise. We infer *according to* a principle but *from* premises. So the Inductive Syllogism in question fails to distinguish between a principle and a premise.

(3) The very principle of Induction states that the premises are by themselves sufficient for the tenability of the conclusion. But if the principle is further to be added to the list of the premises, the principle will then contradict itself. For in that case the premises of the Inductive Syllogism are not considered sufficient for the tenability of the conclusion.

So one great lesson we have is that the Inductive Syllogism is neither an ordinary syllogism nor a syllogistic substitute for Induction. It is only the cogency of the syllogistic form that has been utilised for tracing the different constituents of the Inductive process. The Inductive Syllogism cannot therefore usurp the role of Induction a distinct and non-formal mode of reasoning.

APPENDIX—B

EVIDENCE OF THE GROUND OF INDUCTION : THE SO-CALLED PARADOX OF INDUCTION

According to Mill the very definition of Induction presupposes an assumption with regard to the course of nature and the order of the universe viz., that there are uniformities in nature. The inductive methods also assume the universality of the law of causation. So it can be said that the proposition that the course of nature is uniform or that the law of causation is universal is the fundamental principle or general axiom of Induction. But still "it would be a great error to offer this large generalisation as any explanation of the Inductive process." For this law is neither self-evident nor one that we *must* accept. Some metaphysicians undoubtedly hold that the universality of causation is a truth which we cannot help believing and the proof of this principle according to them is that everybody does believe it. But "belief is not proof and does not dispense with the necessity of proof." Moreover, it is not true that everybody believes or cannot help believing it. For "the Greek philosophers, not even excepting Aristotle, recognised chance and spontaneity as among the agents in nature." The belief we entertain in the uniformity of nature and the universality of causation "is itself an instance of induction and by no means one of the earliest which any of us or which mankind in general can have made." "Far from being the first induction we make, it is one of the last or at all events one of those which are latest in attaining strict philosophical accuracy. As a general maxim, indeed, it has scarcely entered into the minds of any but philosophers ; not even by them has its extents and limits been always very justly conceived." So, it appears that this great generalisation is itself founded upon prior generalisations. We could never

have had entertained the notion of universal causation or thought of affirmiug that all phenomena take place according to general laws unless many cases of causation or partial uniformities had previously become familiar. "The more obvious of the particular uniformities suggest and give evidence of, the general uniformity, and the general uniformity, once established enables us to prove the remainder of the particular uniformities of which it is made up. That is "the obscurer laws of nature were discovered by means of it, but the more obvious ones must have been understood and assented to as general truths before it was ever heard of." Now, as rigorous processes of induction assume the general uniformity, the knowledge of the particular uniformities from which it was first inferred must have been due to Induction by simple Enumeration. And so the principle of uniformity of nature or the law of universal causation cannot itself rest on any better foundation.

This position of Mill with regard to the cvidence of the formal ground of induction has often been severely criticised. Thus, it has been urged against him that such a position is inconsistent. It has also been argued that it lands us in a paradox.

It is held to be inconsistent with Mill's theories concerning the nature of scientific induction and induction by simple enumeration. Scientific induction according to Mill is a rigorous process while induction by simple eaumeration is a loose and uncertain process. But, as the former is indebted to the latter, it is inconsistent to contrast the one with the other in this way. Indeed, it is asked, how can induction by simple enumeration which can yield probable conclusions only be at the basis of scientific induction, that yields certain conclusions?

Again it is held that Mill's view lands us in a paradox and involves a vicious circle. The uniformity of nature is the ground of induction. To say that it is grounded upon induction is to involve a vicious circle. For then, induction depends upon the principle of uniformity of nature and the principle of uniformity of nature depends upon induction.

Now, to be fair, Mill himself anticipated such objections. As against the first objection he pointed out that "the inconsistency is only apparent." For induction by simple enumeration is not an invalid process. It is merely a fallible process, "and fallible in different degrees." And so if "we can substitute for the more fallible forms of the process an operation grounded on the same process in a less fallible form, we shall have effected a very material improvement. And this is what scientific induction does." Indeed "the precariousness of the method of simple enumeration is in an inverse ratio to the largeness of the generalisation. The process is delusive and insufficient, exactly in proportion as the subject matter of the observation is special and limited. As the sphere widens, this unscientific method becomes less and less liable to mislead; and the most universal class of truths, the law of causation, for instance, and the principles of number and geometry, are duly and satisfactorily proved by that method alone."

As against the second objection he held that there is a paradox here provided that in a syllogism the conclusion is drawn from, and not according to the universal major premise. In other words, the principle of uniformity of nature or the law of universal conclusion warrants induction "in the only sense in which the general propositions which we place at the head of our reasonings when we throw them into syllogisms ever really contribute to their validity." "The major premise is not the proof of the conclusion but is proved along with the conclusion from the same evidence." Similarly "any new fact of causation inferred by induction is rightly inferred, if no other objection can be made to the general truth that every event has a cause."

EXERCISES WITH HINTS

1. *What is the paradox of Induction ? Is there really a paradox ?*
2. *What is the proof of the formal ground of Induction ?*
   [See Appendix B and also ch. XIII sec. 8.]